Uncertainties in Modeling and Simulation of Earthquakes, Soils, Structures and their Interaction

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Outline

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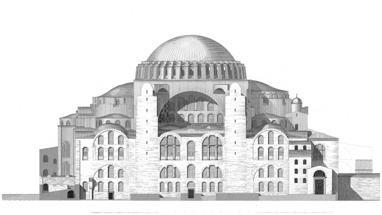
Motivation

- Improve modeling and simulation for infrastructure objects
- Control of modeling, epistemic uncertainty
- Propagate parametric, aleatory uncertainty
- Goal: predict and inform
- Engineer needs to know!
- Design sustainable objects



Numerical Prediction under Uncertainty

- Modeling, Epistemic Uncertainty, simplifying assumptions

  Low, medium, high sophistication modeling and simulation

  Choice of sophistication level for confidence in results

- Parametric, Aleatory Uncertainty, $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$,

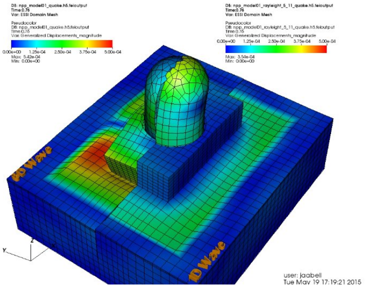
  Uncertain mass M, viscous damping C and stiffness K^{ep}

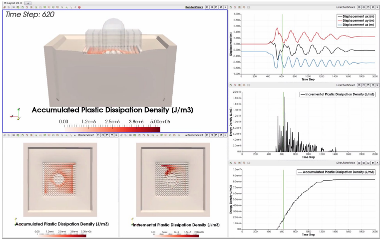
  Uncertain loads, $F(t)$

  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

Modeling, Epistemic Uncertainty

- Important (?!) features are simplified, 1C vs 3C, inelasticity
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that features being simplified out are less or not important





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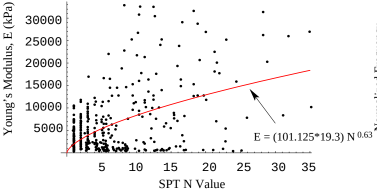
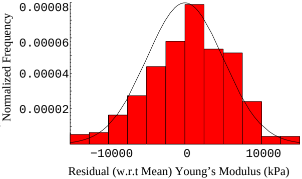
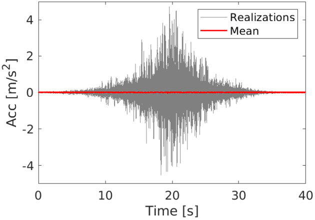
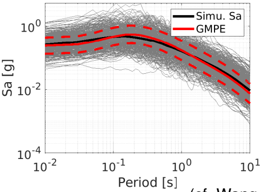
Parametric, Aleatory Uncertainty





(cf. Phoon and Kulhawy (1999B))





(cf. Wang et al. (2019))

Outline

Real-ESSI Simulator System

The Real-ESSI, **Real**istic Modeling and Simulation of **E**arthquakes, **S**oils, **S**tructures and their **I**nteraction Simulator is a software, hardware and documentation system for time domain, linear and nonlinear, elastic and inelastic, deterministic or probabilistic, 3D, modeling and simulation of:

- statics and dynamics of soil,
- statics and dynamics of rock,
- statics and dynamics of structures,
- statics of soil-structure systems, and
- dynamics of earthquake-soil-structure system interaction

Used for:

- Design, linear elastic, load combinations, dimensioning
- Assessment, nonlinear/inelastic, safety margins

Real-ESSI Simulator System

- Real-ESSI System Components
  - Real-ESSI Pre-processor (gmsh/gmESSI, X2ESSI)
  - Real-ESSI Program (local, remote, cloud)
  - Real-ESSI Post-Processor (Paraview/pvESSI, Python)
- Real-ESSI System availability: Windows/iOS/Linux docker, Linux Executables, AWS
- Real-ESSI education and training: theory and applications
- Real-ESSI documentation and program available at `http://real-essi.us/`

Real-ESSI Modeling Features

- Solid elements: dry, un-/fully-saturated, elastic, inelastic
- Structural elements: beams, shells, elastic, inelastic
- Contact/interface/joint elements: bonded, shear/frictional (EPP, EPH, EPS); gap/normal; linear, nonlinear, dry, coupled/saturated,
- Super element: stiffness and mass matrices
- Material models: soil, rock, concrete, steel...
- Seismic input: 1C and 3C/6C, deterministic or probabilistic
- Energy dissipation calculation: elastic-plastic, viscous, algorithmic
- Solid/Structure-Fluid interaction, full coupling, OpenFOAM
- Intrusive probabilistic inelastic modeling

Real ESSI Simulator: Domain Specific Language, DSL

Domain Specific Language (DSL), Yacc & Lex

English like modeling and simulation language

Parser and compiler, can define functions, models, etc.

Can extend models and methods

Requires units!

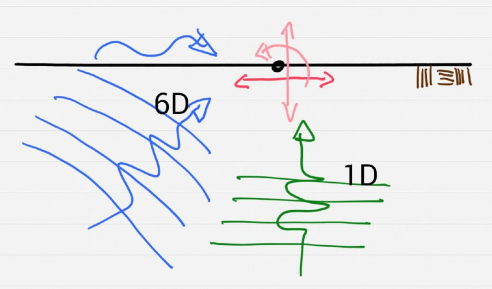
Outline

Earthquake Ground Motions

- Real earthquake ground motions
  - Body, P and S waves
  - Surface, Rayleigh and Love waves
  - Lack of correlation, incoherent motions
  - Inclined seismic waves
  - 3D, 3C/6C waves
- What are the effects of real earthquake ground motions on soil-structure systems ?!

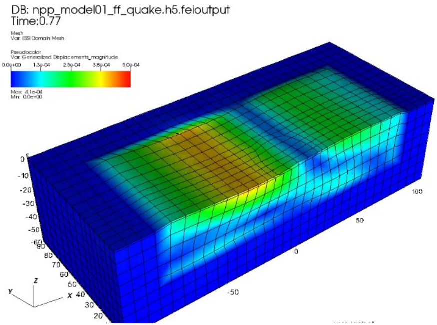
ESSI: 6C or 1C Seismic Motions

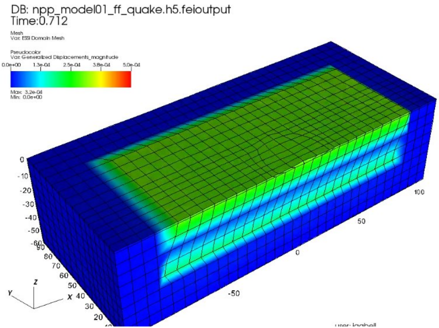
- Assume that a full 6C (3C) motions at the surface are only recorded in one horizontal direction
- From such recorded motions one can develop a vertically propagating shear wave (1C) in 1D
- Apply such vertically propagating shear wave to same soil-structure system



1C vs 6C Free Field Motions

- One component of motions, 1C from 6C
- Excellent fit





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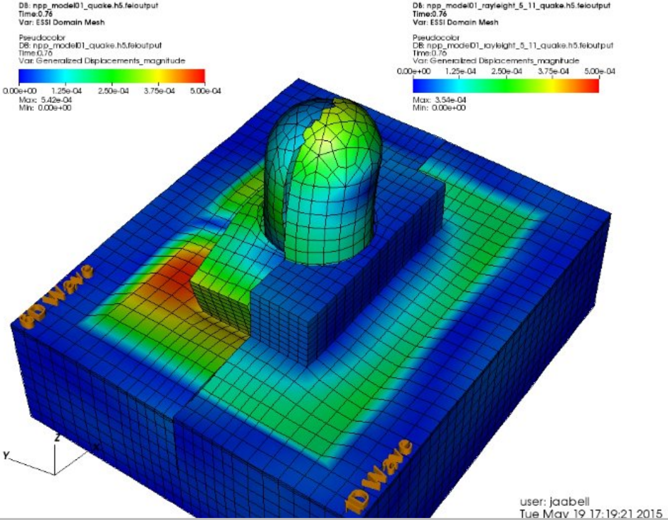
6C vs 1C NPP ESSI Response Comparison



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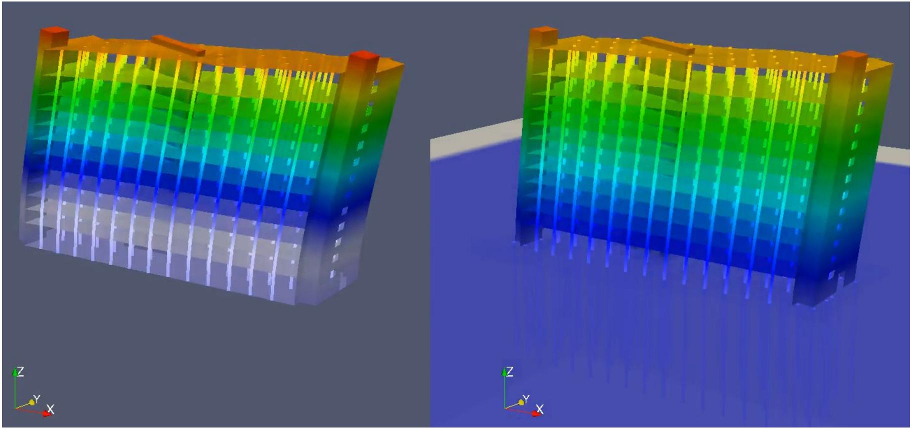
Ventura Hotel, Northridge Earthquake, nonSSI vs SSI



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Outline

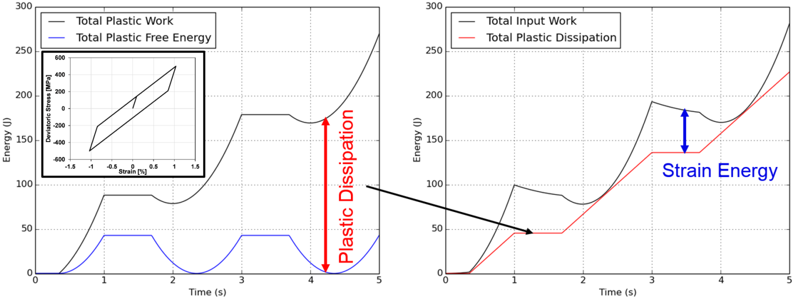
Energy Input and Dissipation

- Energy input, dynamic forcing
- Energy dissipation outside SSI domain:
  - SSI system oscillation radiation
  - Reflected wave radiation
- Energy dissipation/conversion inside SSI domain:
  - Inelasticity of soil, interfaces, structure, dissipators
  - Viscous coupling with internal/pore, and external fluids
- Numerical energy dissipation/production

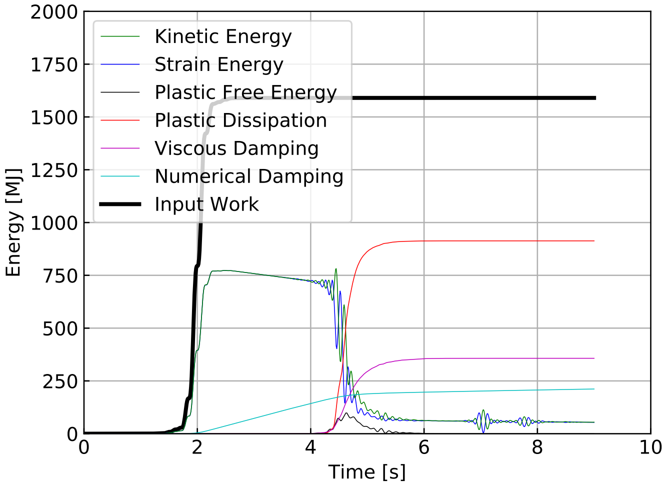
Plastic Energy Dissipation

Single elastic-plastic element under cyclic shear loading

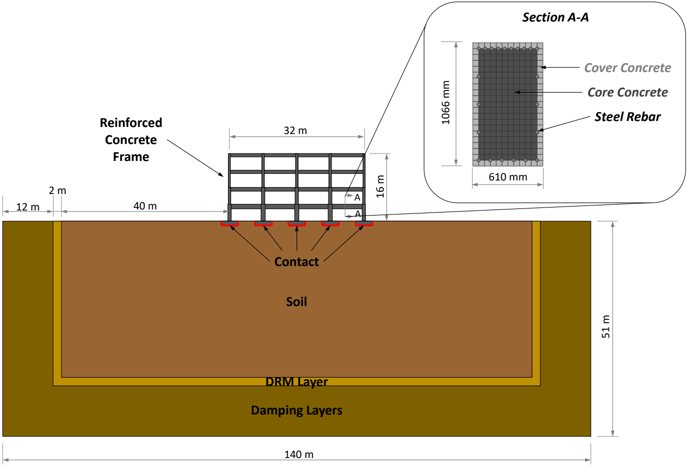
- Difference between plastic work and plastic dissipation
- Plastic work can decrease
- Plastic dissipation always increases



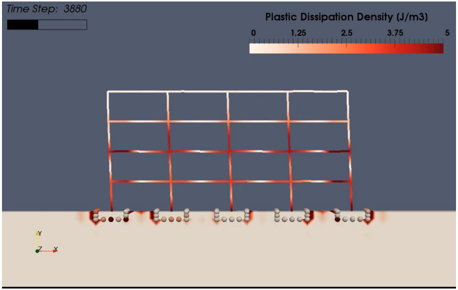
Energy Dissipation Control



Energy Dissipation for Design



Design Alternatives



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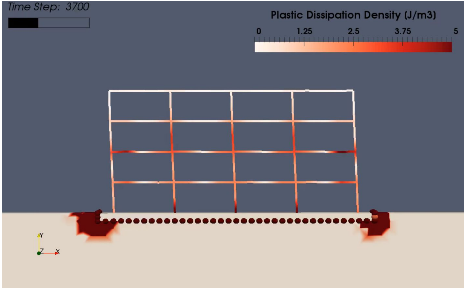

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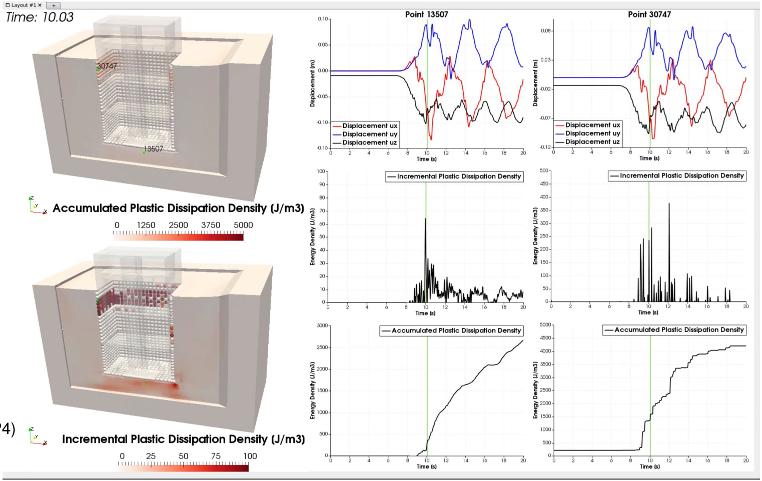
Energy Dissipation for an SMR Model



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Outline

Uncertainty Propagation through Inelastic System

- Incremental el–pl constitutive equation

$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

- Dynamic Finite Elements

$$M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$$

- Material and loads are uncertain

Previous Work

- Linear algebraic or differential equations:
  - Variable Transf. Method (Montgomery and Runger 2003)
  - Cumulant Expansion Method (Gardiner 2004)
- Nonlinear differential equations:
  - Monte Carlo Simulation (Schueller 1997, De Lima et al 2001, Mellah et al. 2000, Griffiths et al. 2005...)
    $\rightarrow$ can be accurate, very costly
  - Perturbation Method (Anders and Hori 2000, Kleiber and Hien 1992, Matthies et al. 1997)
    $\rightarrow$ first and second order Taylor series expansion about mean - limited to problems with small C.O.V. and inherits "closure problem"
  - SFEM (Ghanem and Spanos 1989, Matthies et al, 2004, 2005, 2014...)

3D Fokker-Planck-Kolmogorov Equation

$$
\begin{aligned}
\frac{\partial P(\sigma_{ij}(x_t,t),t)}{\partial t} &= \frac{\partial}{\partial \sigma_{mn}}\left[\left\{\left\langle \eta_{mn}(\sigma_{mn}(x_t,t), E_{mnrs}(x_t), \epsilon_{rs}(x_t,t))\right\rangle\right.\right.\\
&+ \int_0^t d\tau Cov_0\left[\frac{\partial \eta_{mn}(\sigma_{mn}(x_t,t), E_{mnrs}(x_t), \epsilon_{rs}(x_t,t))}{\partial \sigma_{ab}};\right.\\
&\left.\left.\left.\eta_{ab}(\sigma_{ab}(x_{t-\tau}, t-\tau), E_{abcd}(x_{t-\tau}), \epsilon_{cd}(x_{t-\tau}, t-\tau)\right]\right\} P(\sigma_{ij}(x_t,t),t)\right]\\
&+ \frac{\partial^2}{\partial \sigma_{mn}\partial \sigma_{ab}}\left[\left\{\int_0^t d\tau Cov_0\left[\eta_{mn}(\sigma_{mn}(x_t,t), E_{mnrs}(x_t), \epsilon_{rs}(x_t,t));\right.\right.\right.\\
&\left.\left.\left.\eta_{ab}(\sigma_{ab}(x_{t-\tau}, t-\tau), E_{abcd}(x_{t-\tau}), \epsilon_{cd}(x_{t-\tau}, t-\tau))\right]\right\} P(\sigma_{ij}(x_t,t),t)\right]
\end{aligned}
$$

(Jeremić et al. 2007)

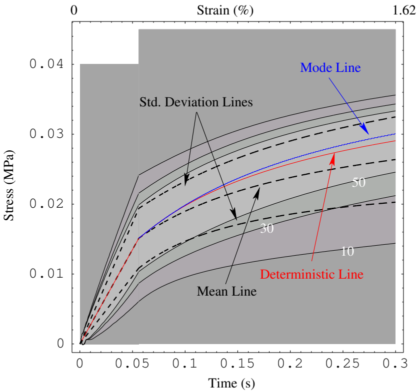
FPK Equation

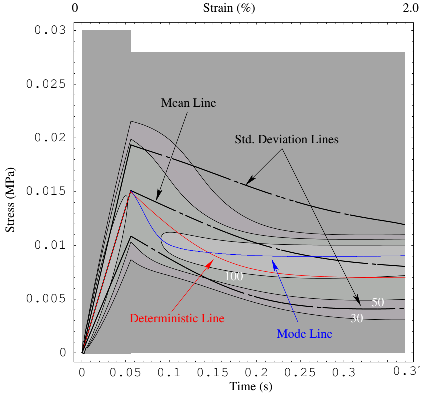
- Advection-diffusion equation

$$\frac{\partial P(\sigma,t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left[N_{(1)}P(\sigma,t) - \frac{\partial}{\partial \sigma}\left\{N_{(2)}P(\sigma,t)\right\}\right]$$

- Complete probabilistic description of response
- Solution PDF is second-order exact to covariance of time (exact mean and variance)
- It is deterministic equation in probability density space
- It is linear PDE in probability density space $\rightarrow$ simplifies the numerical solution process

Cam Clay with Random G, M and p_0





Time Domain Stochastic Galerkin Method

Dynamic Finite Elements $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary)

  Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients

- Output response process

  Multi-dimensional Hermite PC with unknown coefficients

- Galerkin projection: minimize the error to compute unknown coefficients of response process
- Time integration using Newmark's method

  Update coefficients following an elastic-plastic constitutive law at each time step

Polynomial Chaos Representation

Material random field:
$D(x,\theta) = \sum_{i=1}^{P1} a_i(x)\Psi_i(\{\xi_r(\theta)\})$

Seismic motions random process:
$f_m(t,\theta) = \sum_{j=1}^{P_2} f_{mj}(t)\Psi_j(\{\xi_k(\theta)\})$

Displacement response:
$u_n(t,\theta) = \sum_{k=1}^{P_3} d_{nk}(t)\Psi_k(\{\xi_l(\theta)\})$

where $a_i(x), f_{mj}(t)$ are known PC coefficients, while $d_{nk}(t)$ are unknown PC coefficients.

Direct Probabilistic Constitutive Solution in 1D

- Zero elastic region elasto-plasticity with stochastic Armstrong-Frederick kinematic hardening
  $\Delta\sigma = H_a\Delta\epsilon - c_r\sigma|\Delta\epsilon|; \quad E_t = d\sigma/d\epsilon = H_a \pm c_r\sigma$
- Uncertain: init. stiff. H_a, shear strength H_a/c_r, strain $\Delta\epsilon$:
  $H_a = \Sigma h_i\Phi_i; \quad C_r = \Sigma c_i\Phi_i; \quad \Delta\epsilon = \Sigma\Delta\epsilon_i\Phi_i$
- Resulting stress and stiffness are also uncertain

Direct Probabilistic Stiffness Solution

- Analytic product for all the components,
  $$E^{EP}_{ijkl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right]$$
- Stiffness: each Polynomial Chaos component is updated incrementally
  $E^{n+1}_{t_1} = \frac{1}{<\Phi_1 \Phi_1>} \{ \sum_{i=1}^{P_h} \; h_i < \Phi_i \Phi_1 > \pm \sum_{j=1}^{P_c} \sum_{l=1}^{P_\sigma} \; c_j \sigma_l^{n+1} < \Phi_j \Phi_l \Phi_1 > \}$

  $\vdots$

  $E^{n+1}_{t_P} = \frac{1}{<\Phi_1 \Phi_P>} \{ \sum_{i=1}^{P_h} \; h_i < \Phi_i \Phi_P > \pm \sum_{j=1}^{P_c} \sum_{l=1}^{P_\sigma} \; c_j \sigma_l^{n+1} < \Phi_j \Phi_l \Phi_P > \}$
- Total stiffness is :
  $E^{n+1}_{t} = \sum_{l=1}^{P_E} E^{n+1}_{t_i} \Phi_i$

Direct Probabilistic Stress Solution

- Analytic product, for each stress component, $\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl}$
- Incremental stress: each Polynomial Chaos component is updated incrementally
$$\Delta\sigma_1^{n+1} = \frac{1}{<\Phi_1\Phi_1>}\{\sum_{i=1}^{P_h}\sum_{k=1}^{P_e}\ h_i\Delta\epsilon_k^n < \Phi_i\Phi_k\Phi_1 > - \sum_{j=1}^{P_g}\sum_{k=1}^{P_e}\sum_{l=1}^{P_\sigma}\ c_j\Delta\epsilon_k^n\sigma_l^n < \Phi_j\Phi_k\Phi_l\Phi_1 >\}$$
$$\vdots$$
$$\Delta\sigma_P^{n+1} = \frac{1}{<\Phi_P\Phi_P>}\{\sum_{i=1}^{P_h}\sum_{k=1}^{P_e}\ h_i\Delta\epsilon_k^n < \Phi_i\Phi_k\Phi_P > - \sum_{j=1}^{P_g}\sum_{k=1}^{P_e}\sum_{l=1}^{P_\sigma}\ c_j\Delta\epsilon_k^n\sigma_l^n < \Phi_j\Phi_k\Phi_l\Phi_P >\}$$
- Stress update:
$$\sum_{l=1}^{P_\sigma}\sigma_i^{n+1}\Phi_i = \sum_{l=1}^{P_\sigma}\sigma_i^n\Phi_i + \sum_{l=1}^{P_\sigma}\Delta\sigma_i^{n+1}\Phi_i$$

Stochastic Elastic-Plastic Finite Element Method

Material uncertainty expanded into stochastic shape funcs.

Loading uncertainty expanded into stochastic shape funcs.

Displacement expanded into stochastic shape funcs.

Jeremić et al. 2011

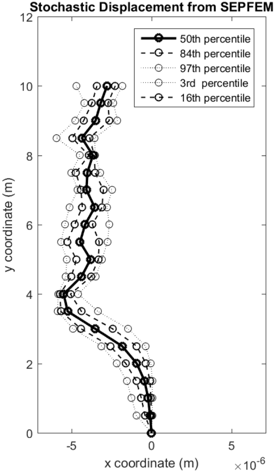
$$\begin{bmatrix} \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} \end{bmatrix} \begin{bmatrix} \Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\ \vdots \\ \sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i > \end{bmatrix}$$

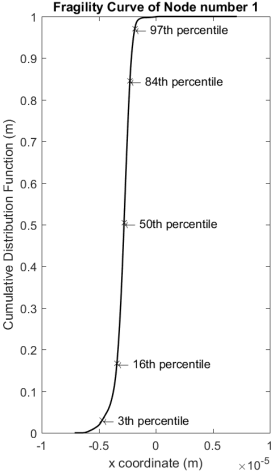
SEPFEM: System Size

- SEPFEM offers a complete solution (single step)
- It is NOT based on Monte Carlo approach
- System of equations does grow (!)

# KL terms material	# KL terms load	PC order displacement	Total # terms per DoF
4	4	10	43758
4	4	20	3 108 105
4	4	30	48 903 492
6	6	10	646 646
6	6	20	225 792 840
6	6	30	$1.1058\ 10^{10}$
8	8	10	5 311 735
8	8	20	$7.3079\ 10^{9}$
8	8	30	$9.9149\ 10^{11}$
...	...	...	...

SEPFEM: Example in 1D





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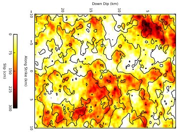
Outline

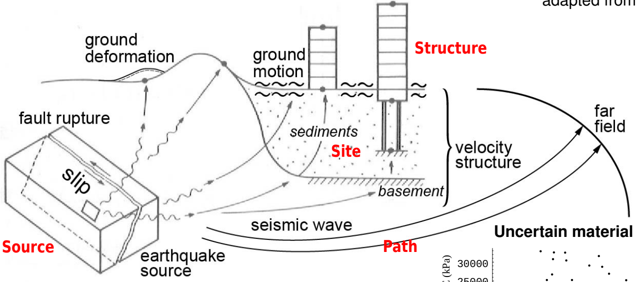
Probabilistic Seismic Risk Analysis (PSRA)

Uncertain source, path, site and structure $\Longrightarrow$ Probabilities of engineering demand parameters (EDP) damage measures (DM), loss, etc

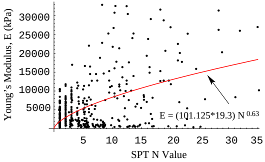
adapted from Taga(1982)



Uncertain source rupture



Uncertain material properties



Probabilistic Seismic Risk Analysis

- Objective, quantitative decision making based on exceedance rate $\lambda(EDP > z)$
- PSRA: convolution of PSHA and fragility

$$\lambda(EDP > z) = \int \underbrace{\left|\frac{d\lambda(IM > x)}{dx}\right|}_{\textbf{PSHA}} \underbrace{G(EDP > z|IM = x)}_{\textbf{fragility analysis}} dx$$

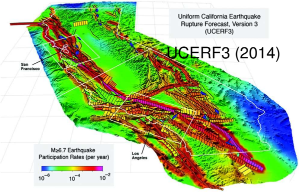
$\lambda(\cdot)$: rate of exceedance
EDP: engineering demand parameter
PSHA: probabilistic seismic hazard analysis
IM: intensity measure

Seismic Risk Analysis Challenges

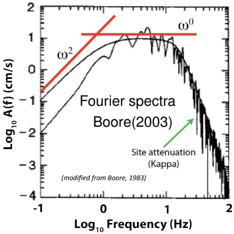
- Intensity Measure(s) (IM) serves as the proxy of damaging ground motions
- Does a single IM, e.g., $Sa(T_0)$, represent all uncertainty?
- IMs difficult to choose, Spectral Acc, PGA, PGV...
- Fragility analysis: incremental dynamic analysis (IDA)
- Use of Monte Carlo method, accuracy, efficiency...
- Monte Carlo, computationally expensive, CyberShake for LA, 20,000 cases, 100Y runtime, (Maechling et al. 2007)

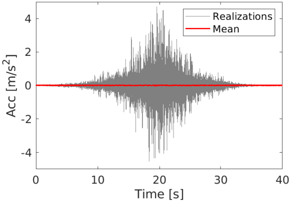
Risk Framework

Seismic source characterization



Stochastic ground motion



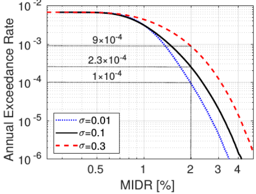


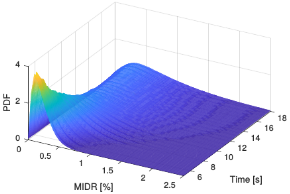
$$\lambda(EDP > z) = \sum N_i(M_i, R_i) P(EDP > z | M_i, R_i)$$

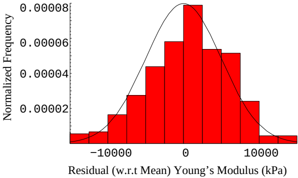
Uncertainty propagation SEPFEM

Uncertainty characterization Hermite polynomial chaos

EDP hazard/risk





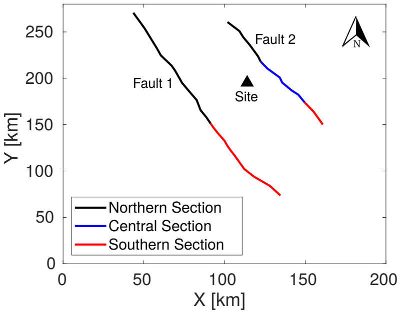


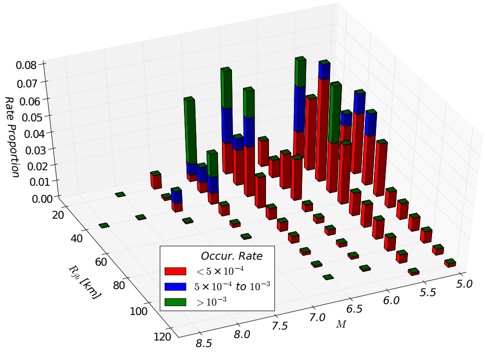
Stochastic Ground Motion Modeling

- Shift from modeling specific IM to fundamental characteristics of ground motions
  - Uncertain Fourier amplitude spectra (FAS)
  - Uncertain Fourier phase spectra (FPS)

- GMPE studies of FAS, (*Bora et al. (2018)*, *Bayless & Abrahamson (2018,2019)*, *Stafford(2017)*,)
- Stochastic FPS by phase derivative (Boore,2005) (Logistic phase derivative model by *Baglio & Abrahamson (2017)*)
- Near future change from $\boldsymbol{Sa}(\boldsymbol{T_0})$ to **FAS**

Risk Example



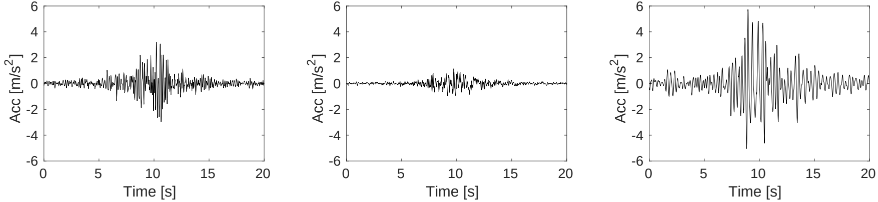


- Fault 1: San Gregorio fault
- Fault 2: Calaveras fault
- Uncertainty: Segmentation, slip rate, rupture geometry, etc.

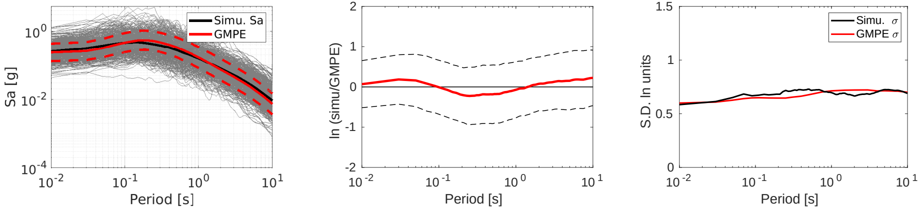
- 371 total seismic scenarios
- M 5 $\sim$ 5.5 and 6.5 $\sim$ 7.0
- R_{jb} 20km $\sim$ 40km

Stochastic Ground Motion Modeling

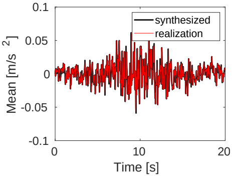
Realizations of simulated uncertain motions for scenario $M = 7$, $R = 15km$:



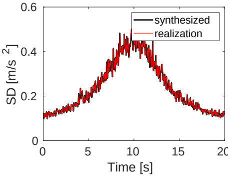
Verification with GMPE:



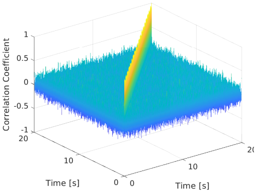
Stochastic Ground Motion Characterization



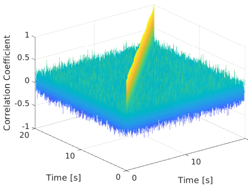
Acc. marginal mean



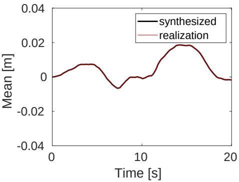
Acc. marginal S.D.



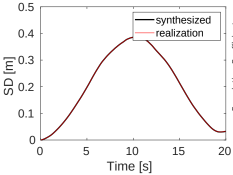
Acc. realization Cov.



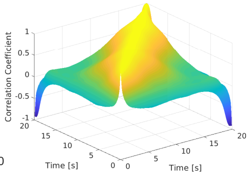
Acc. synthesized Cov.



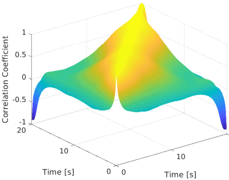
Dis. marginal mean



Dis. marginal S.D.

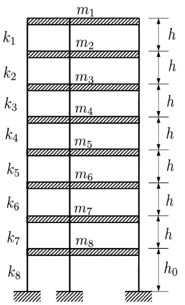


Dis. realization Cov.

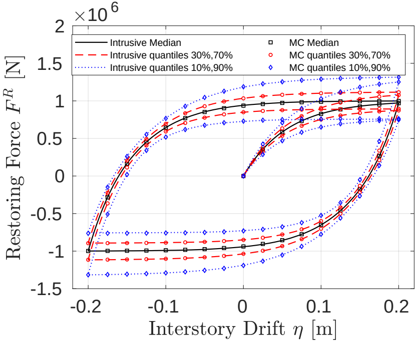


Dis. synthesized Cov.

Stochastic Material Modeling

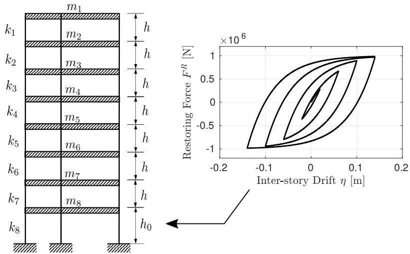


(a) Frame

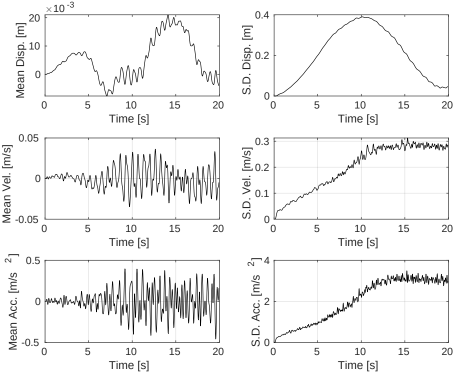


(b) Interstory response

Probabilistic Dynamic Structural Response



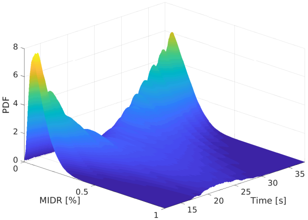
Probabilistic response of top floor from SFEM

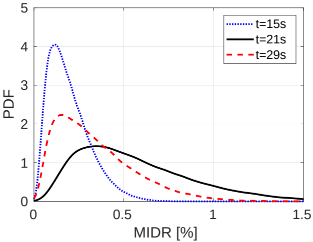


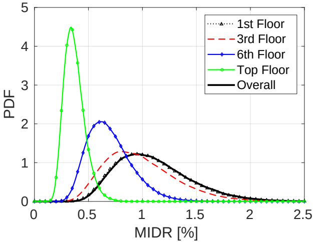
- Coefficient of variation 15% for H_a and C_r
- Time domain stochastic El-Pl FEM analysis (SEPFEM)

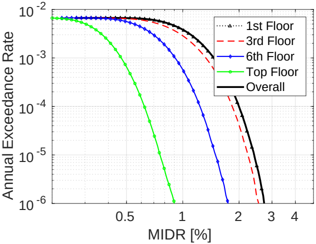
Seismic Risk Analysis

Engineering demand parameter (EDP): Maximum inter-story drift ratio (MIDR)



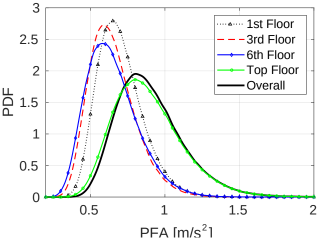


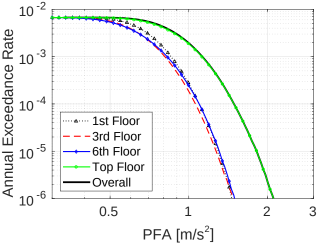


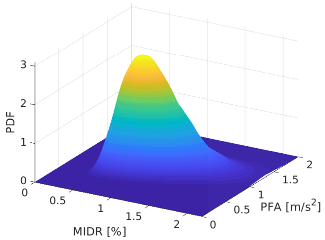


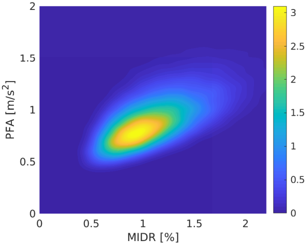
Seismic Risk Analysis

Engineering demand parameter (EDP): Peak floor acceleration (PFA)









Seismic Risk Analysis

- Damage measure (DM) defined on multiple EDPs:

  DM : $\{$MIDR $> 1\% \cup$ PFA $> 1\mathrm{m/s^2}\}$, seismic risk is $\mathbf{4.2 \times 10^{-3}}/\boldsymbol{yr}$

  DM : $\{$MIDR $> 1\% \cap$ PFA $> 1\mathrm{m/s^2}\}$, seismic risk is $\mathbf{1.71 \times 10^{-3}}/\boldsymbol{yr}$

- Damage measure defined on single EDP:

DM	MIDR>0.5%	**MIDR>1%**	MIDR>2%	PFA>0.5$\mathrm{m/s^2}$	**PFA>1$\mathbf{m/s^2}$**	PFA>1.5$\mathrm{m/s^2}$
Risk [/yr]	6.66×10^{-3}	$\mathbf{3.83\times10^{-3}}$	9.97×10^{-5}	6.65×10^{-3}	$\mathbf{1.92\times10^{-3}}$	9.45×10^{-5}

- Seismic risk for DM defined on multiple EDPs can be quite different from that defined on single EDP.

Outline

Appropriate Science Quotes

François-Marie Arouet, Voltaire: "Le doute n’est pas une condition agréable, mais la certitude est absurde."

Niklaus Wirth: "Software is getting slower more rapidly than hardware becomes faster."

Summary

- Numerical modeling to predict and inform
- Engineer needs to know!
- Education and Training is the key!
- Collaborators: Feng, Yang, Behbehani, Sinha, Wang, Karapiperis, Wang, Lacoure, Pisanó, Abell, Tafazzoli, Jie, Preisig, Tasiopoulou, Watanabe, Cheng, Yang.
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- `http://sokocalo.engr.ucdavis.edu/~jeremic`