Uncertainties in Modeling and Simulation of Earthquakes, Soils, Structures and their Interaction

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Outline

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Introduction

Uncertain Inelastic Computational Mechanics

Applications

Summary

Motivation

Improve modeling and simulation for infrastructure objects

Reduction of modeling uncertainty

Choice of analysis level of sophistication

Account for parametric uncertainty

Goal: Predict and Inform rather than (force) fit

Engineer needs to know!

Numerical Prediction under Uncertainty

- Modeling Uncertainty, Simplifying assumptions

  Low, medium, high sophistication modeling and simulation

  Choice of sophistication level for confidence in results

- Parametric Uncertainty, $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$,

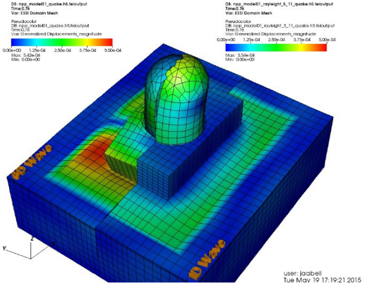
  Uncertain mass M, viscous damping C and stiffness K^{ep}

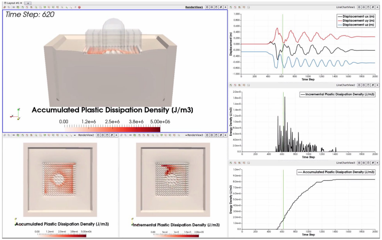
  Uncertain loads, $F(t)$

  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

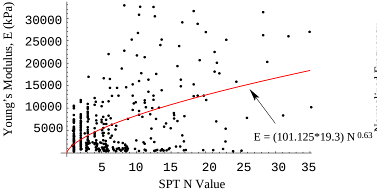
Modeling Uncertainty

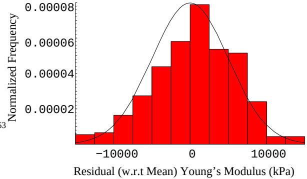
- Important (?!) features are simplified, 1C vs 3C, inelasticity
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that features being simplified out are less or not important



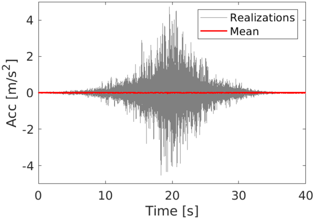


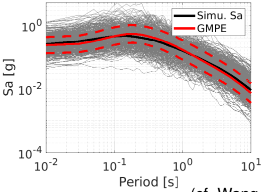
Parametric Uncertainty: Material and Motions





(cf. Phoon and Kulhawy (1999B))





(cf. Wang et al. (2019))

Real-ESSI Simulator System

The Real-ESSI, **Real**istic Modeling and Simulation of **E**arthquakes, **S**oils, **S**tructures and their **I**nteraction. Simulator is a software, hardware and documentation system for time domain, linear and nonlinear, inelastic, deterministic or probabilistic, 3D, modeling and simulation of:

- statics and dynamics of soil,
- statics and dynamics of structures,
- statics of soil-structure systems, and
- dynamics of earthquake-soil-structure system interaction

Used for:

- Design: linear elastic, load combinations, dimensioning
- Assessment: nonlinear/inelastic, risk, safety margins

Outline

Uncertainty Propagation through Inelastic System

- Incremental el–pl constitutive equation

$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

- Dynamic Finite Elements

$$M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$$

- Material and loads are uncertain

Previous Work

- Linear algebraic or differential equations:
  - Variable Transf. Method (Montgomery and Runger 2003)
  - Cumulant Expansion Method (Gardiner 2004)
- Nonlinear differential equations:
  - Monte Carlo Simulation (Schueller 1997, De Lima et al 2001, Mellah et al. 2000, Griffiths et al. 2005...)
    $\rightarrow$ can be accurate, very costly
  - Perturbation Method (Anders and Hori 2000, Kleiber and Hien 1992, Matthies et al. 1997)
    $\rightarrow$ first and second order Taylor series expansion about mean - limited to problems with small C.O.V. and inherits "closure problem"
  - SFEM (Ghanem and Spanos 1989, Matthies et al, 2004, 2005, 2014...)

3D Fokker-Planck-Kolmogorov Equation

$$
\begin{aligned}
\frac{\partial P(\sigma_{ij}(x_t,t),t)}{\partial t} &= \frac{\partial}{\partial \sigma_{mn}}\left[\left\{\left\langle \eta_{mn}(\sigma_{mn}(x_t,t), E_{mnrs}(x_t), \epsilon_{rs}(x_t,t))\right\rangle\right.\right. \\
&+ \int_0^t d\tau \, Cov_0\left[\frac{\partial \eta_{mn}(\sigma_{mn}(x_t,t), E_{mnrs}(x_t), \epsilon_{rs}(x_t,t))}{\partial \sigma_{ab}};\right. \\
&\left.\left.\left.\eta_{ab}(\sigma_{ab}(x_{t-\tau}, t-\tau), E_{abcd}(x_{t-\tau}), \epsilon_{cd}(x_{t-\tau}, t-\tau)\right]\right\} P(\sigma_{ij}(x_t,t),t)\right] \\
&+ \frac{\partial^2}{\partial \sigma_{mn}\partial \sigma_{ab}}\left[\left\{\int_0^t d\tau \, Cov_0\left[\eta_{mn}(\sigma_{mn}(x_t,t), E_{mnrs}(x_t), \epsilon_{rs}(x_t,t));\right.\right.\right. \\
&\left.\left.\left.\eta_{ab}(\sigma_{ab}(x_{t-\tau}, t-\tau), E_{abcd}(x_{t-\tau}), \epsilon_{cd}(x_{t-\tau}, t-\tau))\right]\right\} P(\sigma_{ij}(x_t,t),t)\right]
\end{aligned}
$$

(Jeremić et al. 2007)

FPK Equation

- Advection-diffusion equation

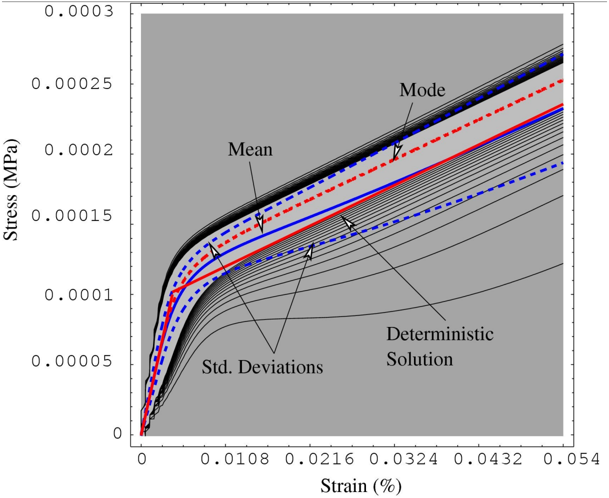
$$\frac{\partial P(\sigma,t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left[N_{(1)}P(\sigma,t) - \frac{\partial}{\partial \sigma}\left\{N_{(2)}P(\sigma,t)\right\}\right]$$

- Complete probabilistic description of response
- Solution PDF is second-order exact to covariance of time (exact mean and variance)
- It is deterministic equation in probability density space
- It is linear PDE in probability density space $\rightarrow$ simplifies the numerical solution process

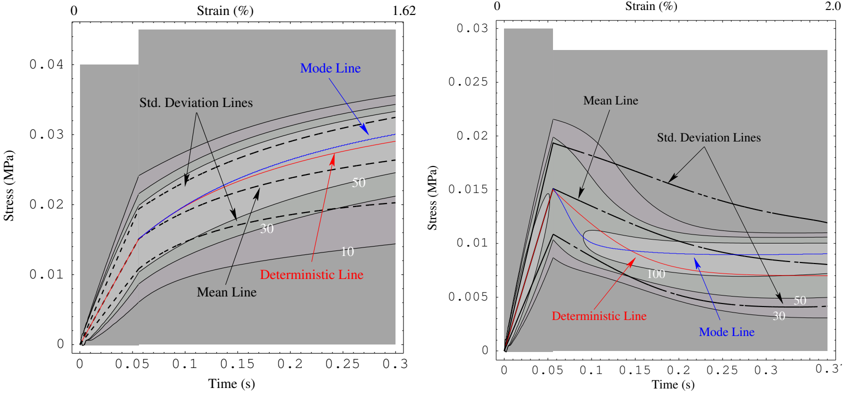
Template Solution of FPK Equation

- FPK diffusion–advection equation is applicable to any material model $\rightarrow$ only the coefficients $N_{(1)}$ and $N_{(2)}$ are different for different material models
- Initial condition
  - Deterministic $\rightarrow$ Dirac delta function $\rightarrow$ $P(\sigma, 0) = \delta(\sigma)$
  - Random $\rightarrow$ Any given distribution
- Boundary condition: Reflecting BC $\rightarrow$ conserves probability mass $\zeta(\sigma, t)|_{At\ Boundaries} = 0$
- Solve using finite differences and/or finite elements
- However (!!) it is a stress solution and probabilistic stiffness is an approximation!

Probabilistic Elastic-Plastic Response



Cam Clay with Random G, M and p_0



Time Domain Stochastic Galerkin Method

Dynamic Finite Elements $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary)

  Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients

- Output response process

  Multi-dimensional Hermite PC with unknown coefficients

- Galerkin projection: minimize the error to compute unknown coefficients of response process
- Time integration using Newmark's method

  Update coefficients following an elastic-plastic constitutive law at each time step

Polynomial Chaos Representation

Material random field:
$D(x,\theta) = \sum_{i=1}^{P1} a_i(x)\Psi_i(\{\xi_r(\theta)\})$

Seismic motions random process:
$f_m(t,\theta) = \sum_{j=1}^{P_2} f_{mj}(t)\Psi_j(\{\xi_k(\theta)\})$

Displacement response:
$u_n(t,\theta) = \sum_{k=1}^{P_3} d_{nk}(t)\Psi_k(\{\xi_l(\theta)\})$

where $a_i(x), f_{mj}(t)$ are known PC coefficients, while $d_{nk}(t)$ are unknown PC coefficients.

Direct Probabilistic Constitutive Solution in 1D

- Zero elastic region elasto-plasticity with stochastic Armstrong-Frederick kinematic hardening
  $\Delta\sigma = H_a \Delta\epsilon - c_r \sigma |\Delta\epsilon|; \quad E_t = d\sigma / d\epsilon = H_a \pm c_r \sigma$
- Uncertain: init. stiff. H_a, shear strength H_a / c_r, strain $\Delta\epsilon$:
  $H_a = \Sigma h_i \Phi_i; \quad C_r = \Sigma c_i \Phi_i; \quad \Delta\epsilon = \Sigma \Delta\epsilon_i \Phi_i$
- Resulting stress and stiffness are also uncertain

Direct Probabilistic Stiffness Solution

- Analytic product for all the components,

$$E^{EP}_{ijkl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right]$$

- Stiffness: each Polynomial Chaos component is updated incrementally

$$E^{n+1}_{t_1} = \frac{1}{<\Phi_1\Phi_1>}\{\sum_{i=1}^{P_h} h_i < \Phi_i\Phi_1 > \pm \sum_{j=1}^{P_c}\sum_{l=1}^{P_\sigma} c_j \sigma_l^{n+1} < \Phi_j\Phi_l\Phi_1 >\}$$

$$\vdots$$

$$E^{n+1}_{t_P} = \frac{1}{<\Phi_1\Phi_P>}\{\sum_{i=1}^{P_h} h_i < \Phi_i\Phi_P > \pm \sum_{j=1}^{P_c}\sum_{l=1}^{P_\sigma} c_j \sigma_l^{n+1} < \Phi_j\Phi_l\Phi_P >\}$$

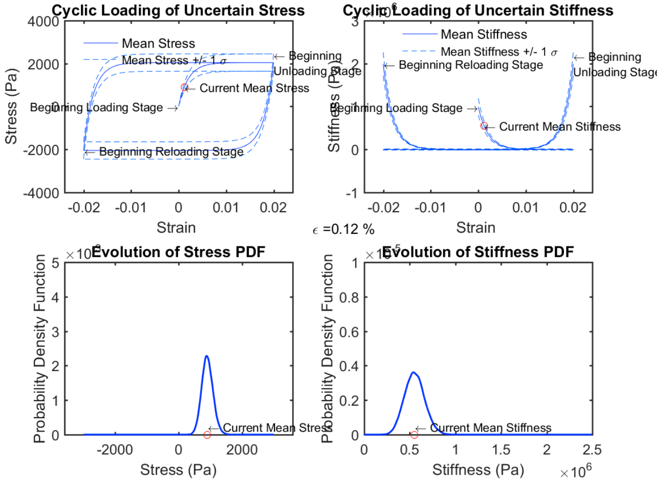
- Total stiffness is :

$$E^{n+1}_{t} = \sum_{l=1}^{P_E} E^{n+1}_{t_i}\Phi_i$$

Direct Probabilistic Stress Solution

- Analytic product, for each stress component,
  $\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl}$
- Incremental stress: each Polynomial Chaos component is updated incrementally
  $\Delta\sigma_1^{n+1} = \frac{1}{<\Phi_1\Phi_1>}\{\sum_{i=1}^{P_h}\sum_{k=1}^{P_e}\ h_i\Delta\epsilon_k^n < \Phi_i\Phi_k\Phi_1 >$
  $-\sum_{j=1}^{P_g}\sum_{k=1}^{P_e}\sum_{l=1}^{P_\sigma}\ c_j\Delta\epsilon_k^n\sigma_l^n < \Phi_j\Phi_k\Phi_l\Phi_1 >\}$
  $\vdots$
  $\Delta\sigma_P^{n+1} = \frac{1}{<\Phi_P\Phi_P>}\{\sum_{i=1}^{P_h}\sum_{k=1}^{P_e}\ h_i\Delta\epsilon_k^n < \Phi_i\Phi_k\Phi_P >$
  $-\sum_{j=1}^{P_g}\sum_{k=1}^{P_e}\sum_{l=1}^{P_\sigma}\ c_j\Delta\epsilon_k^n\sigma_l^n < \Phi_j\Phi_k\Phi_l\Phi_P >\}$
- Stress update:
  $\sum_{l=1}^{P_\sigma}\sigma_i^{n+1}\Phi_i = \sum_{l=1}^{P_\sigma}\sigma_i^n\Phi_i + \sum_{l=1}^{P_\sigma}\Delta\sigma_i^{n+1}\Phi_i$

Probabilistic Elastic-Plastic Response



(MP4)

Stochastic Elastic-Plastic Finite Element Method

- Material uncertainty expanded into stochastic shape funcs.
- Loading uncertainty expanded into stochastic shape funcs.
- Displacement expanded into stochastic shape funcs.
- Jeremić et al. 2011

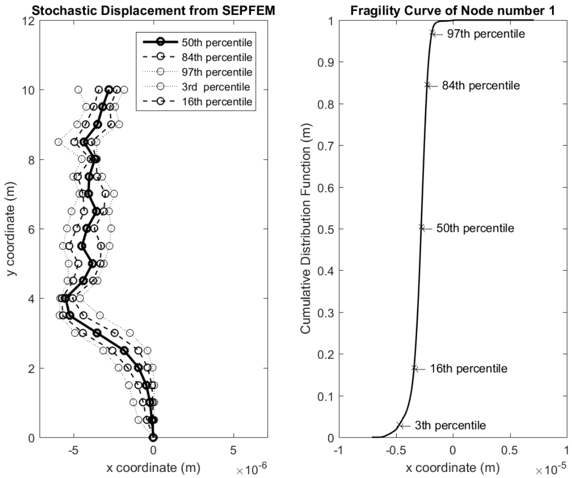
$$\begin{bmatrix} \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} \end{bmatrix} \begin{bmatrix} \Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\ \vdots \\ \sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i > \end{bmatrix}$$

SEPFEM: System Size

- SEPFEM offers a complete solution (single step)
- It is NOT based on Monte Carlo approach
- System of equations does grow (!)

# KL terms material	# KL terms load	PC order displacement	Total # terms per DoF
4	4	10	43758
4	4	20	3 108 105
4	4	30	48 903 492
6	6	10	646 646
6	6	20	225 792 840
6	6	30	$1.1058\ 10^{10}$
8	8	10	5 311 735
8	8	20	$7.3079\ 10^{9}$
8	8	30	$9.9149\ 10^{11}$
...	...	...	...

SEPFEM: Example in 1D



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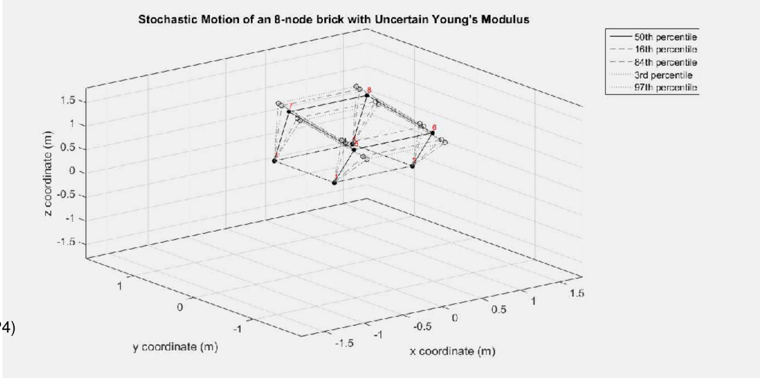
SEPFEM: Example in 3D



(MP4)

Outline

Current State of Art Seismic Risk Analysis (SRA)

- Intensity measure (IM) selected as a proxy for ground motions, usually Spectral acceleration $Sa(T_0)$
- Ground Motion Prediction Equations (GMPEs) need development, ergodic or site specific
- Probabilistic seismic hazard analysis (PSHA)
- Fragility analysis $P(EDP > x|IM = z)$, deterministic time domain FEM, Monte Carlo (MC)

Seismic Risk Analysis Challenges

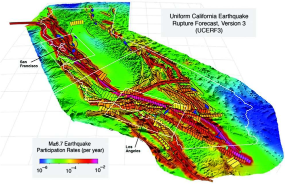
- Miscommunication between seismologists and structural engineers, $Sa(T_0)$ not compatible with nonlinear FEM
- IMs difficult to choose, Spectral Acc, PGA, PGV...
- Single IM does not contain all/most uncertainty
- Monte Carlo, not accurate enough for tails
- Monte Carlo, computationally expensive, CyberShake for LA, 20,000 cases, 100y runtime, (Maechling et al. 2007)

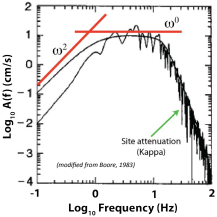
Time Domain Intrusive SRA Framework

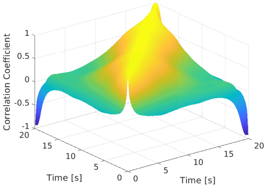
- Stochastic Elastic-Plastic Finite Element Method, SEPFEM, $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$,
- Uncertain seismic loads, from uncertain seismic motions, using Domain Reduction Method
- Uncertain elastic-plastic material, stress and stiffness solution using Forward Kolmogorov, Fokker-Planck equation
- Results, probability distribution functions for σ_{ij}, ϵ_{ij}, u_i...

Stochastic Seismic Motion Development

- UCERF3 (Field et al. 2014)
- Stochastic motions (Boore 2003)
- Polynomial Chaos Karhunen-Loève expansion
- Domain Reduction Method for P_{eff} (Bielak et al. 2003)



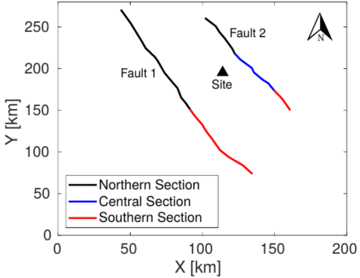




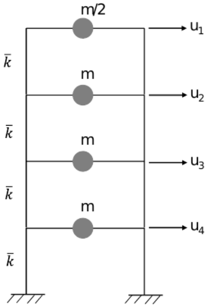
Stochastic Ground Motion Modeling

- Modeling fundamental characteristics of uncertain ground motions, Stochastic Fourier amplitude spectra (FAS). and Stochastic Fourier phase spectra (FPS) and not specific IM
- Mean behavior of stochastic FAS, w^2 source radiation spectrum by Brune(1970), and Boore(1983, 2003, 2015).
- Variability models for stochastic FAS, FAS GMPEs by Bora et al. (2015, 2018), Bayless & Abrahamson (2019), Stafford(2017) and Bayless & Abrahamson (2018).
- Stochastic FPS by phase derivative (Boore,2005), Logistic phase derivative model by Baglio & Abrahamson (2017)

Uncertain Model Description



(a) Faults configuration



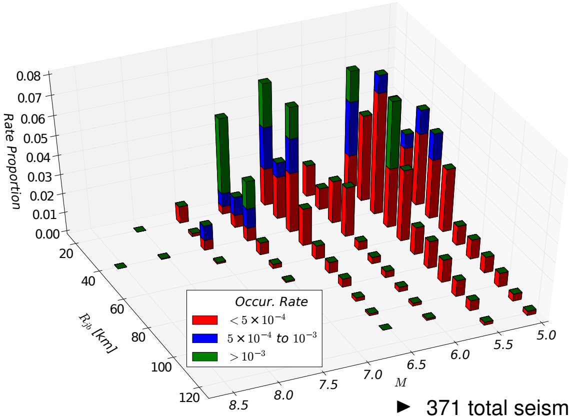
(b) Building structure

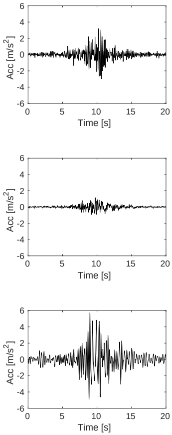
$$Cov_k = \begin{bmatrix} 1.0 & 0.6 & 0.3 & 0.2 \\ 0.6 & 1.0 & 0.5 & 0.2 \\ 0.3 & 0.5 & 1.0 & 0.7 \\ 0.2 & 0.2 & 0.7 & 1.0 \end{bmatrix}$$

- Fault 1: San Gregorio fault
- Fault 2: Calaveras fault
- Uncertainty: Segmentation, slip rate, rupture geometry, etc.

- $Vs_{30} = 620 m/s$
- $m = 100 kips/g$
- $\bar{k} = 168 kip/in$

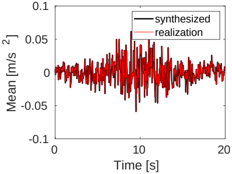
Seismic Source Characterization



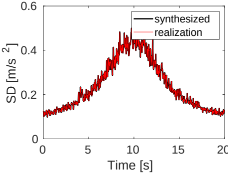


- 371 total seismic scenarios
- M 5 ∼ 5.5 and 6.5 ∼ 7.0
- R_{jb} 20km ∼ 40km

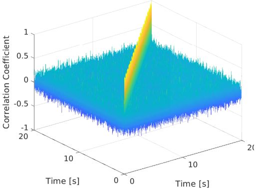
Stochastic Ground Representation



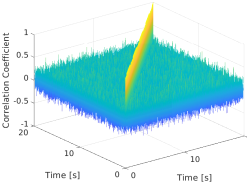
Acc. marginal mean



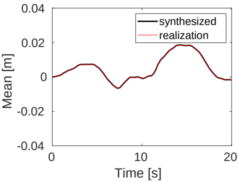
Acc. marginal S.D.



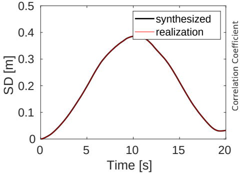
Acc. realization Cov.



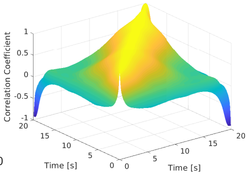
Acc. synthesized Cov.



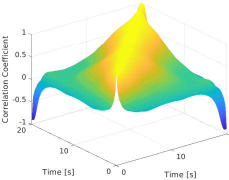
Dis. marginal mean



Dis. marginal S.D.

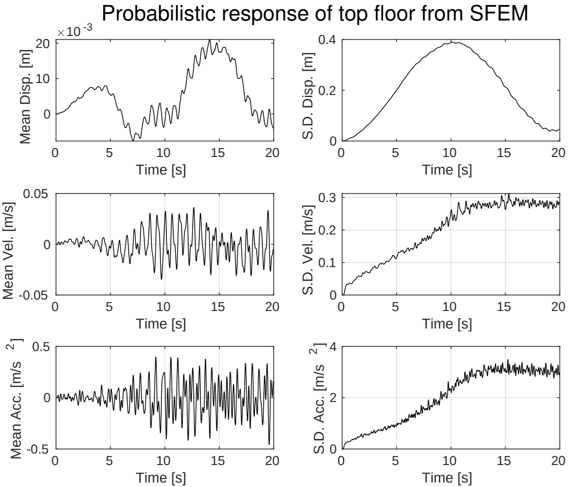


Dis. realization Cov.



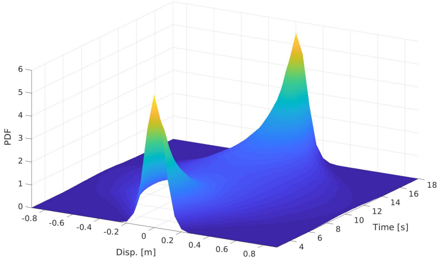
Dis. synthesized Cov.

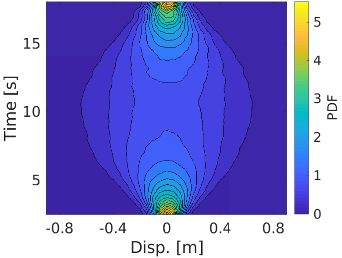
Probabilistic Dynamic Response



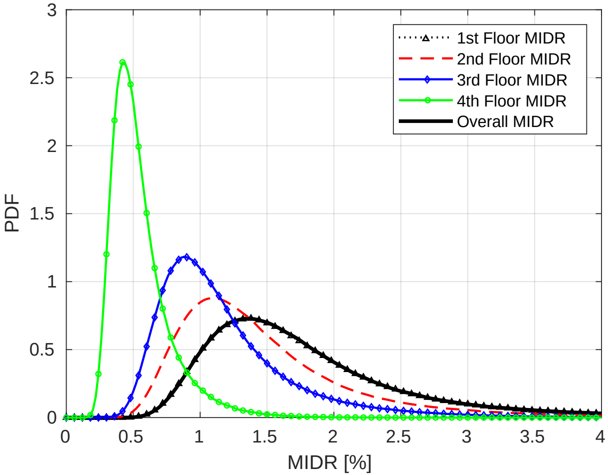
Probabilistic Dynamic Response

Probabilistic density of displacements evolution of top floor

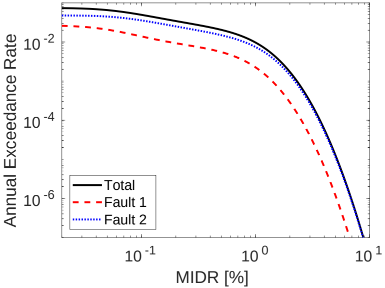


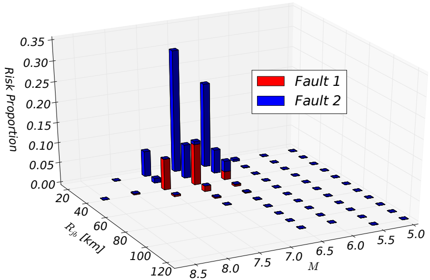


Maximum Inter-Story Drift Ratio (MIDR)



Seismic Risk Analysis



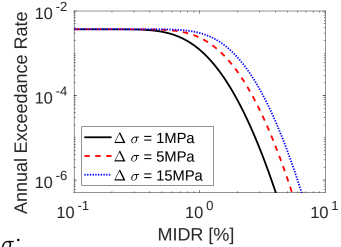


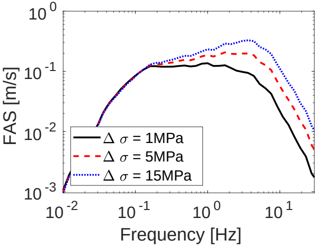
$\lambda(MIDR > 1\%) = 9.7 \times 10^{-3}$
$\lambda(MIDR > 2\%) = 1.7 \times 10^{-3}$
$\lambda(MIDR > 4\%) = 5.9 \times 10^{-5}$

Risk de-aggregation for $\lambda(MIDR > 1\%)$

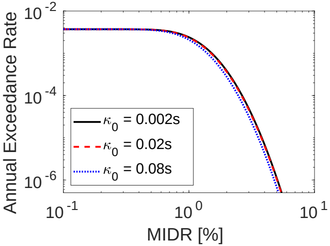
Sensitivity Study

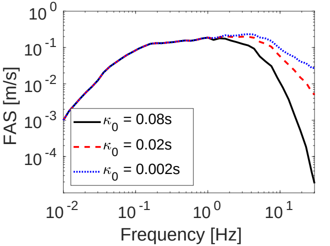
Source $\Delta\sigma$:





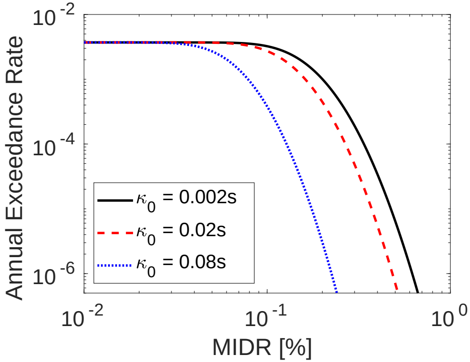
Site κ_0:



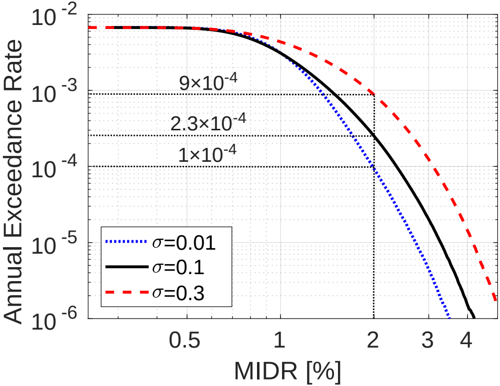


Sensitivity Study

Fundamental frequency *f* increases from 1.6Hz to 8Hz:



Seismic Risk, Uncertain Material



Outline

Appropriate Science Quotes

- François-Marie Arouet, Voltaire: "Le doute n’est pas une condition agréable, mais la certitude est absurde."
- Niklaus Wirth: "Software is getting slower more rapidly than hardware becomes faster."

Summary

- Numerical modeling to predict and inform, rather than fit
- Sophisticated inelastic/nonlinear determinstic/probabilistic modeling and simulations needs to be done carefully and in phases
- Education and Training is the key!
- Collaborators: Feng, Yang, Behbehani, Sinha, Wang, Karapiperis, Wang, Lacoure, Pisanó, Abell, Tafazzoli, Jie, Preisig, Tasiopoulou, Watanabe, Cheng, Yang.
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- `http://sokocalo.engr.ucdavis.edu/~jeremic`