Soil Uncertainty and its Influence on Simulated G/G_{max} and Damping Behavior

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ABSTRACT: In this paper, recently developed probabilistic elasto–plasticity is applied in simulating cyclic behavior of clay. Simple von Mises elastic–perfectly plastic material model is used for simulation. Probabilistic soil parameters, elastic shear modulus (G_{max}) and undrained shear strength (s_u), that are needed for the simulation are obtained from correlations with SPT N-value. It has been shown that the probabilistic approach to geomaterial modeling captures some of the important aspects – modulus reduction, material damping ratio, and modulus degradation – of cyclic behavior of clay reasonably well, even with the simple elastic–perfectly plastic material model.

INTRODUCTION

Behavior of geomaterials is inherently uncertain. This uncertainty stems from natural soil variability, testing and transformation errors (Lacasse and Nadim 1996; Phoon and Kulhawy 1999a). Traditionally, geotechnical engineering community deals with uncertainties in geomaterial by applying factor of safety. However, use of (large) factors of safety not only result in over-expensive design, but also, sometimes, in unsafe design (Duncan 2000). Hence, in recent years, geotechnical engineering practice has seen an increasing emphasis on probabilistic treatment to data and subsequent simulation/design.

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Quantification or mathematical description of uncertainty is usually done within the framework of probability theory, although fuzzy sets (Zadeh 1983), convex models (Ben-Haim and Elishakoff 1990), and interval arithmetic (Moore 1979) have also been used in the past to describe uncertainty mathematically. Under the framework of probability theory, uncertain spatial variability of soil deposit is modeled as random field – a collection of random variables, indexed from space continuum. For complete characterization (up to second order) of a random field, in addition to mean and variance, information on autocovariance function and correlation length or scale of fluctuation are also needed. Thorough descriptions of random field modeling techniques, with procedures for estimating correlation length for geotechnical engineering applications, are given by DeGroot and Baecher (1993) and Fenton (1999a). Testing and transformation uncertainties, on the other hand, are point uncertainties and are usually modeled as random variables, which are completely characterized (up to second order) by their respective means and variances. Over the years researchers have quantified and collected typical variations of different soil properties, ranging from consolidation parameters, laboratory measured strength properties to in-situ properties (Lumb 1966; Lacasse and Nadim 1996; Baecher and Christian 2003; Phoon and Kulhawy 1999a), as well as testing uncertainties, associated with the most commonly used test methods (Hammitt 1966; Phoon and Kulhawy 1999a; Marosi and Hiltunen 2004), and transformation uncertainties, associated with the most common transformation relationships (Phoon and Kulhawy 1999b). A fair amount of work was also done on subsequent probabilistic geotechnical design guidelines (Harr 1987; Kulhawy and Phoon 2002), although the existing geotechnical LRFD codes still do not explicitly consider the soil properties uncertainties. The book by Baecher and Christian (2003) thoroughly describes the current state-of-the-art of probabilistic geotechnical engineering design.

Modeling and simulation under uncertainty, on the other hand, have received much less attention, mainly due to the concern about the the necessity, usefulness, and tractability of probabilistic modeling in geotechnical engineering, when geotechnical problems are difficult to model even deterministically, unless advanced modeling techniques are used. However, published works on this subject (Paice et al. 1996; Rackwitz 2000; Manolis 2002; Griffiths et al. 2002; Fenton and Griffiths 2002; De Lima et al. 2001; Fenton and Griffiths 2003; Borja 2004; Fenton and Griffiths 2005; Popescu et al. 2005) show very promising results, especially in quantifying our confidence in our simulation. Most of the above works are based on Monte Carlo technique (Metropolis and Ulam 1949) in tandem with (deterministic) finite element method. Monte Carlo technique relies on repeated random sampling (of, for example, soil properties) and because of this repeatability, the computational cost associated with it could become extremely huge (and probably intractable), especially for large scale problems, like dynamic soil-structure interaction analysis. Due to the above drawback of Monte Carlo technique, in other fields of science and engineering, stochastic differential equation approach (Gardiner 2004) or numerically, stochastic finite element method (Kleiber and Hien 1992; Ghanem and Spanos 1991) is very popular. However, non-linearities in soil properties prevent direct application of those techniques for probabilistic simulations in geotechnical engineering.

The difficulty in propagating uncertainties in soil properties through the elastic-plastic constitutive equation lies in the high non-linear dependence of the elastic-plastic modulus on stress. Very few published literature exist on this subject. In fact, the first attempt to propagate uncertainties through elastic-plastic constitutive rate equation was published only recently (Anders and Hori 1999). Anders and Hori (1999, 2000) used perturbation approach – a linearized Taylor series expansion with respect to mean – in propagating uncertainties in modulus through the von Mises elastic–perfectly material model. However, inherent to the Taylor series expansion, perturbation technique suffers from small variance requirement (Matthies et al. 1997). A rule of thumb restricts the coefficient of variance (COV) to 20% (Sudret and Der Kiureghian 2000) to minimize the error in perturbation approach. This severely limits the applicability of perturbation approach to geotechnical problems, where soil COVs are rarely less than 20% (cf. Phoon and Kulhawy (1999a, 1999b)). Perturbation approach also suffers from closure problem (Kavvas 2003), which means that information about higher-order statistical moments are necessary to calculate lower-order statistical moments. Griffiths and Fenton (2002, 2003, 2005) used brute force Monte Carlo technique in propagating uncertainties through elastic-perfectly plastic Mohr-Coulomb model. Recently, in circumventing the above drawbacks of Monte Carlo and perturbation techniques, Jeremić et al. (2007) proposed Eulerian–Lagrangian Fokker–Planck–Kolmogorov equation (FPKE; Kavvas 2003) based probabilistic elasto–plasticity. Developed probabilistic elasto–plasticity is compatible with the general theory of (deterministic) plasticity, and hence can be used for probabilistic simulation of a variety of elastic–plastic models. Solution strategies for probabilistic elasto–plasticity were discussed by Sett et al. (2007a, 2007b) for both linear and non-linear hardening models. The concept of probabilistic yielding was introduced by Jeremić and Sett (2009), while Sett and Jeremić (2010) discussed its effect on constitutive simulation under cyclic loading. It was shown that due to uncertainty in yield stress, there is always a possibility that plastic behavior starts at very very low strain and influence of elastic behavior continues far into the plastic domain. Because of this, the average (mean) constitutive response and the most probable (mode) constitutive response show nonlinear behavior with a vanishing linear region, even for the simplest elastic–perfectly plastic material model. This in turn is significant since, by expanding into probability space, one obtains very realistic soil behavior with simple constitutive models, requiring very few soil parameters (and their distributions) that could be obtained directly from in–situ tests (e.g. SPT, CPT etc.).

In this paper, FPKE based probabilistic approach to geomaterial modeling is applied in simulating G/G_{max} and damping behavior of undrained clay. Elastic–perfectly plastic von Mises material model, which requires only two soil parameters, the shear modulus (G_{max}) and the undrained shear strength (s_u), is used. Simulated responses are then compared with published experimental measurements.

PROBABILISTIC FRAMEWORK FOR CONSTITUTIVE SIMULATION

The constitutive behavior of soil can be modeled by elastic–plastic constitutive rate equation, which, in 1–D, can be written as:

$$\frac{d\sigma}{dt} = D\frac{d\epsilon}{dt} \tag{1}$$

where σ is the stress, ϵ is the strain, t is the pseudo time, and D is the stiffness modulus, that can be either elastic or elastic–plastic:

$$D = \begin{cases} D^{el} & \text{elastic} \\ \\ D^{el} - \dfrac{D^{el}\dfrac{\partial U}{\partial \sigma}\dfrac{\partial f}{\partial \sigma}D^{el}}{\dfrac{\partial f}{\partial \sigma}D^{el}\dfrac{\partial U}{\partial \sigma} - \dfrac{\partial f}{\partial q_*}r_*} & \text{elastic-plastic} \end{cases} \tag{2}$$

where, D^{el}, f, U, q_*, and r_* are elastic modulus, yield function, plastic potential function, internal variable(s), and rate(s) of evolution of internal variable(s) respectively. However, due to various uncertainties associated with soil properties, as discussed in the previous section, the modulus, D in Eq. (1) becomes uncertain. In traditional deterministic approach to elastic–plastic geo-material modeling, one typically applies engineering judgment (qualitative) in obtaining the 'most probable' soil parameters and substitute them in Eq. (1) in obtaining the 'most probable' soil constitutive behavior. However, one may note that due to the non-linearity of soil behavior, 'most probable' soil parameters do not necessarily result in 'most probable' constitutive behavior.

Recently, Jeremić et al. (2007) developed a probabilistic approach for elastic–plastic modeling of geomaterials. Proposed approach is based on the extension of constitutive rate equation (Eq. (1)) into probability density space using Eulerian–Lagrangian Fokker–Planck–Kolmogorov approach (Kavvas 2003):

$$\frac{\partial P(\sigma(t),t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left\{N_{(1)}P(\sigma(t),t)\right\} + \frac{\partial^2}{\partial \sigma^2}\left\{N_{(2)}P(\sigma(t),t)\right\} \tag{3}$$

where $P(\sigma(t),t)$ is the probability density of stress, t is the pseudo-time, while $N_{(1)}$ and $N_{(2)}$ are advection and diffusion coefficients, respectively. The advection and diffusion coefficients depend only on the material model used for modeling. It can be shown (see appendix of Jeremić et al. 2007), that the advection and diffusion coefficients for von Mises elastic-perfectly plastic constitutive relationship (used here to model undrained behavior of clay) can be written as:

$$\begin{aligned} {}^{vM}N^{el}_{(1)} &= \frac{d\epsilon}{dt}\langle G\rangle \quad ; \quad {}^{vM}N^{el}_{(2)} = t\left(\frac{d\epsilon}{dt}\right)^2 Var[G] \\ {}^{vM}N^{el-pl}_{(1)} &= 0 \quad ; \quad {}^{vM}N^{el-pl}_{(2)} = 0 \end{aligned} \tag{4}$$

In Eq. (4), G is the elastic shear modulus and ϵ is the shear strain. Furthermore, $\langle\cdot\rangle$ represents the expectation operator, while $Var[\cdot]$ is the variance operator. The superscripts $\cdot^{el}$ and $\cdot^{el-pl}$ on the advection and diffusion coefficients refer to pre–yield elastic region and post-yield, elastic–plastic region.

However, for a heterogeneous material like soil, yield strength is quite uncertain. This is due to the fact that in a representative volume element (RVE; Hashin 1983) of the heterogeneous material, each of the large number of particle contacts has different yield strengths and orientations. Each of these particle contacts will yield differently, depending upon its yield strength. Hence, for material with uncertain yield strength (Σ_y), there exist possibilities that the material (RVE) starts yielding inside the elastic regime and/or elastic behavior continues way into the plastic regime. Under the framework of probability theory, these possibilities are governed by the probability density function of yield strength (Σ_y), which can be quantified by statistically analyzing the test results. Hence, to realistically simulate the probabilistic material behavior, Jeremić and Sett (2009) suggested probability weights, based on probability density function of yield strength (Σ_y), to the elastic and plastic advection and diffusion coefficients in obtaining equivalent advection and diffusion coefficients. For von Mises elastic–perfectly plastic soil with uncertain yield strength (Σ_y), the equivalent advection and diffusion coefficients (${}^{vM}N^{eq}_{(1)}$ and ${}^{vM}N^{eq}_{(2)}$) would become (cf. Sett and Jeremić 2010):

$$\begin{aligned} {}^{vM}N^{eq}_{(1)}(\sigma) &= (1 - P[\Sigma_y \leq \sigma])\,{}^{vM}N^{el}_{(1)} + P[\Sigma_y \leq \sigma]\,{}^{vM}N^{el-pl}_{(1)} \\ &= (1 - P[\Sigma_y \leq \sigma])\frac{d\epsilon}{dt}\langle G\rangle \end{aligned} \tag{5}$$

$$
\begin{aligned}
{}^{vM}N^{eq}_{(2)}(\sigma) &= (1 - P[\Sigma_y \leq \sigma])^{vM}N^{el}_{(2)} + P[\Sigma_y \leq \sigma]^{vM}N^{el-pl}_{(2)} \\
&= (1 - P[\Sigma_y \leq \sigma])t\left(\frac{d\epsilon}{dt}\right)^2 Var[G] \qquad (6)
\end{aligned}
$$

The probability weight ($P[\Sigma_y \leq \sigma]$) in the above equations (Eqs. (5) and (6)), quantifies, at any given stress level, the probability of the material RVE being elastic or elastic–plastic. Using the above equivalent advection and diffusion coefficients (Eqs. (5) and (6)), one can solve the constitutive rate equation, written in probability density space (Eq. (3)) to obtain the complete probabilistic description, in terms of probability density function, of evolutionary stress response with strain (pseudo-time).

One may note that the probabilistic framework presented above is a pure constitutive level (point-location scale) framework and assumes the input soil properties to be random variables. The spatial average (local-average) probabilistic constitutive response, if sought for, for example, in dealing with uncertain spatial variability of soil properties, usually modeled as random fields, would necessitate discretization of random fields into random variables using appropriate technique, for example, Karhunen–Loève expansion (Karhunen 1947; Loève 1948; Ghanem and Spanos 1991). Those random variables could then be propagated through the point-location scale constitutive framework presented above and spatial average constitutive response could be assembled using a stochastic finite element technique. Using polynomial chaos expansion (Wiener 1938; Ghanem and Spanos 1991) and Galerkin technique, Sett and Jeremić (2009) proposed one such stochastic finite element framework and applied the framework in seismic wave propagation through spatially uncertain stochastic soil.

SIMULATION RESULTS AND DISCUSSION

In this section, the constitutive behavior of normally consolidated, high plasticity clay is simulated probabilistically using the FPKE approach described in the previous section. Elastic–perfectly plastic von Mises material model is used for clay. The model requires shear modulus (G_{max}) and undrained shear strength (s_u) as input soil parameters. Both the soil parameters are easily obtainable through transformation from commonly used in-situ tests. In this paper, the above properties are obtained from SPT N-value. In this context, it is important to mention that the authors understand the limitations of using SPT N-value for (deterministic) estimation of s_u and G_{max} for clay. However, the authors' intent here is to demonstrate the power of a simple constitutive model, but with uncertain soil parameters, to simulate the actual response of soil.

Quantification of Uncertainties in Input Soil Parameters

Transformation from measured in-situ properties to mechanical properties usually introduces uncertainty, which is currently (in traditional deterministic analysis) accounted for by applying engineering judgment. Alternatively, under the framework of probability theory, one could quantify the transformation uncertainty by modeling it as a random variable. For example, for alluvial clays in Japan, Phoon and Kulhawy (1999b) proposed the following relationship between SPT N-value and undrained shear strength (s_u):

$$s_u = 0.29\ p_a\ N^{0.72} \tag{7}$$

where $p_a = 101,325$ Pa is the atmospheric pressure. The above relationship (Eq. (7)), along with the data (Hara et al. 1974) from which the relationship is developed, is plotted in FIG. 1. The data-

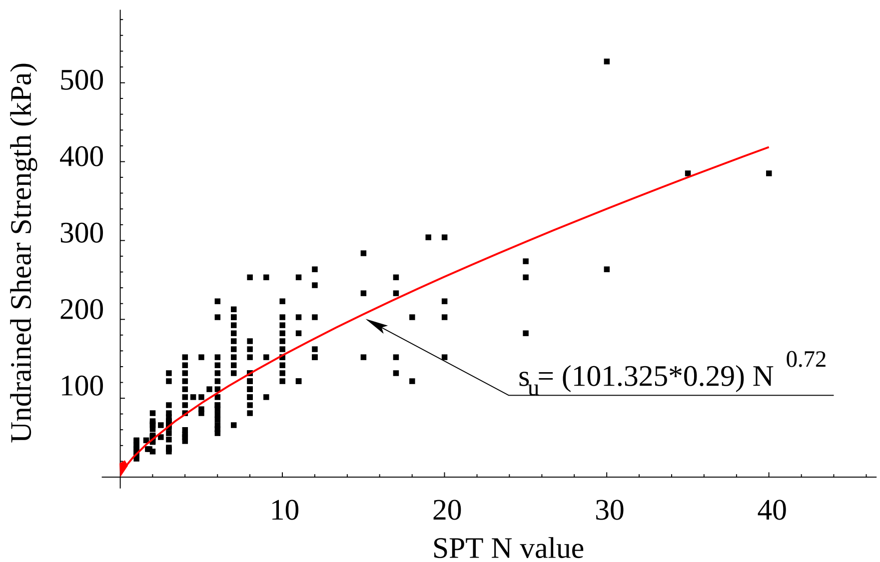

FIG. 1. Transformation relationship between SPT N-value and undrained shear strength, s_u

scatter in FIG. 1 represents knowledge uncertainty in the above transformation equation (Eq. (7)), and under probability theory, can be modeled as a random variable. To this end, Eq. (7) can be written as:

$$s_u = 0.29\ p_a\ N^{0.72} + \chi \tag{8}$$

where, χ is a zero-mean random variable and represent the data-residual with respect to the deterministic transformation equation. The histogram of the residual is plotted in FIG. 2. Regarding the model for the best-fit probability density function (PDF), a Gaussian distribution can be ruled out as the histogram is skewed. After trying few distributions, a Pearson IV type distribution, with Pearson parameters of 0, 2400, -2.75×10^5, and 9×10^8, was found to best fit the residual. The fitted PDF is shown in FIG. 2.

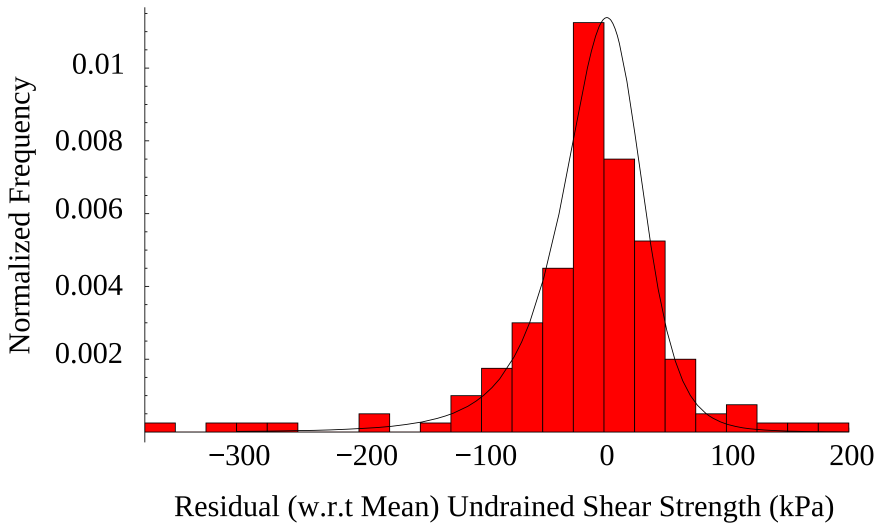

FIG. 2. Histogram of the residual (w.r.t the deterministic transformation equation) undrained strength, along with fitted probability density function

Similarly, for transformation between SPT N-value and Young's modulus (E) for alluvial clays in Japan, Phoon and Kulhawy (1999b) proposed a transformation equation, which can be written in probabilistic form as

$$E = 19.3\, p_a\, N^{0.63} + \chi \tag{9}$$

FIG. 3 shows experimental data (Ohya et al. 1982) along with the deterministic transformation equation that in this case represents the mean trend. The scatter with respect to the mean trend (deterministic transformation equation), plotted as histogram, is shown in FIG. 4. A zero-mean Gaussian random variable with a standard deviation of 4041.8 kPa is found to best fit (FIG. 4) the scatter with respect to the deterministic equation. The above standard deviation is obtained using maximum likelihood technique.

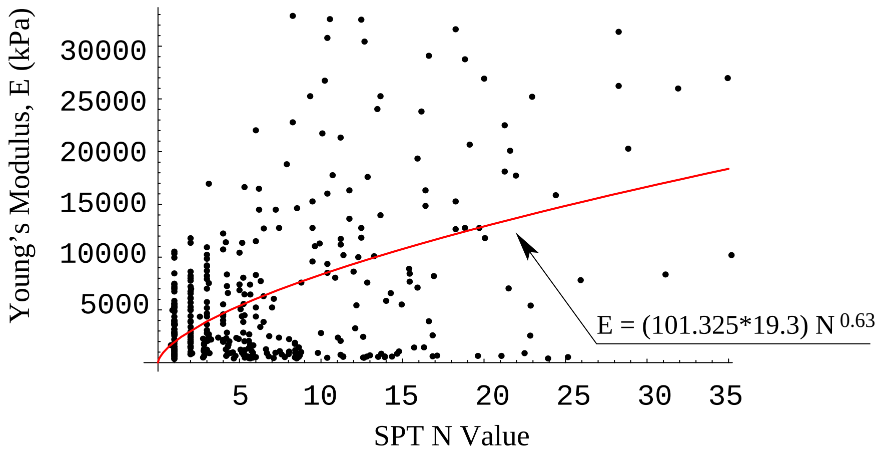

FIG. 3. Transformation relationship between SPT N-value and pressure-meter Young's modulus, E

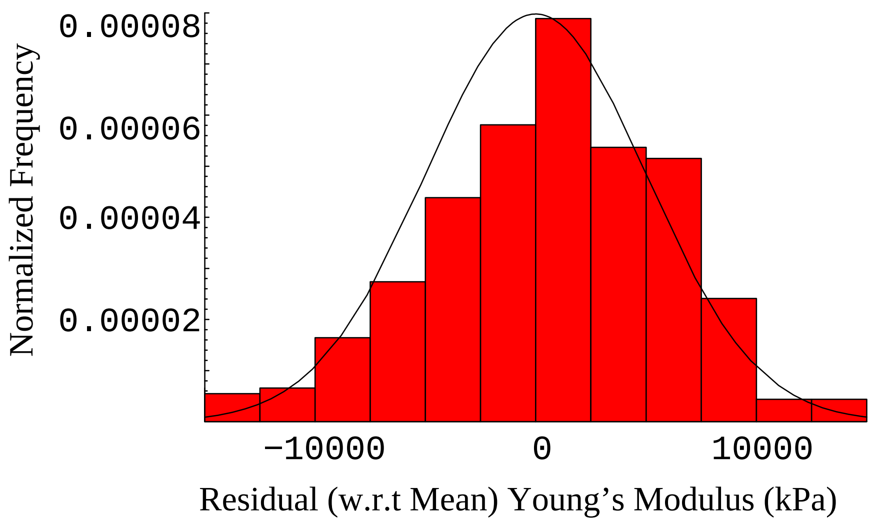

FIG. 4. Histogram of the residual (w.r.t the deterministic transformation equation) Young's modulus, along with fitted probability density function

In this context, one may note that the penetration test is a high strain test (to the order of 10%; refer to Figure 65 in FHWA Geotechnical Engineering Circular No. 5 (Federal Highway Administration, U.S. Department of Transportation 2002)) and, the corresponding estimated modulus (Eq. 9) is a high-strain modulus. Hence, to estimate the corresponding low-strain modulus (needed as the input to the von Mises material model, used in this paper), it is multiplied by 17.25, assuming $(17.25 - 1)/17.25 = 94$ % reduction in modulus at 10% strain, following Idriss (1990) and presuming that the reduction pattern of Young's modulus follows the same that of shear modulus. For example, at $N = 15$, a mean high-strain Young's modulus of 10.735 MPa is predicted by Eq. (9). The corresponding mean low-strain Young's modulus used in this paper is 10.735 MPa $\times 17.25 = 185$ MPa. The multiplying factor (low strain correction factor) for standard deviation should, physically, be less than that of the mean. This is because, as the soil is sheared (or, in other words, as soil plastifies), the micro-structure of soil changes and as a result, our knowledge uncertainty on it increases. This was experimentally observed by Stokoe II et al. (2004). Probabilistic simulations, published elsewhere by the authors (Sett and Jeremić 2010), also show such increase in uncertainty with strain. Hence, in absence of experimental data for clay, following the probabilistic G/G_{max} versus shear strain curve, suggested by Stokoe II et al. (2004), a multiplying factor (low strain correction factor) of $[1-(0.2375-0.05)/0.05] \times 17.25 = 10.7$ is used for standard deviation. At N=15, a standard deviation of 4.04 MPa was estimated, using maximum likelihood technique, for high-strain Young's modulus. The corresponding standard deviation of low-strain Young's modulus, then, becomes $4.04 \times 10.7 = 43.2$ MPa. By assuming undrained condition, one could assume Poisson's ratio to be equal to 0.5 (deterministic) and could transform the Young's modulus to shear modulus as

$$G_{max} = \frac{E_{max}}{2(1+\nu)} \tag{10}$$

One may note that, as the above equation (Eq. (10)) that relates elastic shear modulus and elastic Young's modulus is linear, the elastic shear modulus (G_{max}) would also be a Gaussian distribution. Hence, the statistical properties of the elastic shear modulus (G_{max}) can easily be obtained using standard techniques. For example, at $N = 15$, the mean and the standard deviation of G_{max} are $185\text{MPa}/(2(1+0.5)) = 61.6$ MPa and $43.2\text{MPa}/(2(1+0.5)) = 14.4$ MPa respectively. In this context, it is important to emphasize that the above estimation of the low-strain correction factors would become unnecessary if small-strain shear modulus (G_{max}) is measured directly from geophysical tests or estimated through direct correlation of geophysical test-measured properties (for example shear wave velocity, with SPT N-value). The transformation equations between SPT N-value and shear wave velocity, reported in the literature (Hasancebi and Ulusay 2007; Jafari et al. 2002; Pitilakis et al. 1999; Imai 1977) have not been used in this paper due to lack of reported data points for a meaningful statistical analysis.

In addition to the transformation uncertainty, discussed above, soil properties also include significant testing uncertainties. For example, in SPT, the testing uncertainty arises from equipment, procedure and operator errors. Phoon and Kulhawy (1999a) proposed typical range of COV for SPT as 15-45%. In this paper, an equivalent of 45% COV is added to the undrained shear strength (s_u) and 15% COV is added to the elastic shear modulus (G_{max}) to account for SPT testing uncertainties. Larger uncertainty is used for undrained shear strength (s_u) because the shear strength is not unique but depends on many factors, e.g. direction of loading, strain rate, boundary conditions, stress level, and sample disturbance effects (Ladd 1991; Mayne 2007).

Uncertain spatial variability represents the other important source of uncertainty in soil property. This uncertainty is present because soil properties are measured at few locations and then extrapolated/interpolated to all (some) other points of the soil continuum. In other words, in 'estimating' soil properties between two adjacent boreholes, uncertain spatial variability is incurred. This uncertain spatial variability is traditionally accounted for by applying engineering judgment. Probability theory, on the other hand, deals with uncertain spatial variability through random field modeling (DeGroot and Baecher 1993; Fenton 1999a; Fenton 1999b). Random field modeling characterizes the uncertain spatial variability in terms of standard deviation and correlation structure. The standard deviation is usually added to the uncertainties arising from transformation equation and testing method, while the correlation structure can be accounted for, among others, through stochastic elastic-plastic finite element method (Sett and Jeremić 2009; Sett et al. 2010). In this paper, the uncertain spatial variability has not been explicitly accounted for as the focus of this paper is on point-location (constitutive) behavior. However, one may note that the data of SPT N-value versus undrained shear strength (FIG. 2) and SPT N-value versus Young's modulus (FIG. 3) contain some effects of spatial variability as SPT is performed at approximately every 30 cm (1 foot) and the blow counts obtained in such way represent average values over that length.

Simulation of G/G_{max} and Damping Behavior

The above described uncertain data set is used to analyze, using the probabilistic elastic–plastic constitutive framework (described in Section 2), the undrained cyclic (shear) behavior of clay. Of particular interest is the performance evaluation of a simple, elastic–perfectly plastic von Mises material model, but extended into probability space.

Probabilistic elastic–plastic stress–strain response

FIG. 5(a) shows the mean shear stress versus shear strain hysteresis loop for an undrained clay, simulated using probabilistic von Mises elastic-perfectly plastic material model. The input to the model were the statistics of the soil properties – elastic shear modulus, G_{max} and undrained shear strength, s_u – corresponding to SPT N-value of 15. The uncertain clay material is cyclically sheared to ±1.026% shear strain. The cyclic evolution of standard deviation of shear stress is shown in FIG. 5(b). Results shown in Figures 5(a) and (b) are obtained by solving the Fokker–Planck–Kolmogorov equation (FPKE; Eq. (3), with advection and diffusion coefficients given by Eqs. (5) and (6)) numerically, with appropriate initial and boundary conditions. The solution to the FPKE, the evolutionary PDF of shear stress with shear strain, is then integrated by standard techniques to obtain the evolutionary mean and standard deviation behavior. The details of the solution technique for governing FPKE can be found in Sett and Jeremić (2010).

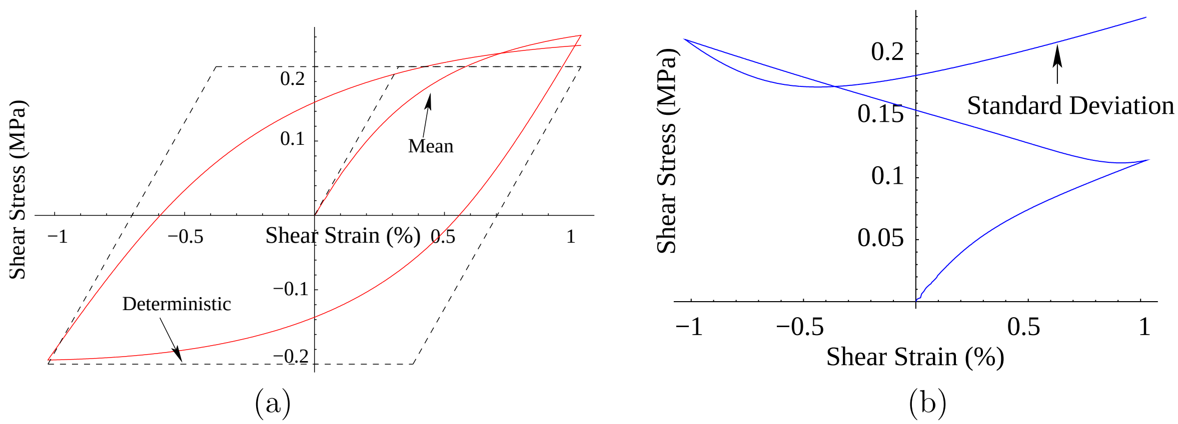

FIG. 5. Simulated hysteresis shear stress versus shear strain loop at ± 1.026% shear strain: (a) mean and (b) standard deviation behavior

It is important to note that simulation results shown in FIG. 5 are obtained using elastic – perfectly plastic von Mises material model and require only two probabilistic soil parameters (their probability distribution) – elastic shear modulus (G_{max}) and undrained shear strength (s_u). If probability distribution of material parameters were neglected and only mean values were used (thus simplifying to deterministic von Mises elastic–perfectly plastic model) simple bi-linear response would result. Such (deterministic) bi-linear response is also shown in FIG. 5(a).

In FIG. 5(a), it is also interesting to observe that the mean response (of the full probability response described by the stress PDF) is non–linear even at very small strain, although, the deterministic model assumes linearity till yielding and then behaves as perfectly plastic material. Such nonlinear mean response is due to the uncertainty in the yield stress, as there is always a probability (however small) that elastic–plastic response starts at a very small strain. In addition to that, there also exist a probability that material is elastic at strains past the (mean) yield point, and since the mean solution is an ensemble average of all the possibilities, such probable elastic influence is extended into plastic range as well. One can visualize this probabilistic yielding effect by observing that within a laboratory specimen (considered generally as a representative volume element (RVE; Hashin 1983)), each of large (infinite) number of particle contacts has different and unknown yield strengths. At a given strain, some of those particle contacts might be elastic while others might be fully yielding. What is observed in any laboratory experiment is the ensemble average (mean) behavior of all the particle contacts. Similar conclusion was developed by Einav and Collins (2008), using probabilistic micro-mechanical simulation. It may be noted that the point-location scale constitutive simulation presented in this paper doesn't account for the scale effect. Such scale effects could be accounted for by quantifying the uncertain spatial variability (for example, through random field modeling) of soil and accounting for it in our simulation. This can be done, for example, through stochastic elastic–plastic finite element method in obtaining local-average constitutive behavior.

Evolution of secant shear modulus

Elastic–plastic constitutive simulation, described above, is used to obtain the evolution of secant shear modulus with shear strain. The deterministic evolution of the secant shear modulus is shown in FIG. 6(a). The deterministic shear modulus remains constant, equal to G_{max} = 61.6 MPa until ≈ 0.3% strain, representing deterministic yield point, before suddenly dropping after yield point. FIG. 6(a) also shows mean and mean±standard deviation of the evolutionary (probabilistic) secant shear modulus. Compared with the deterministic evolution of secant shear modulus, the mean solution predicts a realistic reduction with cyclic shear strain. The region between mean and mean±standard deviation contains the most likely values of evolutionary shear modulus. Coefficient of Variation (COV), which can also be used to visualize uncertainty, is shown in FIG. 6(b). The initial COV of secant shear modulus was [(14.4 MPa + 0.15 × 61.6 MPa)/61.6 MPa]×100% =

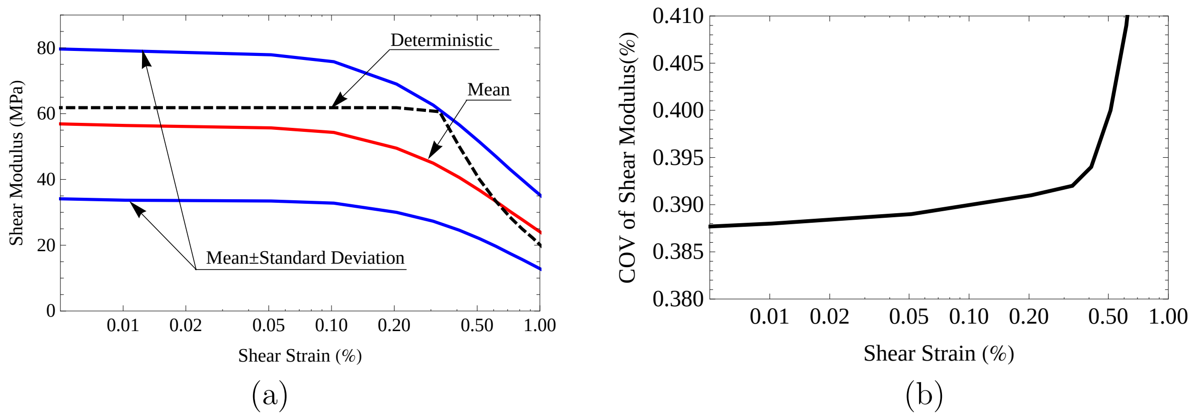

FIG. 6. Simulated (a) probabilistic reduction and (b) evolution of coefficient of variation (COV) of secant shear modulus with cyclic shear strain.

38.3%. It was calculated from the mean and standard deviation values for G_{max}, obtained earlier as 61.6 MPa and 14.4 MPa, respectively. The second term in the numerator (0.15×61.6 MPa) represents the contribution of the testing uncertainty, which was assumed to have a COV of 15%. It is interesting to observe that COV of secant shear modulus increases with cyclic shear strain. This increase in uncertainty comes from the fact that as the material plastifies, this simple two parameter model becomes less and less accurate. In other words, more detailed investigation of the soil (micro) structure is needed and more advanced modeling technique needs to be used if one wishes to reduce such uncertainty.

The above probabilistic evolution of secant shear modulus (FIG. 6) is shown in FIG. 7 in a more common form, in terms of variation of G/G_{max} versus shear strain. It is important to note that, in FIG. 7, the normalizations of evolutionary mean and mean±standard deviation are done by dividing each of those by the mean of elastic shear modulus (Mean[G_{max}]). In other words, the upper and lower limits of normalized secant shear modulus, shown in FIG. 7 represent (Mean[G]±Standard Deviation[G])/Mean[G_{max}], rather than G/G_{max}±standard deviation. The probabilistic evolution of material damping ratio versus shear strain is shown in FIG. 8. The mean damping ratio, shown in FIG. 8 is obtained from the hysteresis loop of mean shear stress versus shear strain. Likewise, the upper and lower bounds of damping ratio in FIG. 8, are obtained from the hysteresis loops of mean±standard deviation of shear stress versus shear strain.

In both FIGs. 7 and 8, the deterministic solutions are also plotted. Though the deterministic

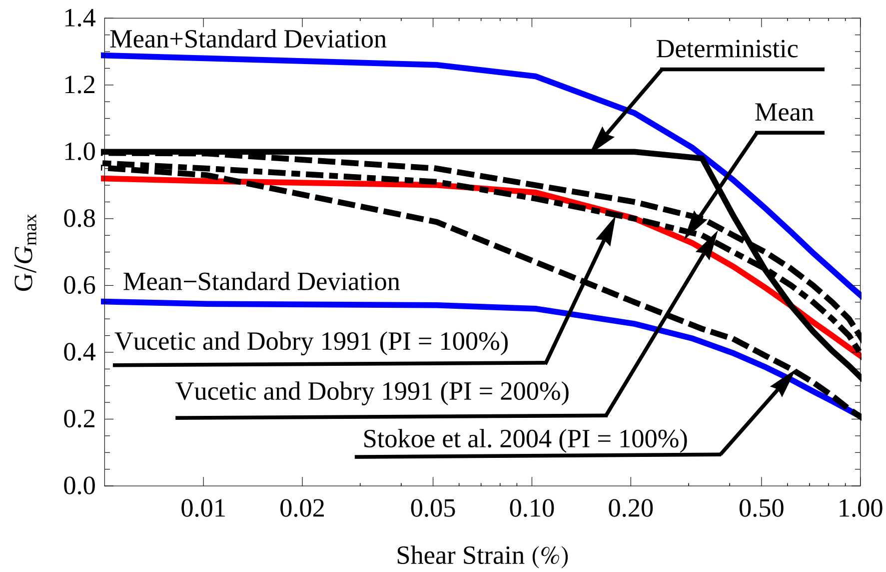

FIG. 7. Simulated probabilistic G/G_{max} behavior

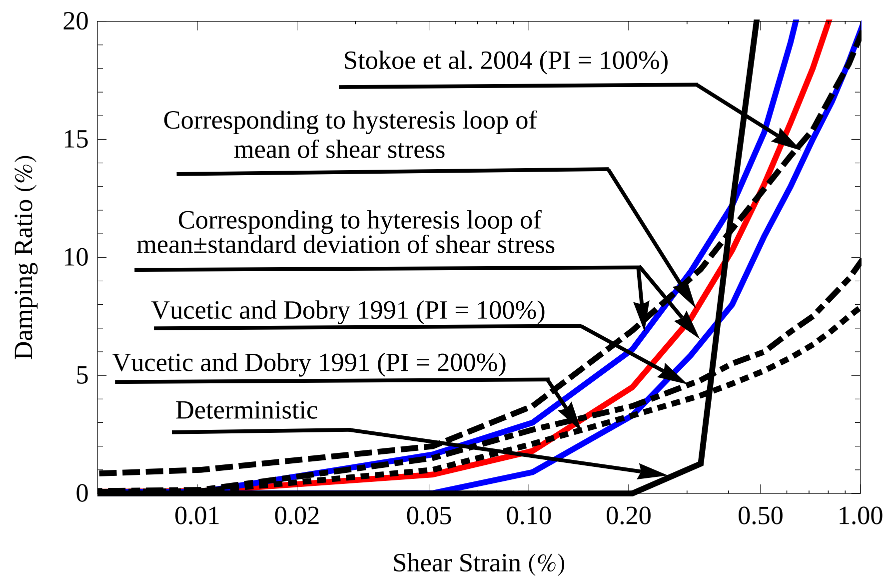

FIG. 8. Simulated probabilistic material damping behavior

solutions fail to predict realistic soil behavior, probabilistic solutions, even with the simplest elastic–perfectly plastic model, are comparable to the experimental observations reported in the literature. For example, the probabilistic G/G_{max} and damping ratio curves, presented in FIGs.. 7 and 8, compared well with the experimental data reported by Vucetic and Dobry (1991) and Stokoe II et al. (2004) for high plasticity clay.

In addition to modulus reduction, the probabilistic approach also captures modulus degradation when the clay material is cyclically sheared repeatedly. The simulated hysteresis loops when the clay material is sheared repeatedly to ±0.1026% strain, is shown in FIG. 9. Only the first four

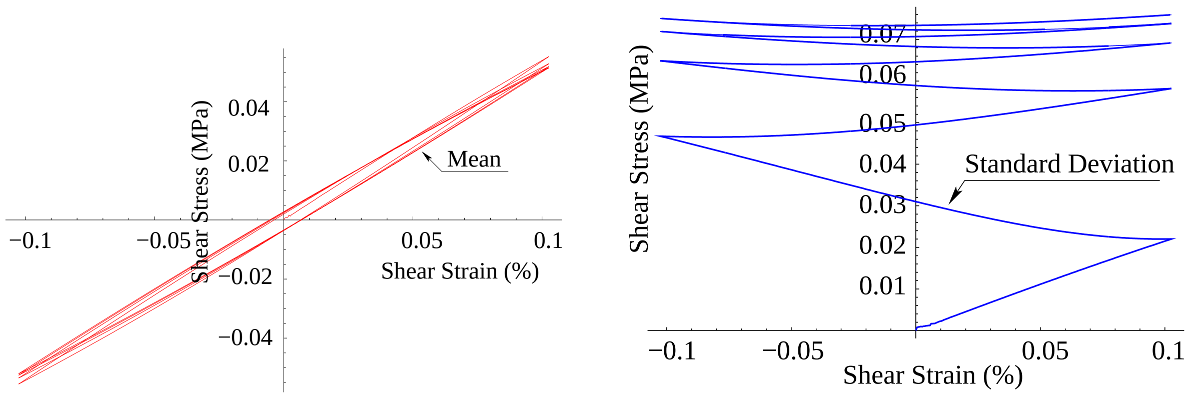

FIG. 9. Simulated hysteresis shear stress versus shear strain loop, when sheared repeatedly at ± 0.1026% strain: (a) mean and (b) standard deviation behavior; First four cycles are shown

loops are shown in FIG. 9 for clarity. The absolute values of mean and standard deviation of secant shear modulus after each cycle are plotted in FIGs. 10(a) and (b), respectively. The mean shear modulus degrades 8% after 10 cycles at 0.1026% strain. The rate of degradation of mean secant shear modulus is higher initially, but stabilizes as the number of cycles increases. The standard deviation of secant shear modulus, on the other hand, increases (275% increase after 10 cycles at 0.1026% strain) with number of cycles. It, however, also stabilizes as the number of cycles increases.

The explanation for increased uncertainty in the secant shear modulus is based on mechanics. With repeated shearing, soil (micro) structure is continuously disturbed and hence, our knowledge uncertainty on it increases. In other words, after repeated shearing, simplistic two-parameter elastic-perfectly plastic model, used here, cannot model such changes accurately. The elastic-

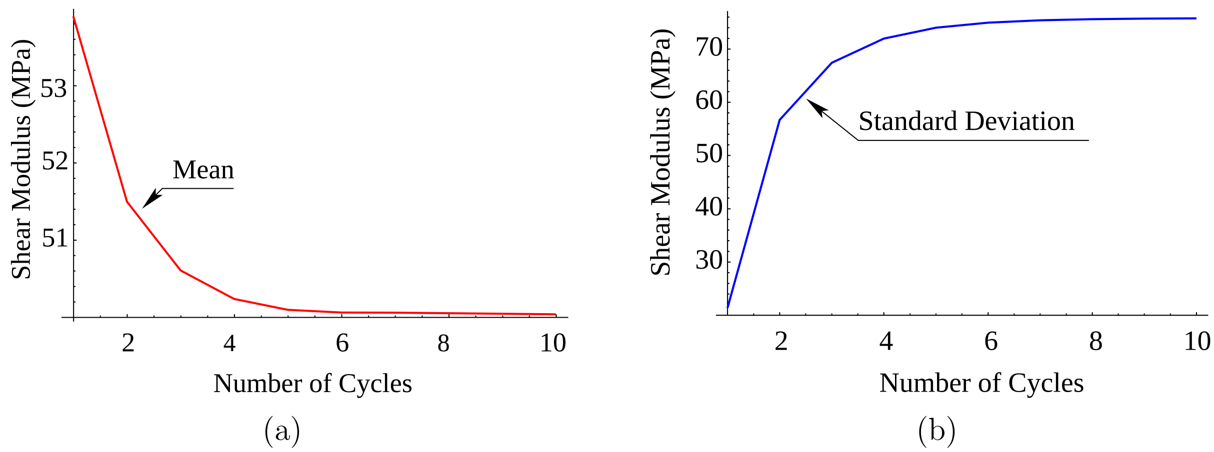

FIG. 10. Simulated probabilistic degradation of shear modulus, when sheared repeatedly at $\pm$ 0.1026% strain: (a) mean and (b) standard deviation behavior

plastic probabilistic solution (advection-diffusion equation) aptly captures that fact. The diffusion component, which controls the spread of the response (stress) probability density function, keeps evolving continuously with strain, irrespective of the direction of loading (shearing) until plasticity is fully mobilized with 100% probability, when the diffusion coefficient becomes zero. The advection component, on the other hand, controls the translation of the response (stress) PDF in the stress-strain domain. This component is a function of loading (shearing) direction, advection coefficient and initial condition at the beginning of each loading direction, which in turn, is a function of the uncertainty present in the system at that state of strain/shearing. The advection component hence gives rise to the degraded modulus after each cycle, until plasticity is fully mobilized with 100% probability. The modulus degradation is, therefore, appearing as a direct consequence of probabilistic yielding of material (clay).

CONCLUSIONS

Presented in this paper was a probabilistic approach to constitutive simulation of undrained clay behavior. It had been shown that probabilistic approach allowed for not only quantification of our confidence in numerical prediction, but also modeling modulus reduction, modulus degradation and damping behavior with simple elastic-perfectly plastic (two-parameter) material model. This is particularly significant since in geotechnical engineering practice, due to various constraints, advanced laboratory tests are rarely performed, while in–situ tests are usually preferred, data from which can be used to calibrate probabilistic material models, one of which was presented here. In particular, shown here was (probabilistic) calibration of a simple, elastic–perfectly plastic model but extended into probability space, by using number of in–situ tests. Such calibrated probabilistic elastic–perfectly plastic model was then used to predict various aspects of undrained shear behavior of clay. It was shown that simulation results compared well with published data (within mean ± standard deviation). More importantly, as results contained full PDFs of the responses, they might have many (other) uses in research and practice.

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