Stochastic Site Response Analysis Through Uncertain Elastoplastic Soil

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Outline

Introduction

Stochastic ESSI

Conclusion

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Stochastic ESSI

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Motivation

Improve modeling and simulation for infrastructure objects

Reduction of modeling uncertainty

Choice of analysis level of sophistication

Goal: Predict and Inform rather than fit

Engineer needs to know!

System for **Real**istic modeling and simulation of **E**arthquakes, **S**oils, **S**tructures and their **I**nteraction:

Real-ESSI Simulator `http://real-essi.info/`

Prediction under Uncertainty

- Modeling Uncertainty, Simplifying assumptions

  Low, medium, high sophistication modeling and simulation

  Choice of sophistication level for confidence in results

- Parametric Uncertainty, $M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$

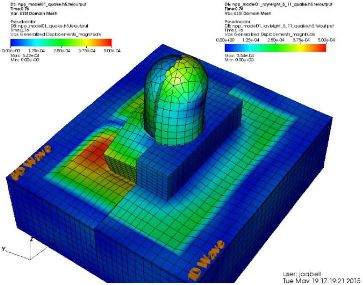
  Uncertain mass M, viscous damping C and stiffness K^{ep}

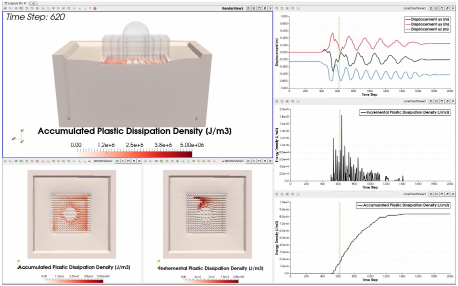
  Uncertain loads, $F(t)$

  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

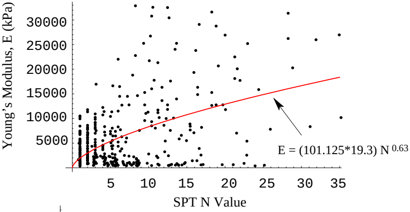
Modeling Uncertainty

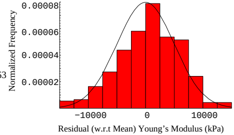
- Important (?!) features are simplified, 1C vs 6C, inelasticity
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that features being simplified out are not important

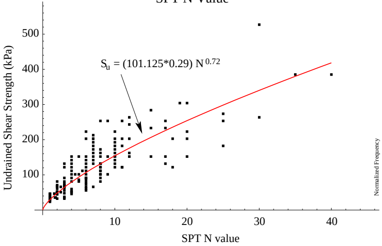


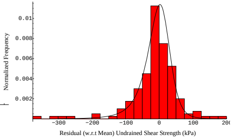


Parametric Uncertainty: Soil Stiffness and Strength









(cf. Phoon and Kulhawy (1999B

Outline

Simulation Methods for Stochastic PDEs

- Analytical, stochastic differential equation approach: difficult to solve with complex random coefficients
- Monte Carlo method : Computationally expensive, will show some results at SMiRT 43026
- Perturbation approach: Small variation with respect to mean, closure problem
- Stochastic collocation method: Global error minimization
- Stochastic Galerkin method: Local error minimization, Stochastic Elastic-Plastic Finite Element Method (SEPFEM)

Uncertainty Propagation through Inelastic System

- Incremental el–pl constitutive equation

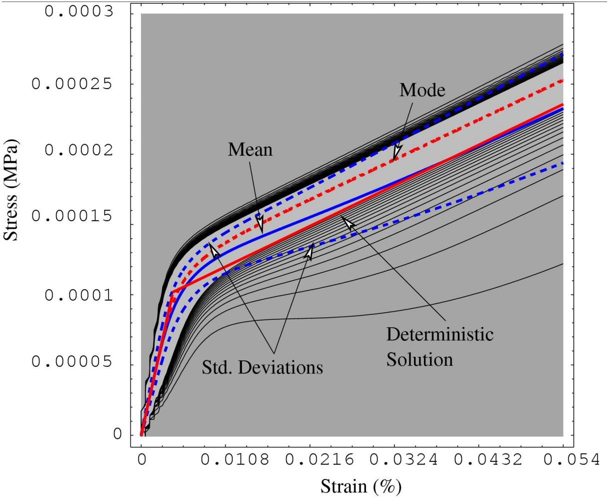
$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

- Dynamic Finite Elements

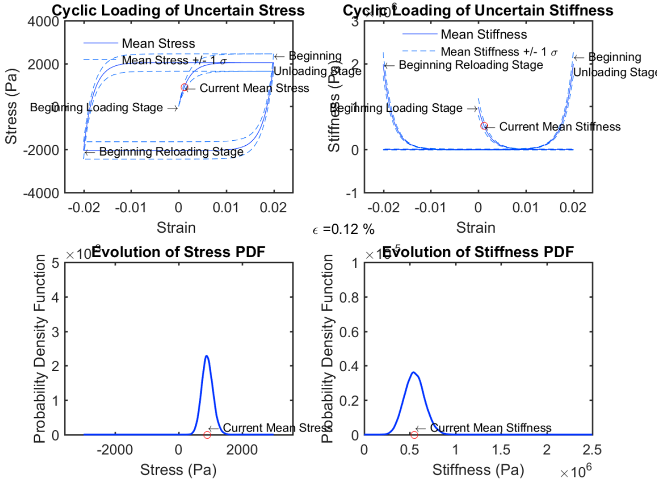
$$M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$$

- Material and load parameters are uncertain

Probabilistic Elastic-Plastic Response



Probabilistic Elastic-Plastic Modeling



Time Domain SEPFEM

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary)
  - Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients
- Output response process
  - Multi-dimensional Hermite PC with unknown coefficients
- Galerkin projection: minimize the error to compute unknown coefficients of response process
- Time integration using Newmark's method
  - Update coefficients following an elastic-plastic constitutive law at each time step

Polynomial Chaos Representation

Material random field: $D(x, \theta) = \sum_{i=1}^{P1} a_i(x)\Psi_i(\{\xi_r(\theta)\})$

Motion random process: $f_m(t, \theta) = \sum_{j=1}^{P_2} f_{mj}(t)\Psi_j(\{\xi_k(\theta)\})$

Displacement response: $u_n(t, \theta) = \sum_{k=1}^{P_3} d_{nk}(t)\Psi_k(\{\xi_l(\theta)\})$

$a_i(x), f_{mj}(t)$ are known PC coefficients,

$d_{nk}(t)$ are unknown PC coefficients, results.

Stochastic Elastic-Plastic Finite Element Method

- Material uncertainty expanded into stochastic shape funcs.
- Loading uncertainty expanded into stochastic shape funcs.
- Displacement expanded into stochastic shape funcs.

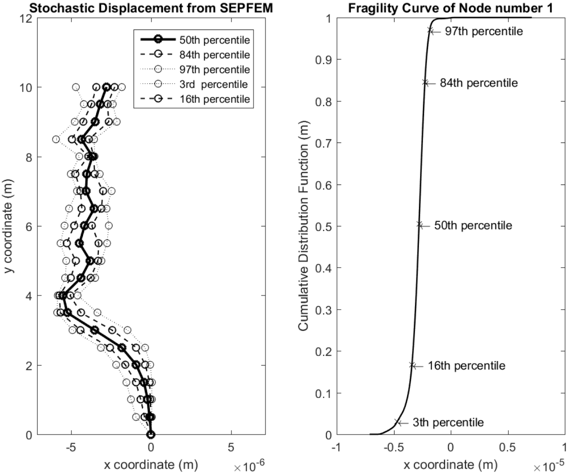
$$\begin{bmatrix} \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} \end{bmatrix} \begin{bmatrix} \Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\ \vdots \\ \sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i > \end{bmatrix}$$

SEPFEM: System Size

- SEPFEM offers a complete solution (single step)
- It is NOT based on Monte Carlo approach
- System of equations does grow (!)

# KL terms material	# KL terms load	PC order displacement	Total # terms per DoF
4	4	10	43,758
4	4	20	3,108,105
4	4	30	48,903,492
6	6	10	646,646
6	6	20	225,792,840
...	...	...	...

SEPFEM: Example in 1D



Outline

Summary

- Numerical modeling to predict and inform, rather than fit
- Analytic modeling and simulation of uncertain ESSI
- Engineer needs to know!
- François-Marie Arouet, Voltaire: "Le doute n'est pas une condition agréable, mais la certitude est absurde."
- Education and Training is the key!
- Real-ESSI short course this Fall!
- Funding from and collaboration with the US-DOE, US-NRC, US-NSF, Caltrans, US-BR, US-FEMA, CNSC-CCSN, UN-IAEA, Shimizu C. and ENSI/Basler&Hofmann is greatly appreciated,
- **`http://real-essi.info/`**