Dynamics of Soils and Structures under Uncertainty

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Outline

Motivation

- Improve seismic design of soil structure systems
- Earthquake Soil Structure Interaction (ESSI) in time and space, plays a major role in successes and failures
- Accurate following and directing (!) the flow of seismic energy in ESSI system to optimize for
  - Safety and
  - Economy
- Development of high fidelity numerical modeling and simulation tools to analyze realistic ESSI behavior: Real ESSI simulator

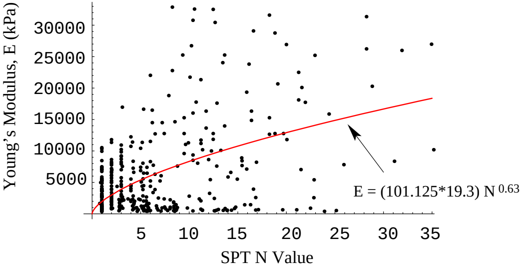
Predictive Capabilities

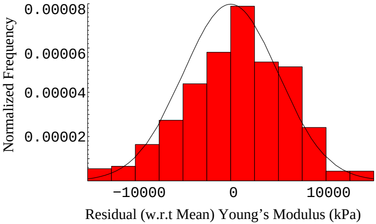
- Verification provides evidence that the model is solved correctly. Mathematics issue.
- Validation provides evidence that the correct model is solved. Physics issue.
- Prediction under Uncertainty (!): use of computational model to foretell the state of a physical system under consideration under conditions for which the computational model has not been validated.
- Modeling and Parametric Uncertainties
- Predictive capabilities with low Kolmogorov Complexity
- Modeling and simulation goal is to inform, not fit

Modeling Uncertainty

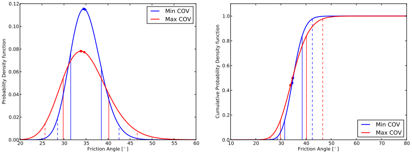
- Simplified modeling: Features (important ?) are neglected (6D ground motions, inelasticity)
- Modeling Uncertainty: unrealistic and unnecessary modeling simplifications
- Modeling simplifications are justifiable if one or two level higher sophistication model shows that features being simplified out are not important

Parametric Uncertainty: Material Stiffness

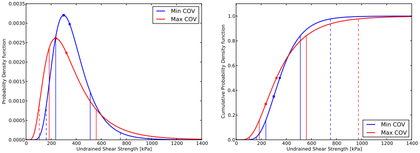




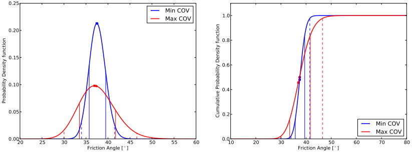
Parametric Uncertainty: Material Properties



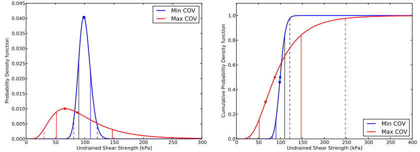
Field ϕ



Field c_u



Lab ϕ



Lab c_u

Realistic ESSI Modeling Uncertainties

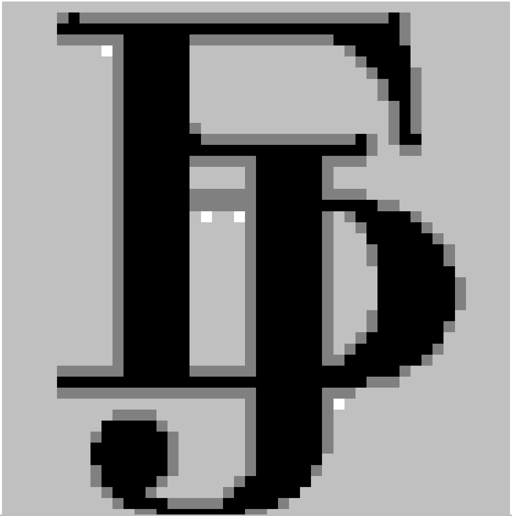
- Seismic Motions: 6D, inclined, body and surface waves (translations, rotations); Incoherency
- Inelastic material: soil, rock, concrete, steel; Contacts, foundation–soil, dry, saturated slip–gap; Nonlinear buoyant forces; Isolators, Dissipators
- Uncertain loading and material
- Verification and Validation $\Rightarrow$ Predictions

Outline

Real ESSI Models

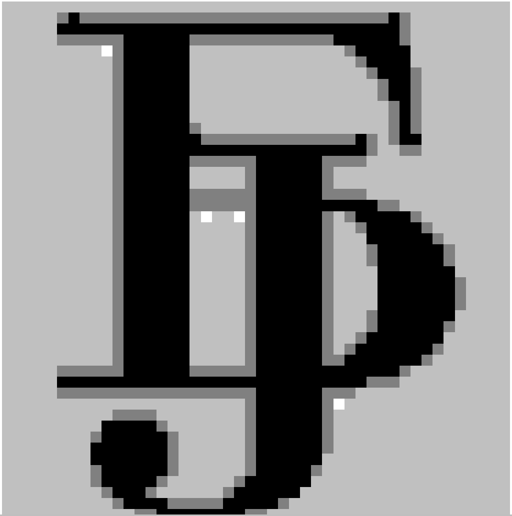
- Full seismic motion input (body and surface waves) using the Domain Reduction Method (Bielak et al.)
- Inelastic (saturated or dry) soil/rock
- Inelastic (saturated or dry) contact (foundation – soil/rock)
- Buoyant (nonlinear) forces
- Inelastic structural modeling (elastic requested?!)
- Verification (extensive) and Validation (in progress)

6D Free Field Motions



(MP4)

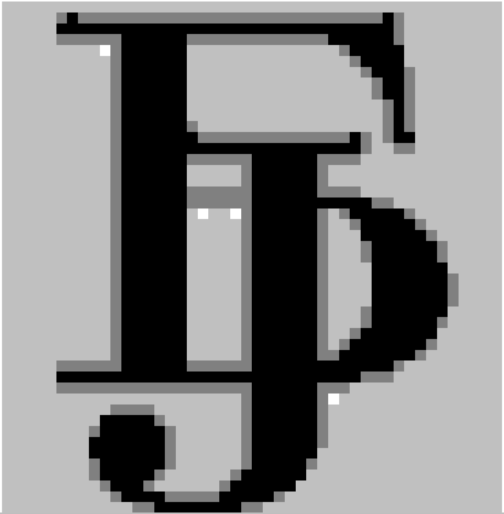
6D Free Field Motions (closeup)



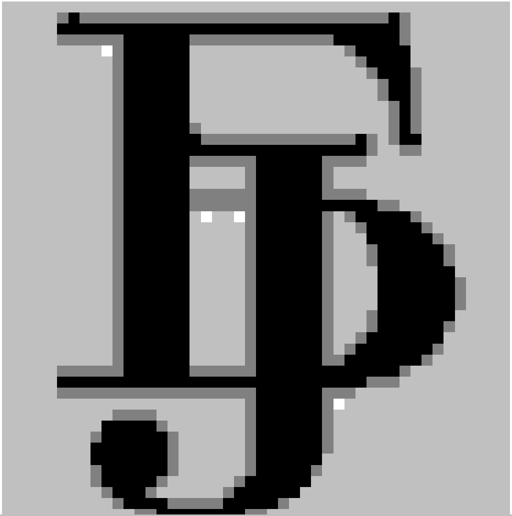
(MP4)

6D Free Field at Location

(MP4)



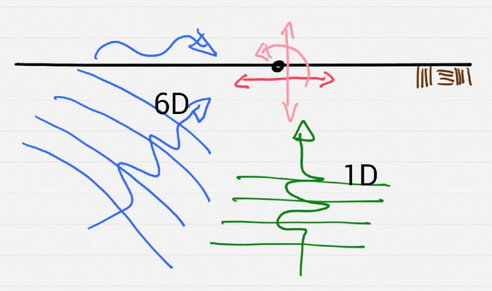
6D Earthquake Soil Structure Interaction



(MP4)

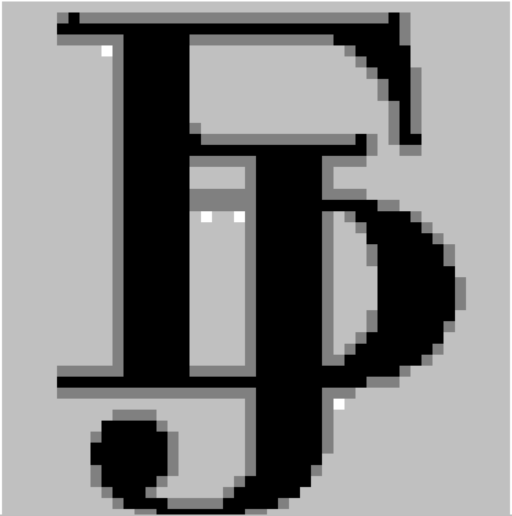
From 6D to 1D?

- Assume that a full 6D (3D) motions at the surface are only recorded in one horizontal direction
- From such recorded motions one can develop a vertically propagating shear wave in 1D
- Apply such vertically propagating shear wave to the same soil-structure system

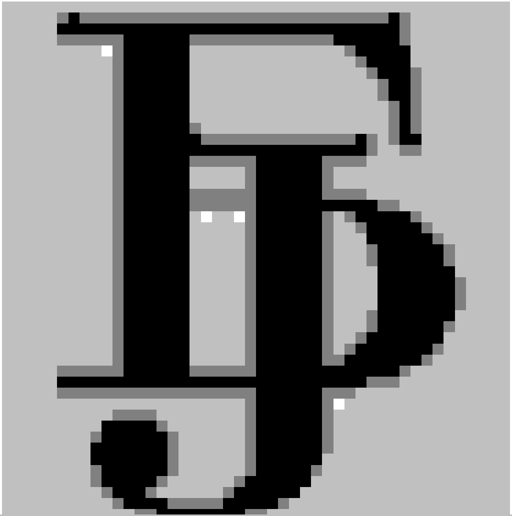


1D Free Field at Location

(MP4)



1D ESSI of NPP



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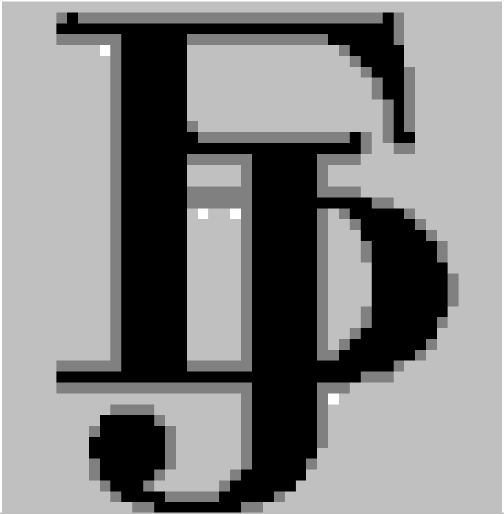
6D vs 1D NPP ESSI Response Comparison



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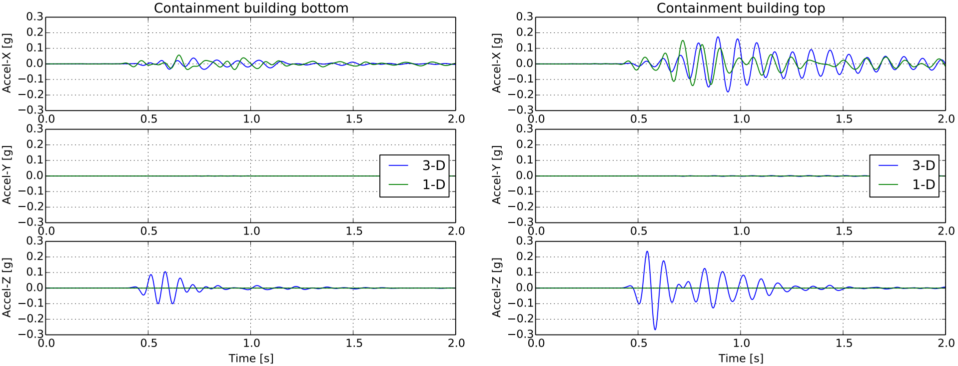
6D vs 1D: Containment Acceleration Response



Outline

Uncertain Material Parameters and Loads

- Decide on modeling complexity
- Determine model/material parameters
- Model/material parameters are uncertain!
  - Measurements
  - Transformation
  - Spatial variability

Uncertainty Propagation through Inelastic System

- Incremental el–pl constitutive equation

$$\Delta\sigma_{ij} = E^{EP}_{ijkl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

- Dynamic Finite Elements

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}^{ep}\mathbf{u} = \mathbf{F}$$

Critique of our Previous Work, PEP and SEPFEM

- Constitutive weighted coefficients N_1 and N_2 do not work well for stress solution!
- We suggested that $\sigma(t)$ be considered a δ-correlated, and based on that simplified stiffness equations. Both the assumption and the resulting equation were not right.
- On a SEPFEM level, stiffness needs update basis functions and KL coefficients in each step. We updated the eigenvalues λ_i and kept the same structure (Karhunen-Loeve) in the approximation of the stiffness, which is not physical
- Implicitly assumed that the stiffness remains Gaussian, which is not the case

Gradient Flow Theory of Probabilistic Elasto-Plasticity

Decomposition of an elastoplastic random process:

$$\left(\frac{\partial}{\partial t} - \mathcal{L}_{rev}\right) P(\boldsymbol{\sigma}, t) = 0 \qquad \text{if } \boldsymbol{\sigma} \in \Omega^{el}$$

$$\left(\frac{\partial}{\partial t} - \mathcal{L}_{irr}\right) P(\boldsymbol{\sigma}, t) = 0 \qquad \text{if } \boldsymbol{\sigma} \in \Omega^{el} \cup \Omega^{pl}$$

- Reversible ($\mathcal{L}_{rev}$) and Irreversible ($\mathcal{L}_{irr}$) operators
- Yield PDF is an attractor, similar to plastic corrector
- Ergodicity of the elastic-plastic process can be proven!

Gradient Flow Theory of Probabilistic Elasto-Plasticity

- Elastic, reversible process, Fokker-Planck (forward Kolmogorov) equation

$$\frac{\partial P(\boldsymbol{\sigma}, t)}{\partial t} = -\nabla \cdot (\langle \boldsymbol{E}\dot{\epsilon} \rangle P(\boldsymbol{\sigma}, t)) + t \, \mathrm{Var}[\boldsymbol{E}\dot{\epsilon}] \Delta P(\boldsymbol{\sigma}, t)$$

$$\mathcal{L}_{rev} = \nabla \cdot (t \, \mathrm{Var}[\boldsymbol{C}\dot{\epsilon}] \nabla - \langle \boldsymbol{C}\dot{\epsilon} \rangle)$$

- Plastic, irreversible process, Fokker-Planck (forward Kolmogorov) equation

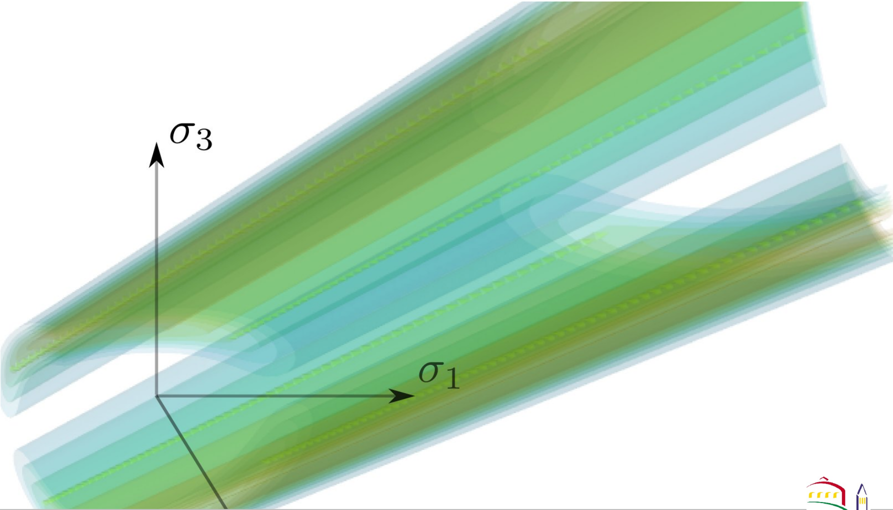
$$\frac{\partial P(\boldsymbol{\sigma}, t)}{\partial t} = \nabla \cdot (\nabla \Psi(\boldsymbol{\sigma}) P(\boldsymbol{\sigma}, t)) + D \Delta P(\boldsymbol{\sigma}, t)$$

$$\mathcal{L}_{irr} = D \nabla \cdot \left(\nabla - \frac{\nabla P_y(\boldsymbol{\sigma})}{P_y(\boldsymbol{\sigma})} \right)$$

Gradient Flow Theory of Probabilistic Elasto-Plasticity

- Limiting (final) distribution is considered to be known
- Underlying potential leading to this distribution is sought
- Transition from uncertain elastic to uncertain plastic response
- Only in a 1D elastoplastic problem does one end up with a stationary distribution
- In higher dimensional problems this yield stress distribution is only "marginally" stationary along one or a combination of the stress components.

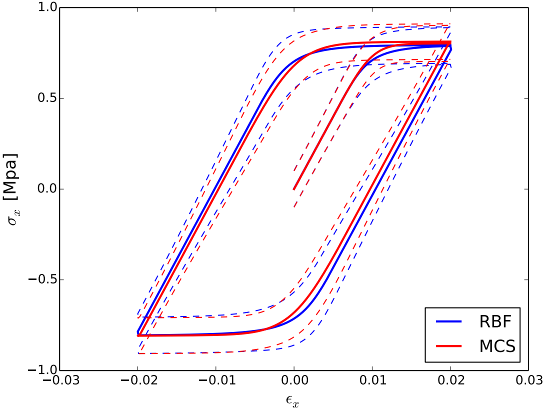
Probabilistic Elasto-Plasticity: von Mises Surface



Gradient Theory of Probabilistic Elasto-Plasticity: Numerical Solution

- Using radial basis functions (a meshless method) for solving Fokker-Planck equations for uncertain elastic-plastic response
- Details in a talk by Mr. Karapiperis later this afternoon (room 2, MS-6, 17:00-19:00, last talk)

Gradient Theory of Probabilistic Elasto-Plasticity: Verification, Elastic-Perfectly Plastic



Stochastic Elastic-Plastic FEM (SEPFEM)

- KL-PC expansion of material random fields (stiffness, etc)
  $\mathbb{D}(\mathbf{x},\theta) = \sum_{i=0}^{M} r_i(\mathbf{x})\Phi_i[\{\xi_r(\theta)\}]$
- PC expansion of displacement field

$$u(\mathbf{x},\theta) = \sum_{i=0}^{p} d_i(\mathbf{x})\psi_i[\{\xi_r(\theta)\}]$$

- Stochastic Galerkin

$$\sum_{n=1}^{N} K'_{mn} d_{ni} + \sum_{n=1}^{N}\sum_{j=0}^{P} d_{nj} \sum_{k=1}^{M} b_{ijk} K''_{mnk} = \Phi_m \langle \psi_i[\{\xi_r\}]\rangle$$

SEPFEM Statistical linearization

Update the FE stiffness in the elastoplastic regime:

- Solve elastoplastic FPE for each integration point

$$\frac{\partial P^{nl}(\sigma,t)}{\partial t} = \frac{\partial}{\partial\sigma}\left(\left\langle D^{k}(1-P[\Sigma_{y}\leq\sigma])\frac{\Delta\epsilon}{\Delta t}\right\rangle P\right) + \frac{\partial^{2}}{\partial\sigma^{2}}\left(tVar\left[D^{k}(1-P[\Sigma_{y}\leq\sigma])\frac{\Delta\epsilon}{\Delta t}\right]P\right)$$

- Consider an equivalent linear FPE

$$\frac{\partial P^{lin}(\sigma,t)}{\partial t} = N^{eq}_{(1)}\frac{\partial P}{\partial\sigma} + N^{eq}_{(2)}\frac{\partial^{2}P}{\partial\sigma^{2}}$$

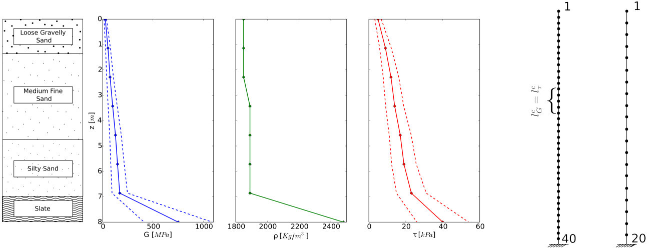
- Linearization of the PC coeff. as an optimization problem

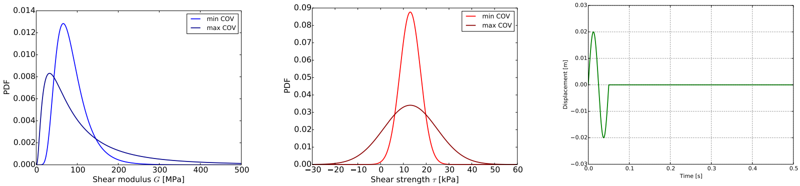
$$\frac{\partial P^{lin}(\sigma,t)}{\partial t} = \frac{\partial P^{nl}(\sigma,t)}{\partial t}$$

Dynamic, Time Domain, SEPFEM

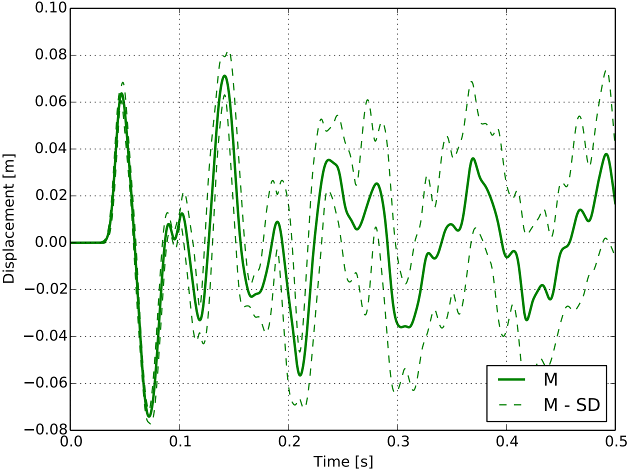
- Gaussian formulation inadequate due to occurrence of "probabilistic softening" modes $\Rightarrow$ Need for positive definite kernel
- Stochastic forcing (e.g. uncertain earthquake)
- Stability of time marching algorithm (Newmark, Rosenbrock, Cubic Hermitian) analyzed using amplification matrix
- Long-integration error and higher order statistics phase shift

Wave Propagation Through Uncertain Soil

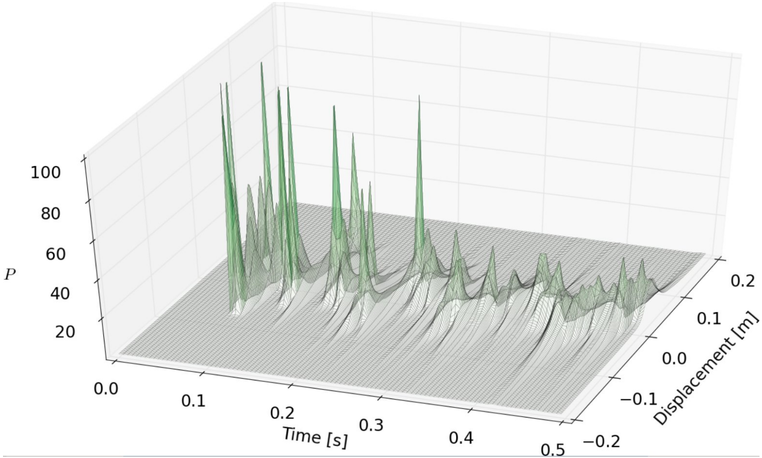




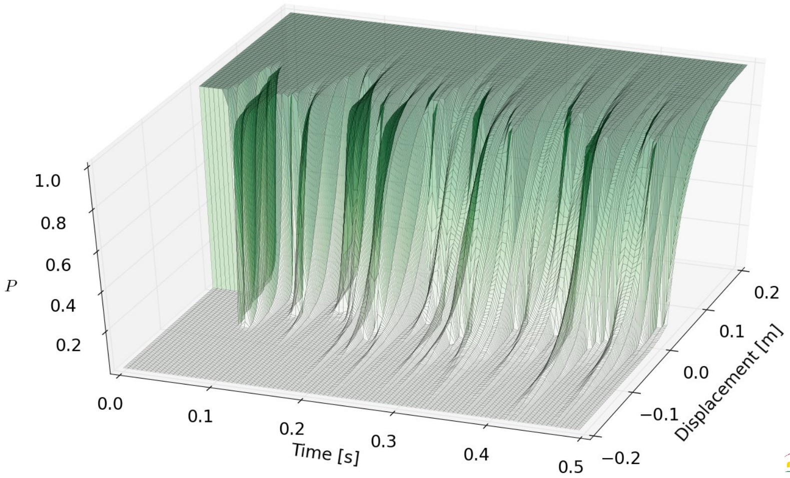
Uncertain Elastic Response at the Surface (COV = 120%)



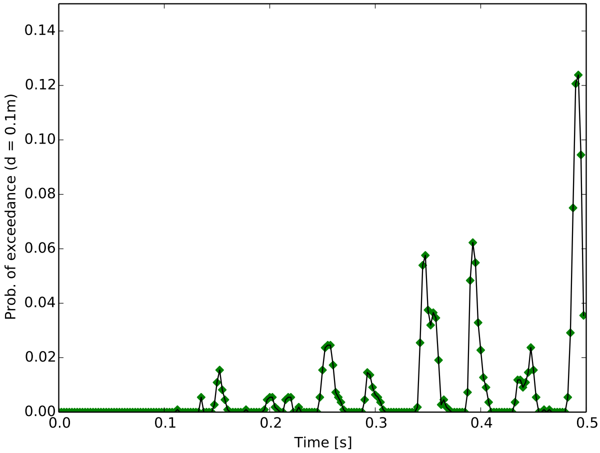
Displacement PDFs at the Surface (COV = 120%)



Displacement CDFs (Fragilities) at the Surface (COV = 120%)



Probability of Exceedance, $disp = 0.1m$ (COV = 120%)



Concluding Remarks

- Uncertainty influences results of numerical predictions
- Uncertainty (modeling and parametric) must be taken into account
- Goal is to predict and inform, not fit
- Philosophy of modeling and simulation system **Real ESSI** simulator (aka: Врло Просто, Muy Fácil, Molto Facile, 真简单 , 本当に簡単 , Πραγματικά Εύκολο, बहुत ही आसान, آسان واقعی, Très Facile, Вистински Лесно, Wirklich Einfach)

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