ZERO THICKNESS INTERFACE ELEMENTS—NUMERICAL STABILITY AND APPLICATION

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SUMMARY

Many methods have been proposed to model joints in rocks or the interface between soil and a structure. Many analysts have reported numerical problems when using zero thickness interface elements while others have presented satisfactory results without comment of such difficulties. The numerical behaviour of zero thickness interface elements is further investigated in this paper. Some simple examples illustrate the application of interface elements to practical situations and highlight the numerical difficulties that may be encountered. Both ill-conditioning of the stiffness matrix and high stress gradients were found to cause numerical instability. Ill-conditioning can be reduced by careful selection of the size of the 2D elements adjacent to the interface. The problem of steep stress gradients is entirely one of inadequate mesh design. Contrary to other reports, this paper shows that the Newton–Cotes integration scheme has no benefit over Gaussian integration.

Analyses of a retaining wall using interface elements confirm the analytical values of active and passive earth pressure coefficients which are commonly used in analysis and design of retaining walls.

1. INTRODUCTION

In any soil–structure interaction situation relative movement of the structure with respect to the soil can occur. The use of continuum 2D elements with compatibility in a finite element analysis of these situations prohibits relative movement at the soil–structure interface. Nodal compatibility of the finite element method constrains the adjacent structural and soil elements to move together. Interface or joint elements can be used to model the soil–structure boundary such as the sides of a wall or pile, or the underside of a footing. Particular advantages being the ability to vary the constitutive behaviour of the soil–structure interface (e.g. the maximum wall friction angle) and to allow differential movement of the soil and the structure, i.e. slip and separation.

1.1. Interface models

Many methods have been proposed to model discontinuous behaviour at the soil–structure interface. These include:

1. Use of thin 2D finite elements with standard constitutive laws
2. Linkage elements in which only the connections between opposite nodes are considered

Usually opposite nodes are connected by discrete springs.
(3) Special interface or joint elements of either zero or finite thickness.\(^5\)\(^--\)\(^10\)

(4) Hybrid methods where the soil and structure are modelled separately and linked through constraint equations to maintain compatibility of force and displacement at the interface.\(^11\)\(^--\)\(^14\)

Wilson\(^10\) has demonstrated that ill-conditioning of the interface element stiffness matrix due to large off-diagonal terms causes loss of accuracy. Ghaboussi \textit{et al.}\(^9\) and Wilson\(^10\) therefore advocate the use of relative displacement as independent degrees of freedom for interface elements. The use of this relative degree-of-freedom formulation requires modification of the adjacent 2D elements only on one side of the interface, so that they use the same relative degrees-of-freedom. Thus, incorporation into a finite element program is complex. This formulation also causes problems at joint intersections.\(^8\)

The benefit of the relative degree-of-freedom formulation is uncertain. Pande and Sharma\(^2\) compared a series of analyses using the relative degree-of-freedom formulation with similar analyses using the absolute degree-of-freedom formulation. They found that little ill-conditioning is experienced with the use of very thin 2D elements. The additional effort involved in the use of the relative displacement element may not be worthwhile.

Griffiths\(^1\) recommends the use of special interface elements instead of thin standard 2D elements for more accurate modelling of the interface when slippage is of prime concern.

Desai \textit{et al.}\(^6\) propose a thin 2D joint/interface element called a thin-layer element for soil–structure interaction. A special constitutive law is used to model the principle deformation modes of shear and opening. They report that with the use of zero thickness elements it is often difficult to obtain constant and stable stress in the interface elements themselves, and therefore the stress in the adjacent 2D elements is often adopted rather than the interface element stress itself. It is argued that the thin-layer element is computationally more reliable than the zero thickness element. The choice of element thickness is however important, and can effect the behaviour of the interface given the same constitutive parameters.\(^6\) An extensive parametric study has recently resulted in guidelines and empirical criteria for the determination of element thickness for the thin-layer element.\(^15\)

Zero thickness elements however have been used by many others\(^5,7,16--19\)\ without reporting similar problems. The element used has been based on that proposed by Goodman \textit{et al.}\(^9\) The essential modifications to the original element of Goodman \textit{et al.} are the extension to four and six node isoparametric formulations to make the interface element compatible with isoparametric quadrilateral 2D elements.

\subsection*{1.2. Constitutive laws}

The development of interface or joint elements has been motivated by the desire to model the behaviour of joints in fractured rock. The proposed constitutive laws are therefore strongly oriented towards describing this behaviour.

Non-linear elastic hyperbolic relationships are generally used to describe the shear stress–strain behaviour.\(^5,6,16,19\) Similar relationships have been used to describe the normal stress–strain behaviour in rock joints.\(^5,6,17\) For soil–structure interaction, linear elastic normal stress–strain behaviour has often been considered sufficient.\(^7,18\)

The yield behaviour of interface elements has not generally been formulated in terms of plasticity theory. In these simple formulations the Mohr–Coulomb strength criterion is used to define the maximum permissible shear stress. When the shear stress reaches this limit, the value of the shear stiffness is set to zero. The normal stiffness remains unchanged. Tensile stress in the
interface element is generally not permitted. If tensile stress occurs both the normal and shear stiffness are set to zero and the tensile stress is redistributed by the solution algorithm.

In more complex models dilation has been included. These models are based on the sawtooth model for rock joint behaviour. The models include the effect of dilation during the pre-failure shearing phase of the joint.5,20

Models based on the theory of plasticity in which the dilation is controlled by a yield criterion and a plastic potential function have been proposed by Ghaboussi et al.,8 Desai et al.,5 Gens et al.21 A more complex plasticity model incorporating the disturbed-state concept for more realistic modelling of joints and interfaces is introduced by Desai and Ma.21 Their model allows for the elastic-plastic, dilation, roughness, and hardening and softening response of joints. Navayogarajah et al.22 present a model based on the Hierarchical Single-Surface (HISS) approach to describe the behaviour of interfaces under static and cyclic loadings, including strain softening.

1.3. Soil-structure and soil-reinforcement analysis

The zero thickness element formulation using a simple linear elastic perfectly plastic constitutive model appears to have been successfully used for retaining wall-soil problems and for soil-reinforcement interaction.7,17,18 This paper highlights and examines some of the numerical problems the authors have encountered when using zero thickness interface elements in finite element analysis. The linear elastic perfectly plastic model was adopted for the analyses presented in this paper.

2. INTERFACE ELEMENT FORMULATION USED IN THIS STUDY

The isoparametric interface element is described by Beer7 and Carol and Alonso.5 The element (Figure 1) with four or six nodes is fully compatible with four and eight-node isoparametric 2D elements. The interface stress is characterized by the normal and shear stresses. The normal stress, \( \sigma \), and the shear stress, \( \tau \), are related by the constitutive law to the normal and tangential interface element 'strains', \( \varepsilon \) and \( \gamma \).

\[
\begin{pmatrix}
\tau \\
\sigma
\end{pmatrix} = [D] \begin{pmatrix}
\gamma \\
\varepsilon
\end{pmatrix}
\] (1)

The interface element 'strain' is defined as the relative displacement of the top and bottom of the interface element, i.e.

\[
\gamma = \Delta u_i = u_i^{\text{top}} - u_i^{\text{bot}}
\] (2)

\[
\varepsilon = \Delta v_i = v_i^{\text{top}} - v_i^{\text{bot}}
\] (3)

where

\[
u_i = u \cos \alpha + v \sin \alpha
\] (4)

\[
v_i = -u \sin \alpha + v \cos \alpha
\]

and, \( u \) and \( v \) are the global displacements in the \( x \) and \( y \) directions, respectively. Hence,

\[
\gamma = (u^{\text{top}} - u^{\text{bot}}) \cos \alpha + (v^{\text{top}} - v^{\text{bot}}) \sin \alpha
\] (5)

\[
\varepsilon = -(u^{\text{top}} - u^{\text{bot}}) \sin \alpha + (v^{\text{top}} - v^{\text{bot}}) \cos \alpha
\] (6)
2.1. Constitutive laws

A linear elastic perfectly plastic model using a Mohr–Coulomb failure criterion as the yield surface is used in the analyses presented here (Figure 2). The formulation of the constitutive behaviour was based on plasticity theory. The elastic constitutive $[D]$ matrix is

$$[D] = \begin{bmatrix} K_s & 0 \\ 0 & K_n \end{bmatrix}$$

where $K_s$ and $K_n$ are the elastic shear stiffness and normal stiffness respectively (in units of $F/L^3$).

The failure criteria are formulated in terms of effective stress. Pore fluid may exist in the interface. Undrained behaviour is modelled by including the effective bulk modulus of the pore fluid in the stiffness matrix.
The Mohr–Coulomb failure criterion defines the yield function, $F$, and the gradient of the plastic potential function, $G$.

$$F = |\tau| + \sigma' \tan \phi' - c'$$

$$\frac{\partial G}{\partial \sigma'} = \tan \nu; \quad \frac{\partial G}{\partial \gamma} = \pm 1$$

where $\sigma$ is the effective normal stress, $\phi'$ is the maximum angle of shearing resistance, $c'$ is the cohesion (Figure 2) and $\nu$ is the dilation angle. Dilation of the interface is represented by a value of $\nu < 0$.

**Opening and closing**

If the interface moves such that the maximum normal tensile strength is exceeded ($c'/\tan \phi'$) the interface is allowed to subsequently open and close and the residual tensile stress is redistributed. When the interface is open the normal stress remains equal to $c'/\tan \phi'$ and shear stress remains equal to zero. This is essentially the same as setting both normal and shear stiffnesses to zero. The amount of opening of the interface is recorded. When the interface recloses and reforms contact, the constitutive model again defines the interface behaviour.

### 3. NUMERICAL BEHAVIOUR OF INTERFACE ELEMENT

Numerical problems such as ill-conditioning, poor convergence of solution and unstable integration point stresses have been experienced by the authors when using the zero thickness interface element. To investigate these problems three simple examples, crudely representing practical situations, have been analyzed to highlight and illustrate the behaviour of the interface element.

In the analyses results using full (three point) and reduced (2 point) Gaussian integration were compared. Results using the Newton–Cotes (3 point) integration scheme are also discussed. The interface is assumed fully drained with pore pressure equal to zero.

**Example 1** (simply supported split beam). This example demonstrates the ability of an interface element to model frictionless slip between 2D elements (e.g. rock joints). The beam consists of two rows of ten equal length 8-node 2D elements, separated by ten, 6-node interface elements (Figure 3). Only half of the beam was modelled because of symmetry. The problem is specified in dimensionless units. The Young's modulus and Poisson's ratio of the beam are, $E = 5.0 \times 10^4$ and $\mu = 0$. A uniform load of 10/unit length was applied to the top surface of the upper row of elements.

**Example 2** (simple pull-out test). In view of potential applications to soil reinforcement and pile analysis this example demonstrates the interface element in a problem controlled by the sliding mode of deformation. This problem consists of a thin strip of 2D elements, representing a reinforcing membrane, on a rigid base (Figure 4). The contact between the membrane and the base is modelled by interface elements. Five, 8-node 2D elements and five 6-node interface elements were used to model the test. Initially a uniformly distributed load of 10 kPa was applied to the top of the membrane. End A of the membrane was then moved horizontally by the amount $\delta$, in the direction shown under displacement control. This problem involves a sliding front that moves progressively from the point of load application towards the other end of the membrane. This problem is a simplified version of the pull-out test studied by Gens et al.17
Figure 3. Split beam–finite element mesh

Figure 4. Pull-out test–finite element mesh

Figure 5. Overturning of elastic block
Example 3. (overturning of an elastic block). This problem is crudely representative of a gravity retaining wall. It consists of an elastic block, Young's modulus $E = 10^6$ kN/m$^2$ and Poisson's ratio $\mu = 0$, separated from a rigid footing along the base A-B by interface elements (Figure 5). For clarity, the interface elements are shown artificially expanded in Figure 5. Ten, 6-node interface elements and fifty, 8-node 2D elements were used to model the problem. A downwards vertical stress (200 kN/m$^2$) and a shear stress in the positive $x$ direction were applied to the top boundary C-D. The interface was assumed to be non-dilatant ($v = 0$) with strength parameters $\phi' = 20^\circ$ and $c' = 0$.

Since tension in the interface is not permitted separation first occurs in the interface at point A when the applied horizontal stress is $16.66$ kN/m$^2$. As the shear stress is increased the zone of separation spreads towards B. Overturning instability of the block occurs when the applied shear stress is $50$ kN/m$^2$. Figure 6 illustrates the load-displacement behaviour of the point D at the top of the block. When all horizontal load is removed there is a net horizontal displacement of the block due to the plastic shear strain that occurs during loading and unloading.

3.1. Ill-conditioning

The analysis of the overturning block (Example 3) was used to investigate the reported problems of ill-conditioning.$^{10}$ A large number of elastic analyses were undertaken and in these the interface element stiffness was varied from $K_s$ and $K_n = 10^6$ to $K_s$ and $K_n = 10^{10}$ kN/m$^3$, with Young's modulus of the 2D elements, $E = 10^6$ /m$^2$ and from $K_s$ and $K_n = 10^3$ to $K_s$ and $K_n = 10^7$ kN/m$^3$ with $E = 10^3$ kN/m$^2$. Note that in general $K_s$ was not equal $K_n$. The shear stress applied to the top of the block was 50 kPa.

Ill-conditioning of the stiffness matrix caused fluctuating stress in the interface element when the interface element stiffness was large with respect to the stiffness of the 2D elements. The results of the analysis with $K_s = K_n = 10^9$ kN/m$^3$ and $E = 10^6$ kN/m$^2$ using 3 point Gaussian integration are shown in Figure 7 in which the integration point stresses are plotted. The effect of ill-conditioning was noticeable if either $K_s$ or $K_n$ was greater than about $100E$. (This is dependent on
the units used. The units used here are kN and m.) The use of reduced integration (2 point) does not prevent the fluctuation in interface stress which is observed in Figure 7 for full (3 point) integration.

The linear, normal and shear stress distributions shown in Figure 7 are the theoretical distributions for a rigid block on elastic springs. Since the vertical faces of the block are stress free, the shear stress within the base of the block must reduce to zero at the corners (A and B), however the shear stress applied by the interface element to the base does not. To satisfy continuum theory a singularity therefore occurs in the shear stress and vertical stress distributions at the corners within the block. The theoretical shear stress distribution is parabolic for a long flexible block on a rigid foundation. The shear stress distribution in the interface elements lies between the two extremes.

An analysis with a thin layer of 2D elements in place of the interface elements was also performed. The elastic properties of the thin layer are $E_i$ and $\mu_i$ and the thickness is $t$. The
equivalent normal and shear stiffnesses of the thin layer are

\[ K_{in} = \frac{E_i}{t(1 - \mu_i^2)} , \quad K_{is} = \frac{E_i}{2t(1 + \mu_i)} \]

It is noted that when \( K_{in} = K_{is} , \quad (1 - \mu_i^2) = 2(1 + \mu_i) \). The solution of this equation gives \( \mu_i = -1 \), in which case the stiffnesses are infinite. To avoid this paradox and to be consistent with the overlying elements the Poisson’s ratio was chosen to be zero (\( \mu_i = 0 \)). With the thickness chosen to be equal to 0.02 m and \( E_i = 2 \times 10^7 \) the normal stiffness of the thin elements is equivalent to that of the interface element and the shear stiffness is half that of the interface elements. Fluctuations also occurred in the 2D element stresses. The integration point stress along the centre line of the 2D elements is also shown in Figure 7.

Reducing the size of the 2D elements adjacent to the interface elements reduced the effect of ill-conditioning even though the total number of elements remained the same. Figure 8 shows the results of the analysis when different size 2D elements are used adjacent to the interface element.

![Figure 8. Reduced ill-conditioning with smaller elements (full integration)](image-url)
(see insert on Figures 7 and 8). In this case the contributions to the global stiffness matrix from the 2D elements is increased in the locations concerning the interface element degrees of freedom. The contributions from the interface elements remains unchanged. Thus there is less difference between the stiffness of the interface elements and the stiffness of the 2D elements and hence less ill-conditioning.

3.2. Convergence of the non-linear solution algorithm

For analyses in which the interface was elasto-plastic, a modified Newton-Raphson algorithm incorporating a sub-stepping stress point algorithm with error control has been used to solve the finite element equations. It is not uncommon to use a constant elastic stiffness matrix for all iterations with this approach. The benefit being that the global (elasto-plastic) stiffness matrix does not have to be recalculated and inverted at every stage of the analysis. This method works well in cases where the changes in the elasto-plastic stiffness are not large. Where the behaviour of a problem is highly non-linear the use of the constant elastic stiffness matrix can result in large numbers of iterations and convergence problems.

During analyses of the overturning block (Example 3), convergence of the solution become slower on successive increments. In this problem the global stiffness is rapidly changing as successive interface elements open (Figure 6). In an attempt to increase the speed of convergence, the elastic global stiffness matrix used in the solution algorithm was replaced by the tangent elasto-plastic stiffness matrix \( D_{ep} \) calculated the beginning of each increment. This dramatically increased the speed of convergence up to a shear stress of 35 kPa, but caused divergence at later stages. The solutions for Example 3 were obtained by using the \( D_{ep} \) matrix calculated at a load level of 30 kPa for the remaining loading increments (35, 40 and 45 kPa) and first two unloading increments (40 and 35 kPa). For the subsequent unloading increments the elastic stiffness matrix was used. This technique was used since the 'stiffness' of the problem remains very low in early stages of unloading until reclosure of the interface occurs over a significant portion of the base of the block.

The solution algorithm used also assumed the global stiffness matrix is symmetric and therefore stores only the upper triangular part. (The benefit being a large saving in storage space.) If a non-symmetric \( D_{ep} \) matrix is encountered, an 'averaged' global stiffness matrix is used in the solution algorithm. The full non-symmetric \( D_{ep} \) is used when updating the iterative out-of-balance load for successive iterations so that the final solution fully satisfies the non-symmetric constitutive equations. The \( D_{ep} \) matrix for a non-associated interface element is non-symmetric. In the analysis of the overturning block, it is because the incorrect 'averaged' stiffness matrix was being used at later stages of the analysis that the solution diverged. When the interface element properties were changed so that the plastic flow was associated (the global stiffness was symmetric), the solution converged very rapidly when the \( D_{ep} \) matrix calculated at the beginning of each increment was used.

Poor convergence also occurred when analysing the split beam (Example 1). In this problem the two beams separated over a short distance near the simply supported end. This caused large changes in the stiffness matrix. In this case the use of the \( D_{ep} \) matrix, updated at the beginning of each increment, allowed rapid convergence of the solution.

3.3. Steep stress gradients

Gens et al. found the behaviour of the interface element in the pull-out test was unsatisfactory when Gaussian integration was used. Large stress oscillations occurred near the sliding front. Gens et al. reported that if the Newton–Cotes integration scheme was adopted the results were
quite satisfactory. The pull-out test in Example 2 was therefore analysed with a range of interface and membrane stiffness to investigate whether similar problems would be encountered. The analyses were carried out using 3 point Gaussian and Newton–Cotes integration.

The difference between Gaussian and Newton–Cotes integration is the location of the integration points and their corresponding weights. The integration points for the 3 point Newton–Cotes method correspond to the positions of the three nodal pairs (i.e. at each end and the midpoint). The stress in the element—at each integration point—is then given by the relative displacement between the pair of nodes at the integration point location. The displacement of the other nodes in the element has no influence. The element is therefore essentially a linkage element. Three point Newton–Cotes integration is equivalent to Simpson’s rule for integration. For Gaussian integration the integration points are located between the end and the midpoint of the element, and at the midpoint of the element. The relative displacement of all nodes affects the stress at each integration point.

In the results presented below the Poisson’s ratio of the elastic membrane is zero and the interface element parameters are $K_s = K_n = 10^6$ kN/m$^3$, $\psi' = 30^\circ$, $v = 0^\circ$ and $c' = 0$. Analyses were also performed with $K_s = 10^6$ and $K_n = 10^9$. The results of these analyses are the same as those with $K_s = K_n$ and therefore are not presented. This is a problem dominated by sliding behaviour. The membrane is free to move up and down as the interface dilates or contracts. The normal stress in the interface element is controlled by the boundary condition of 10 kPa applied to the top of the membrane. In this example the value of the normal stiffness is not important.

The integration point stress was found to oscillate widely near the sliding front when the membrane stiffness, $E$, was reduced with respect to the interface stiffness. Figure 9 shows the shear stress distribution in the interface elements when the displacement, $\delta$, of point A (Figure 4) is 0·12 mm. The Newton–Cotes integration scheme appears to greatly improve the behaviour of the interface element. However further investigation of the pull-out test with $E = 10^4$ kN/m$^2$ shows that the stress distribution given by the Newton–Cotes integration is actually a poor approximation to the correct solution.

![Figure 9. Shear stress in interface elements](image-url)
Figure 10. Effect of element size on pull-out results

(a) Shear stress on interface

(b) Axial force in membrane

(c) Displacement of membrane
Figure 9 indicates that as the stiffness of the membrane reduces, the gradient of the shear stress distribution on the interface becomes steeper. When $E = 10^6$ kN/m$^2$ the stress reduces from the maximum value, 5.8 kPa, to nearly zero over a distance of 0.4 m. This reduction therefore occurs over 2 elements or 6 integration points. When $E = 10^5$ kN/m$^2$ the stress reduces over a distance of 0.2 m, which is the size of a single element. When $E = 10^4$ kN/m$^2$ the same stress change probably occurs over a distance less than the represented by one element. This is likely to cause the stress oscillation seen in Figure 9. Smaller elements (less than 0.2 m) are clearly necessary for the analysis of this problem if $E < 10^3$ kN/m$^2$.

Further analyses were performed with $E = 10^4$ kN/m$^2$ but with smaller elements at end A of the membrane and interface. Elements with lengths of 0.05 m and 0.025 m were placed at the end of the membrane and interface. Figure 10 shows the results of these analyses when $\delta = 0.12$ mm and also the results taken from Figure 9 where the element size is 0.2 m.

At end A of the interface the shear stress is greater than the expected value of 5.77 kPa ($10\tan30^\circ$) and the distribution of shear stress in the first 0.05 m is rather surprising. The membrane is experiencing a shear stress acting on its underside only. Being thin, the bending stiffness of the membrane is very small. The shear stress acting on the side of the membrane causes the end A of the membrane to bend downwards increasing the normal stress in the interface. Thus the maximum permissible shear stress in increased. At a short distance from the end of the membrane the curvature of the membrane causes a reduction in normal stress in the interface element resulting in a complex distribution of shear stress. When the membrane was constrained so that it could not bend, the maximum normal stress in the interface element remained at 10 kPa and the maximum shear stress at 5.77 kPa. This confirms that bending of the membrane causes the unusual distribution of shear stress, shown in Figure 10.

The problem of oscillating stress is due to the use of elements too large to model adequately the steep stress gradient that occurs in this problem. Stress oscillation does not occur when small elements are used. The numerical problems encountered here are not due to poor performance of the interface element but are due to inadequate modelling of the problem at hand. Only high-order elements that allow for a complex distribution of stress across the element will be able to accurately model the sliding front with the use of larger elements. Newton–Cotes integration is unnecessary and also undesirable as it has the effect of 'glossing over' or 'smoothing out' the steep stress gradient, thus hiding the real solution. When sufficiently small elements are used to describe adequately the stress gradient, Newton–Cotes and Gaussian integration give similar results.

Steep stress gradients across the interface element were also noted to cause poor convergence in the split beam analysis (Example 1). In this problem the two halves of the beam separate near the simply supported end. There is a large stress gradient in the normal stress distribution at the end of the beam where the two halves make contact. The stress gradient becomes greater as the normal stiffness of the interface element is increased. Oscillation in the stress occurs and convergence is inhibited. Reducing the size of the elements at the end of the beam allowed more accurate representation of the steep stress gradient in this region and improved the speed of convergence.

4. USING INTERFACE ELEMENTS WITH RETAINING WALLS

To assess the performance of interface elements in retaining wall analysis a series of analyses of the passive and active failure modes of deformation were carried out. A rigid wall embedded 5 m into the soil was translated uniformly in the horizontal direction. Interface elements placed on both sides of the wall allowed the magnitude of the maximum angle of wall friction, $\delta$, to be
varied. The minimum active force, the maximum passive force and the failure surfaces are compared. Similar analyses were carried out by Potts and Fourie.\textsuperscript{26} Interface elements were not available to them and consequently they only studied the two extremes of a fully rough wall, $\delta = \phi'$ and a smooth wall, $\delta = 0$.

4.1. Details of analyses

The finite element mesh used in the analysis and the applied boundary constraints are shown in Figure 11. The rigid wall was modelled as part of the boundary. Details of the mesh around the

![Figure 11. Retaining wall analysis—finite element mesh](image1)

![Figure 12. Interface element details](image2)
Table I. Interface element properties

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Elastic stiffness $K_s, K_p$ (kN/m$^3$)</th>
<th>Angle of friction $\delta$ (degrees)</th>
<th>Angle of dilation $\psi$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E^*$</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>2</td>
<td>100$E$</td>
<td>25.0</td>
<td>25.0</td>
</tr>
<tr>
<td>3</td>
<td>$E$</td>
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<tr>
<td>6</td>
<td>$E$</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>7</td>
<td>$E$</td>
<td>5.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

* $E$ is the Young's modulus of the soil

bottom of the wall is shown in Figure 12. The nodes on the boundaries A-B and A'-B' and nodes C, D, and E were moved horizontally to the right at a rate of 1 mm per increment. A total of 120 mm displacement was applied. Vertical displacement of these nodes was not permitted.

The Mohr–Coulomb failure criterion was adopted as the yield function for the soil. The material properties of the soil are the same as those used by Potts and Fourie. These are: $E = 60,000$ kN/m$^2$, $\mu = 0.2$, $c' = 0$, $\phi' = 25^\circ$. Fully associated plastic flow was assumed. The bulk unit weight $\gamma = 20$ kN/m$^3$. The initial horizontal stress was assumed to be equal to the vertical stress ($K_0 = 1.0$). Fully drained plane strain analyses were performed.

The zero thickness interface element with elastic perfectly behaviour described in Section 2 was used to model the interface between the wall and the soil. Table I gives the interface element properties used in each analysis. In all analyses the Mohr–Coulomb failure model was used and the effective cohesion $c' = 0$.

An analysis in which there were no interface elements was also performed.

4.2. Results of analyses

Earth pressure coefficients. The equivalent earth pressure coefficient, $K$, is calculated from $K = 2P_h/(\gamma H^2)$ where $H$ is the height of the wall and $P_h$ is the sum of the horizontal nodal reactions along the boundary representing the wall. Calculation of the reaction considered the stress only in the elements adjacent to the wall (i.e. on boundaries AB and A'B'). The limiting values of $K$, i.e. $K_s$, $K_{sb}$, $K_p$ and $K_{ph}$, are listed in Table II.

The angle of wall friction given in Table II is calculated from $\arctan(P_v/P_h)$ where $P_v$ and $P_h$ are the sum of the vertical and horizontal reactions along the boundary representing the wall. It is therefore the angle from the horizontal of the direction of the resultant reaction. On the passive side of the wall, friction is fully mobilized over the full length of the wall. On the active side the average mobilized wall friction is slightly less than the maximum wall friction angle. This is likely to be caused by fluctuations in the soil stress (see discussion below on earth pressure) due to large stress gradients in the elements around the base of the wall.

The magnitude of the average wall friction in the analysis without interface elements is greater than 25°. This does not satisfy the soil constitutive model. The friction angle has been calculated from the horizontal and vertical reactions on the boundary representing the face of the wall. In finite element analysis, the reactions are indirectly determined by extrapolating the integration point stresses to the boundary. The integration point stresses satisfy the constitutive law. The
extrapolated boundary stresses and reactions do not necessarily satisfy the constitutive law, particularly if high stress gradients exist. The integration points for the zero thickness interface elements are on the interface and therefore no extrapolation towards the wall is necessary. Hence the interface stress and the average angle of wall friction satisfy the constitutive law.

The ultimate earth pressure coefficients are dependent on the maximum wall friction angle but are independent, for all practical purposes, on the dilation and the elastic stiffness of the interface.

The calculated values of $K_{ph}$ and $K_{ah}$ are compared with various approximate analytical methods in Table III. The analytical values of $K_{ah}$ are all very similar and have little variation due to wall friction. The finite element values are slightly lower than all of the analytical methods for all values of wall friction angle. The differences are however quite small. The various analytical values of $K_{ph}$ are similar at low wall friction angles (for $\delta = 0$ they are all equal) but have a

Table III. Comparison of $K_{ph}$ and $K_{ah}$ with other methods

<table>
<thead>
<tr>
<th>Wall friction angle</th>
<th>(5^\circ)</th>
<th>(12.5^\circ)</th>
<th>(17.5^\circ)</th>
<th>(25^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive pressure coefficients, $K_{ph}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finite element analysis</td>
<td>2.84$^a$</td>
<td>3.37</td>
<td>3.69</td>
<td>4.00$^b$</td>
</tr>
<tr>
<td>Caquot and Kerisel$^{27}$</td>
<td>2.85</td>
<td>3.38</td>
<td>3.73</td>
<td>3.89</td>
</tr>
<tr>
<td>Chen$^{28}$</td>
<td>2.81</td>
<td>3.36</td>
<td>3.71</td>
<td>4.19</td>
</tr>
<tr>
<td>Coulomb$^{30}$</td>
<td>2.82</td>
<td>3.47</td>
<td>4.01</td>
<td>5.07</td>
</tr>
<tr>
<td>Packshaw$^{29}$</td>
<td>2.79</td>
<td>3.28</td>
<td>3.58</td>
<td>3.90</td>
</tr>
</tbody>
</table>

| Active pressure coefficients, $K_{ah}$ |
| Finite element analysis | 0.37$^a$ | 0.34 | 0.33 | 0.32$^b$ |
| Caquot and Kerisel$^{27}$ | 0.38$^c$ | 0.36 | 0.35 | 0.33 |
| Chen$^{28}$ | 0.386 | 0.360 | 0.346 | 0.327 |
| Coulomb$^{30}$ | 0.385 | 0.359 | 0.343 | 0.322 |
| Packshaw$^{29}$ | 0.385 | 0.359 | 0.343 | 0.322 |

\(^a\) Analysis No. 1
\(^b\) Analysis No. 6
\(^c\) Interpolated value
substantial variation for a fully rough wall. The values calculated by the finite element analyses with the interface element are in reasonable agreement with those calculated by Caquot and Kerisel, Chen, and Packshaw at all values of wall friction. The values given by Packshaw are slightly less than the finite element values at all wall friction angles. The values given by Coulomb overestimate \( K_{ph} \) considerably for higher wall friction angles. This is widely agreed. The values of Caquot and Kerisel are practically the same as the finite element calculations for low wall friction angles but is about 3 percent less than the finite element result when \( \delta = \phi' \).

**Earth pressure.** The ultimate stress distribution on both sides of the wall is nearly linear in each analysis. A linear distribution is generally assumed in the analytical earth pressure theories. Towards the base of the wall the passive pressure tends to be greater than, and the active pressure less than, an equivalent linear distribution. The distribution of stress in the 2D elements adjacent to the interface is essentially the same as the distributions in the interface elements. Some oscillation of the integration point stress occurs in the bottom two elements. This is likely to be due to very large strains and high stress gradients in the soil elements around the base of the wall. At the base of the wall the horizontal stress in the soil changes from about 400 kPa on the passive side to about 40 kPa on the active side over the width of the wall (0.2 m).

![Figure 13. Normal stress in interface elements (Analyses 1 and 2)](image-url)
In Figure 13 the stress distribution in the interface elements for analysis No. 2 \( (K_s = 100E) \) is compared with the distribution in the interface from analysis No. 1 \( (K_s = E) \). Oscillation in the stress occurs in analysis No. 2, particularly in the lower half of the wall.

**Failure mechanism.** The failure surfaces calculated by the analyses are close to planar in the case of \( \delta = 5^\circ \) and spiral shaped in the case of \( \delta = \phi' \). The results of the other analyses indicate failure surfaces between these two extremes. These results agree with analytical limit equilibrium\(^2\) and limit analysis\(^2\) methods which also indicate failure on plane surfaces for smooth walls \( (\delta = 0) \) and on curved surfaces (e.g. log spiral) for rough walls \( (\delta = \phi') \). The zone of failure appears independent of the stiffness and angle of dilation of the interface elements.

**Interface properties.** In the analyses of the retaining wall the elastic shear stiffness is assumed equal to the elastic normal stiffness. In reality this may often be unrealistic.

However, in retaining wall problems the elastic parameters of the interface are not important except for the oscillations reported above if \( K_s \) is large. The interface and surrounding soil yields very quickly and the elastic-plastic behaviour then dominates. The use of a much smaller value of \( K_s \) will result in greater shear displacement prior to yield but will not effect the value of the limiting earth pressure. The value of the limiting earth pressure is governed by the soil properties and the strength parameters of the interface.

Reduction in \( K_s \) will decrease the shear stiffness contribution of the interface in the global stiffness matrix. This is similar to the effect on the elasto-plastic stiffness matrix of reducing the friction angle. It does not result in ill-conditioning of the stiffness matrix or numerical difficulties. The results presented here are also valid for the case when \( K_n \gg K_s \).

5. **CONCLUSIONS**

The zero thickness interface element is useful for modelling relative slip and opening and closing on predefined surfaces. Numerical problems can however occur through ill-conditioning of the stiffness matrix and high stress gradients in the interface elements. In many situations stress gradients are likely to be high and are increased with increased interface stiffness. This problem can therefore easily be confused with ill-conditioning. Ill-conditioning was noticed in the problems analysed here when the stiffness of the interface element was greater than 100 times the Young's modulus of the surrounding soil. This however depends on the units used in the problem and the size of the surrounding soil elements.

In analyses in which the interface is opening and closing, and where non-dilatant interface properties are assumed, large changes in the stiffness matrix occur. This can cause convergence difficulties with a modified Newton–Raphson solution algorithm in which a constant elastic stiffness matrix is being used. The use of the tangent elasto-plastic stiffness at each stage of the analysis will in most cases significantly accelerate convergence. When non-associated behaviour of the interface or 2D elements is assumed, the \( D_{sp} \) matrix is non-symmetric. For some problems it may be necessary to store and solve the full non-symmetric global stiffness matrix in order to achieve convergence.

Reduced (2 point) Gaussian integration does not help to solve the numerical problems illustrated. Reduced and full Gaussian integration produce similar results.

Newton–Cotes integration tends to improve the numerical behaviour of interface elements at the possible expense of hiding the true solution. Where sufficiently small elements are used to model the interface behaviour, Newton–Cotes and Gaussian integration give similar results. No advantage is therefore gained through the use of Newton–Cotes integration.
The Use of interface elements with an elasto-plastic Mohr–Coulomb failure criterion has predicted ultimate values of earth pressure for various angles of wall friction in good agreement with other analytical solutions. These analyses have provided numerical confirmation of the analytical solutions of Caquot and Kerisel which are widely accepted and commonly used in retaining wall design. These values however apply only to the situation of uniform wall translation. Different modes of wall movement, for example rotation, will affect the ultimate value and distribution of the earth pressure on the wall.

In the analysis of the active and passive failure modes of a retaining wall, the ultimate earth pressure is independent of the elastic stiffness and the angle of dilation of the interface elements. The results obtained for a rough wall modelled with interface elements assuming associated plastic flow are essentially the same as those obtained if no interface element is used to model the soil–wall boundary.

In the retaining wall problem failure occurs through the soil mass. The ultimate failure load for given interface behaviour is governed by the soil properties rather than the interface properties. The interface element simply provides a stress boundary condition to the soil mass. Hence it is only the strength parameter (i.e. interface friction angle) that is important. A different result will be obtained when different soil properties are used with the same interface properties. Conversely, for given soil properties and interface strength the same result is obtained with different interface stiffness and dilation.

In problems in which the failure mode is by sliding on the soil–structure interface, this would not be expected to be so. In analysis of piles and ground anchors for example, the ultimate load for given interface element properties could conceivably be largely independent of the soil properties if the strength of the interface is exceeded before the strength of the soil. The normal stiffness and dilation properties of the interface, and the boundary conditions in this type of problem would be expected to play a dominant role. The use of the more complex elasto-plastic models may be necessary in these situations to obtain more realistic and accurate results.

The zero thickness interface element provides a useful means to model reduced interface friction in finite element analysis of retaining walls. The simple elasto-plastic Mohr–Coulomb constitutive model appears to be satisfactory for the modelling of interface behaviour in this type of analysis.

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