Forward and Backward Uncertainty Propagation in Computational Earthquake Engineering

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Outline

Introduction

Uncertain Inelastic Dynamics
- Forward Uncertainty Propagation
- Backward Uncertainty Propagation, Sensitivities

Summary

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Introduction

Uncertain Inelastic Dynamics
- Forward Uncertainty Propagation
- Backward Uncertainty Propagation, Sensitivities

Summary

Motivation

Improve modeling and simulation of infrastructure objects

Modeling, epistemic uncertainty

Parametric, aleatory uncertainty

Goal is to Predict and Inform

Engineer needs to know!

Numerical Prediction under Uncertainty

- Modeling, Epistemic Uncertainty

  Modeling Simplifications

  Modeling sophistication for confidence in results

  Verification and Validation

- Parametric, Aleatory Uncertainty

  $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$,

  Uncertain: mass M, viscous damping C and stiffness K^{ep}

  Uncertain loads, $F(t)$

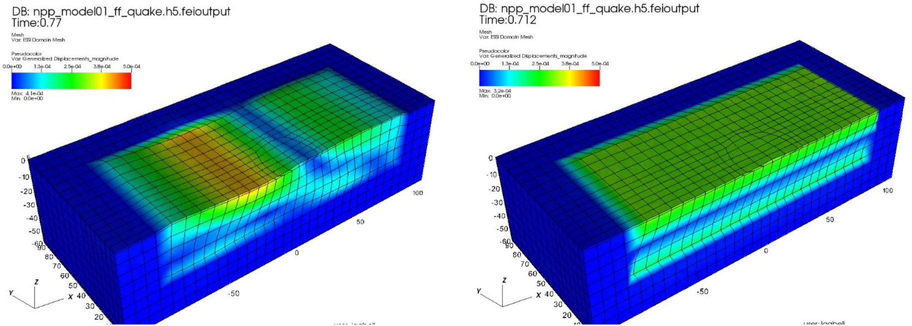
  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

Modeling, Epistemic Uncertainty

- Important (?!) features are simplified
  - 1C vs 3C seismic motions
  - Elastic vs Inelastic behavior
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that features being simplified out are less or not important

1C vs 6C Free Field Motions

- One component of motions, 1C from 6C
- Excellent fit, wrong mechanics



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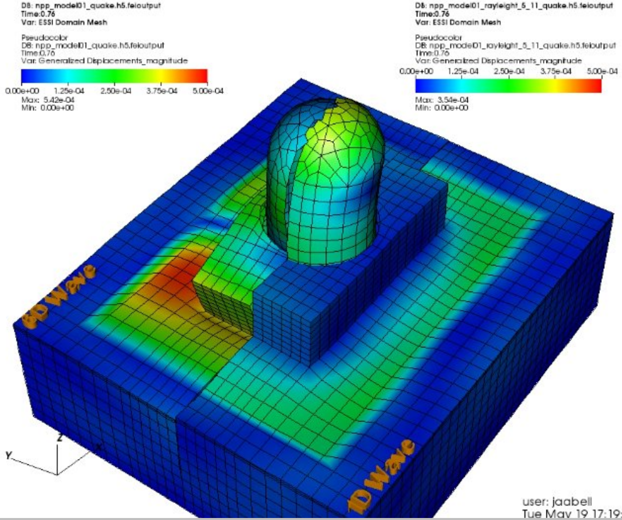
6C vs 1C NPP ESSI Response Comparison



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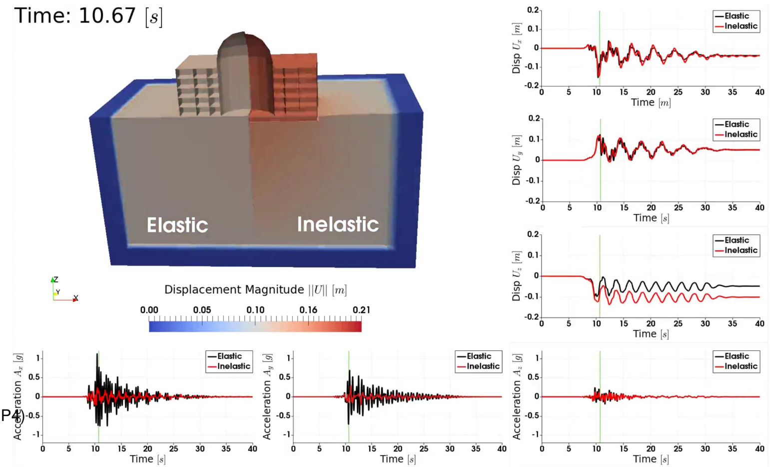
Elastic vs Inelastic NPP Response



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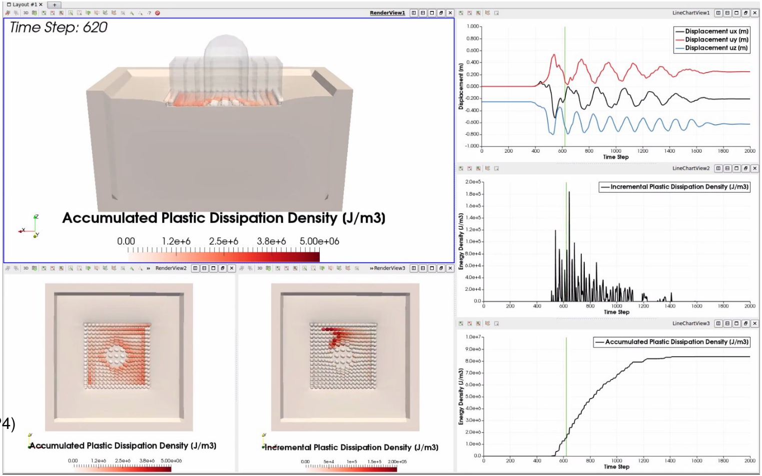
Energy Dissipation in a Large-Scale Model



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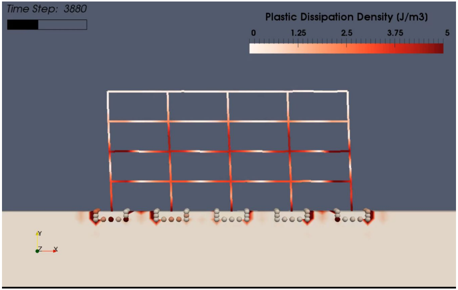
Design Alternatives



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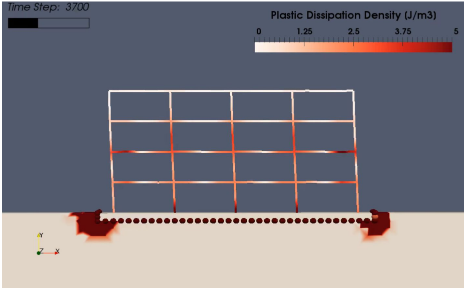

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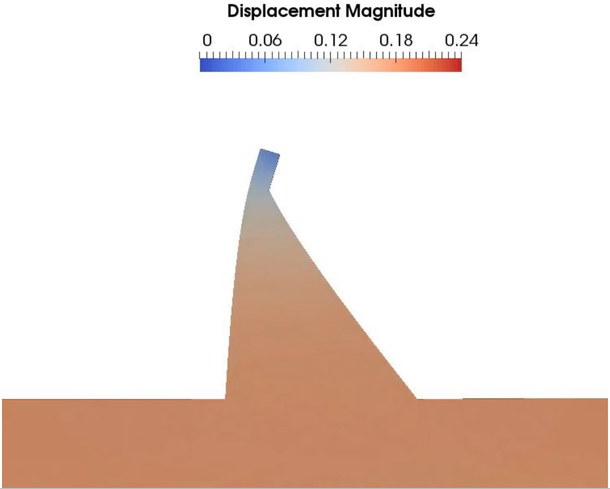
Pine Flat Dam, Dynamic Response with Reservoir



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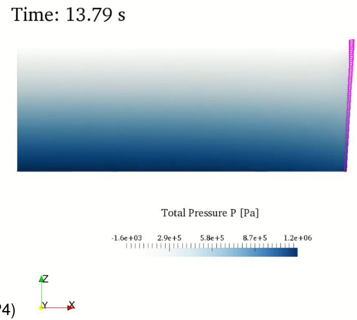
Pine Flat Dam, Hydrodynamic Pressure



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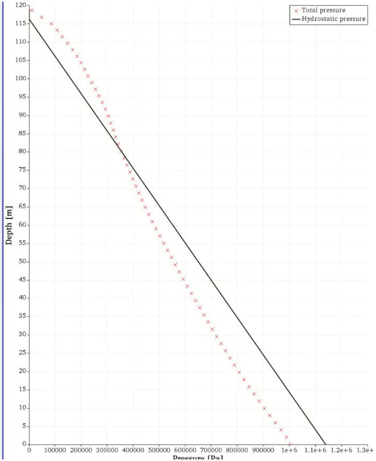
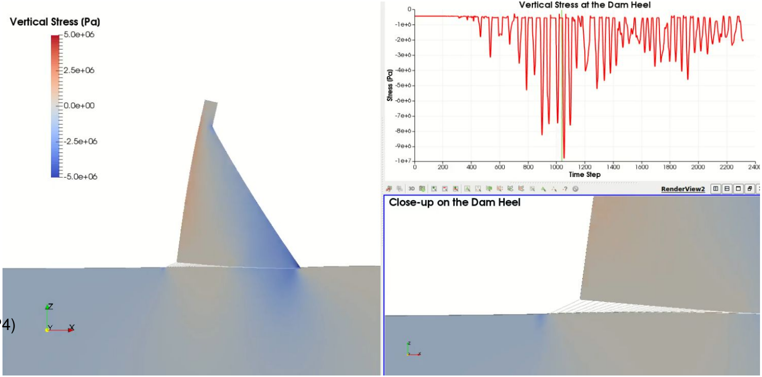


Pine Flat Dam, Inelastic Interface



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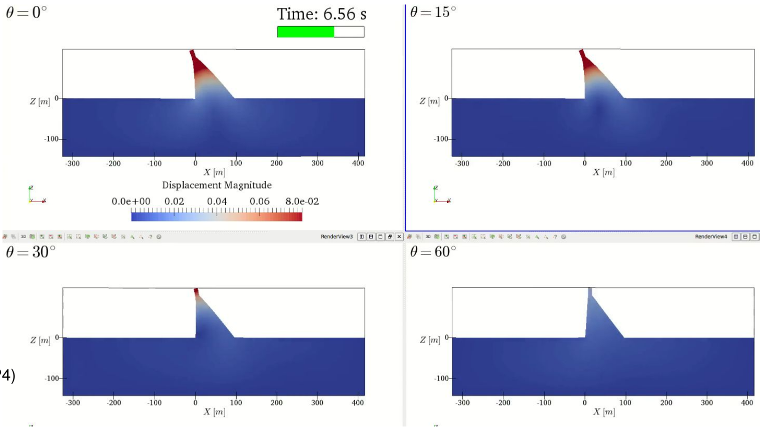
Pine Flat Dam, Inclined Plane Waves



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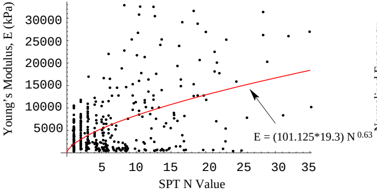
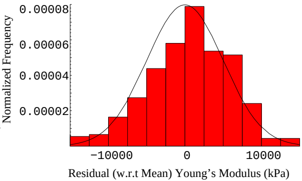
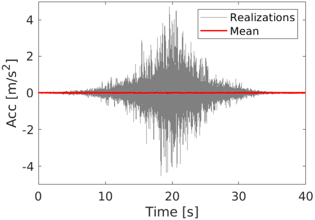
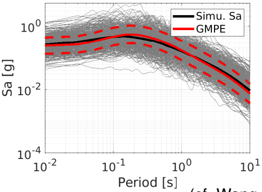
Parametric, Aleatory Uncertainty





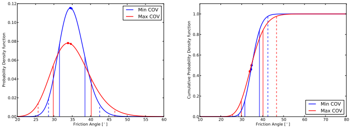
(cf. Phoon and Kulhawy (1999B))



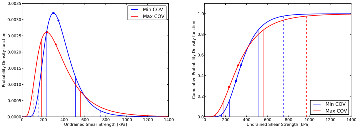


(cf. Wang et al. (2019))

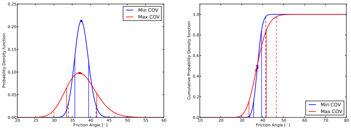
Parametric Uncertainty: Material Properties



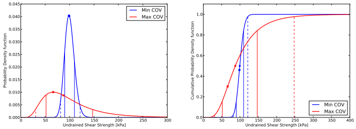
Field ϕ



Field c_u



Lab ϕ



Lab c_u

Real-ESSI Simulator System

The Real-ESSI, **Real**istic Modeling and Simulation of **E**arthquakes, **S**oils, **S**tructures and their **I**nteraction Simulator is a software, hardware and documentation system for time domain, linear and nonlinear, elastic and inelastic, deterministic or probabilistic, 3D, modeling and simulation of:

- statics and dynamics of soil and rock,
- statics and dynamics of rock,
- statics and dynamics of structures,
- statics of soil-structure systems, and
- dynamics of earthquake-soil-structure system interaction

Used for:

- Design, linear elastic, load combinations, dimensioning
- Assessment, nonlinear/inelastic, safety margins

Real-ESSI Modeling Features

- Solid elements: dry, (un-)saturated, elastic, inelastic
- Structural elements: beams, shells, elastic, inelastic
- Contact/interface/joint elements: Bonded, Shear/Frictional (EPP, EPH, EPS); Gap/Normal; linear, nonlinear, dry, coupled/saturated,
- Super element: stiffness and mass matrices
- Material models: soil, rock, concrete, steel...
- Seismic input: 1C and 3C, deterministic or probabilistic
- Energy dissipation: elastic-plastic, viscous, algorithmic
- Solid/Structure-Fluid interaction, full coupling, OpenFOAM
- Intrusive, forw. and backw. probabilistic inelastic modeling
- Detailed Verification and partial Validation
- Real-ESSI system: http://real-essi.info/

Outline

Forward Uncertainty Propagation

- Given uncertain material and uncertain loads
- Determine uncertain response, $u_i, \dot{u}_i, \ddot{u}_i, \epsilon_{ij}, \sigma_{ij}$, PDFs/CDFs
- Intrusive, analytic development, to circumvent Monte Carlo inefficiencies

Forward Uncertain Inelasticity

- Incremental el–pl constitutive equation

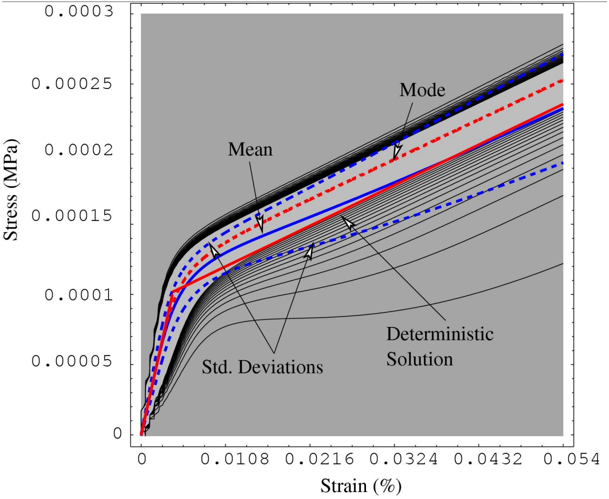
$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

- Dynamic Finite Elements

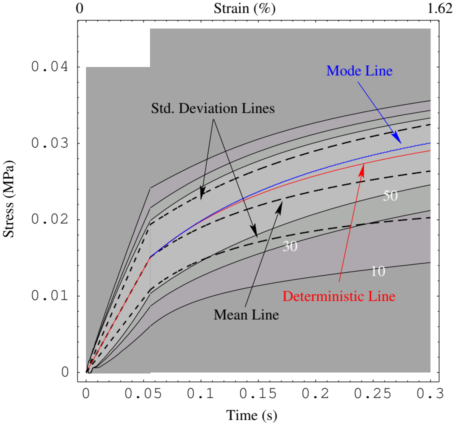
$$M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$$

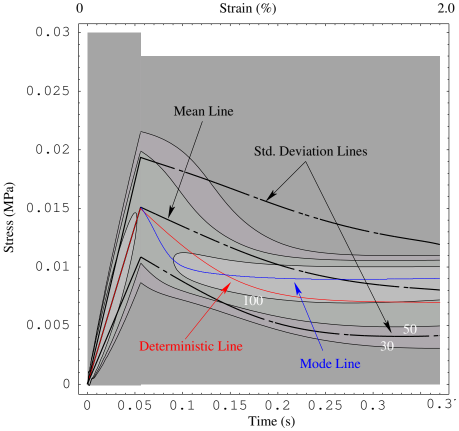
- Material and loads are uncertain

Probabilistic Elastic-Plastic Response



Cam Clay with Random G, M and p_0





Time Domain Stochastic Galerkin Method

Dynamic Finite Elements $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary): Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients
- Output response process: Multi-dimensional Hermite PC with unknown coefficients
- Galerkin projection: minimize the error to compute unknown coefficients of response process

Polynomial Chaos Representation

Material random field:
$D(x,\theta) = \sum_{i=1}^{P1} a_i(x)\Psi_i(\{\xi_r(\theta)\})$

Seismic loads/motions random process:
$f_m(t,\theta) = \sum_{j=1}^{P_2} f_{mj}(t)\Psi_j(\{\xi_k(\theta)\})$

Displacement response:
$u_n(t,\theta) = \sum_{k=1}^{P_3} d_{nk}(t)\Psi_k(\{\xi_l(\theta)\})$

where $a_i(x), f_{mj}(t)$ are known PC coefficients, while $d_{nk}(t)$ are unknown PC coefficients.

Direct Probabilistic Constitutive Solution in 1D

- Zero elastic region elasto-plasticity with stochastic Armstrong-Frederick kinematic hardening
  $\Delta\sigma = H_a\Delta\epsilon - c_r\sigma|\Delta\epsilon|; \quad E_t = d\sigma/d\epsilon = H_a \pm c_r\sigma$

- Uncertain: init. stiff. H_a, shear strength H_a/c_r, strain $\Delta\epsilon$:
  $H_a = \Sigma h_i\Phi_i; \quad C_r = \Sigma c_i\Phi_i; \quad \Delta\epsilon = \Sigma\Delta\epsilon_i\Phi_i$

- Resulting stress and stiffness are also uncertain

Direct Probabilistic Stiffness Solution

- Analytic product for all the components,
  $$E_{ijkl}^{EP} = \left[E_{ijkl}^{el} - \frac{E_{ijmn}^{el} m_{mn} n_{pq} E_{pqkl}^{el}}{n_{rs} E_{rstu}^{el} m_{tu} - \xi_* h_*} \right]$$
- Stiffness: each Polynomial Chaos component is updated incrementally
  $$E_{t_1}^{n+1} = \frac{1}{<\Phi_1 \Phi_1>} \{ \sum_{i=1}^{P_h} \; h_i < \Phi_i \Phi_1 > \pm \sum_{j=1}^{P_c} \sum_{l=1}^{P_\sigma} \; c_j \sigma_l^{n+1} < \Phi_j \Phi_l \Phi_1 > \}$$
  $$\ldots$$
  $$E_{t_P}^{n+1} = \frac{1}{<\Phi_1 \Phi_P>} \{ \sum_{i=1}^{P_h} \; h_i < \Phi_i \Phi_P > \pm \sum_{j=1}^{P_c} \sum_{l=1}^{P_\sigma} \; c_j \sigma_l^{n+1} < \Phi_j \Phi_l \Phi_P > \}$$
- Total stiffness is :
  $$E_t^{n+1} = \sum_{l=1}^{P_E} E_{t_i}^{n+1} \Phi_i$$

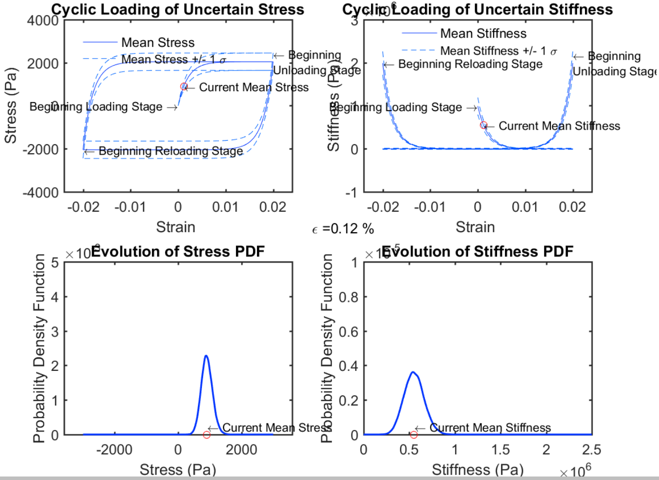
Direct Probabilistic Stress Solution

- Analytic product, for each stress component,
  $\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl}$

- Incremental stress: each Polynomial Chaos component is updated incrementally
  $\Delta\sigma_1^{n+1} = \frac{1}{<\Phi_1\Phi_1>}\{\sum_{i=1}^{P_h}\sum_{k=1}^{P_e}\ h_i\Delta\epsilon_k^n < \Phi_i\Phi_k\Phi_1 >$
  $-\sum_{j=1}^{P_g}\sum_{k=1}^{P_e}\sum_{l=1}^{P_\sigma}\ c_j\Delta\epsilon_k^n\sigma_l^n < \Phi_j\Phi_k\Phi_l\Phi_1 >\}$
  ...
  $\Delta\sigma_P^{n+1} = \frac{1}{<\Phi_P\Phi_P>}\{\sum_{i=1}^{P_h}\sum_{k=1}^{P_e}\ h_i\Delta\epsilon_k^n < \Phi_i\Phi_k\Phi_P >$
  $-\sum_{j=1}^{P_g}\sum_{k=1}^{P_e}\sum_{l=1}^{P_\sigma}\ c_j\Delta\epsilon_k^n\sigma_l^n < \Phi_j\Phi_k\Phi_l\Phi_P >\}$

- Stress update:
  $\sum_{l=1}^{P_\sigma}\sigma_i^{n+1}\Phi_i = \sum_{l=1}^{P_\sigma}\sigma_i^n\Phi_i + \sum_{l=1}^{P_\sigma}\Delta\sigma_i^{n+1}\Phi_i$

Probabilistic Elastic-Plastic Response



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Stochastic Elastic-Plastic Finite Element Method

- Material uncertainty expanded into stochastic shape funcs.
- Loading uncertainty expanded into stochastic shape funcs.
- Displacement expanded into stochastic shape funcs.
- Jeremić et al. 2011

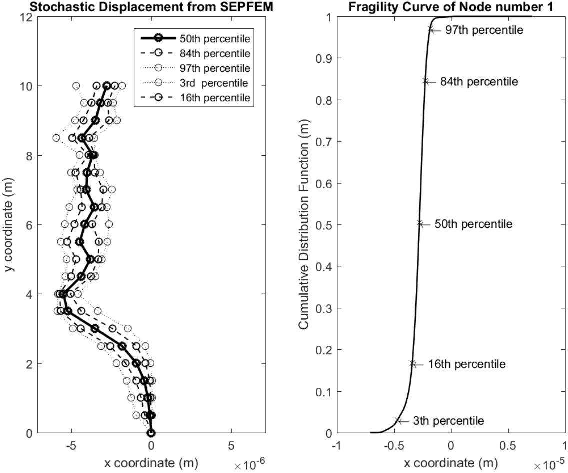
$$\begin{bmatrix} \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} \end{bmatrix} \begin{bmatrix} \Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\ \vdots \\ \sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i > \end{bmatrix}$$

SEPFEM: System Size

- SEPFEM offers a complete probabilistic solution
- It is NOT based on Monte Carlo approach
- System of equations does grow (!)

# KL terms material	# KL terms load	PC order displacement	Total # terms per DoF
4	4	10	43758
4	4	20	3 108 105
4	4	30	48 903 492
6	6	10	646 646
6	6	20	225 792 840
6	6	30	$1.1058\ 10^{10}$
8	8	10	5 311 735
8	8	20	$7.3079\ 10^{9}$
8	8	30	$9.9149\ 10^{11}$
...	...	...	...

SEPFEM: Example in 1D



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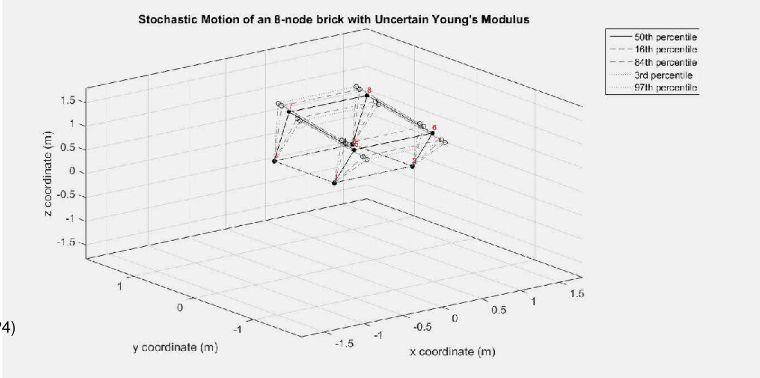
SEPFEM: Example in 3D



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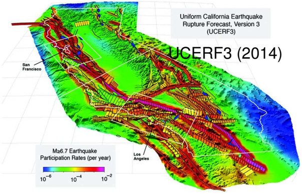
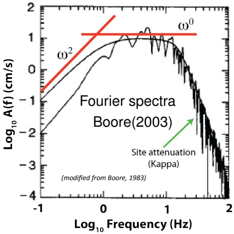
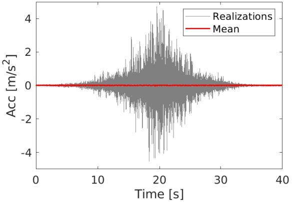
Application: Seismic Hazard

Seismic source characterization



Stochastic ground motion



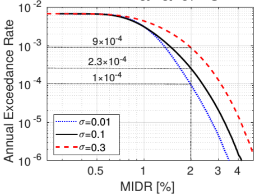


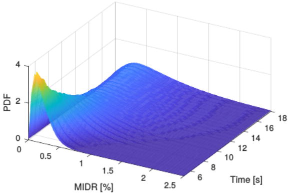
$$\lambda(EDP > z) = \sum N_i(M_i, R_i) P(EDP > z | M_i, R_i)$$

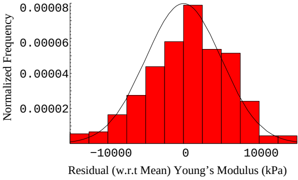
Uncertainty propagation SEPFEM

Uncertainty characterization Hermite polynomial chaos

EDP hazard/risk



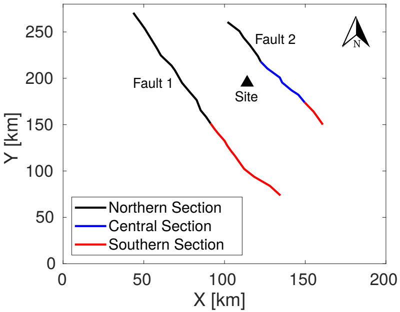


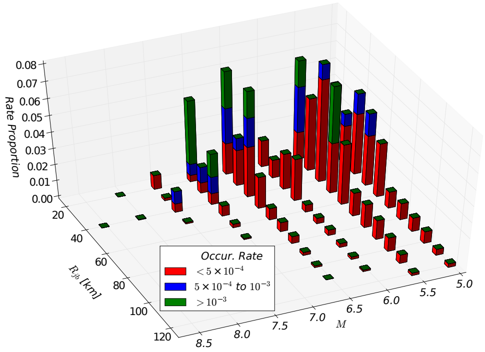


Stochastic Ground Motion Modeling

- Shift from modeling specific IM to fundamental characteristics of ground motions
  - Uncertain Fourier amplitude spectra (FAS)
  - Uncertain Fourier phase spectra (FPS)
- GMPE studies of FAS, (*Bora et al. (2018)*, *Bayless & Abrahamson (2018,2019)*, *Stafford(2017)*,)
- Stochastic FPS by phase derivative (Boore,2005) (Logistic phase derivative model by *Baglio & Abrahamson (2017)*)
- Near future change from $\boldsymbol{Sa}(\boldsymbol{T_0})$ to **FAS** and **FPS**

Example Object



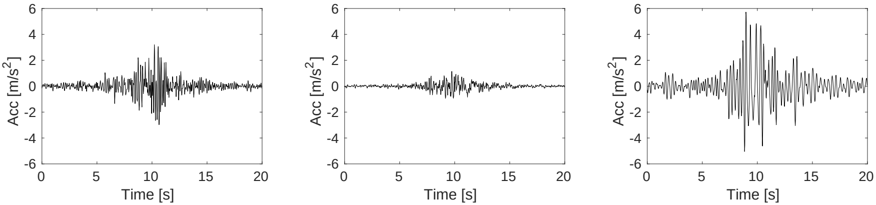


- Fault 1: San Gregorio fault
- Fault 2: Calaveras fault
- Uncertainty: Segmentation, slip rate, rupture geometry, etc.

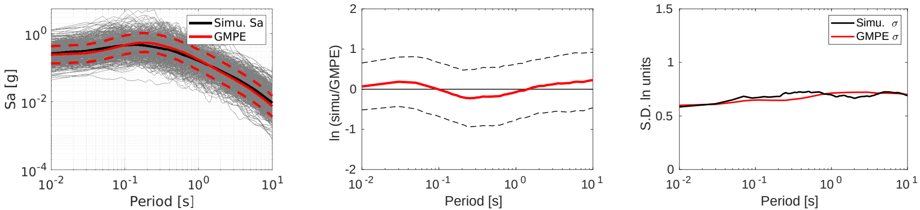
- 371 total seismic scenarios
- M 5 $\sim$ 5.5 and 6.5 $\sim$ 7.0
- R_{jb} 20km $\sim$ 40km

Stochastic Ground Motion Modeling

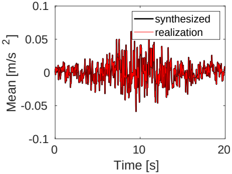
Realizations of simulated uncertain motions for scenario $M = 7$, $R = 15km$:



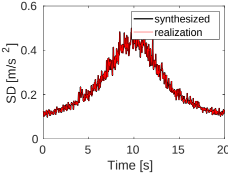
Verification with GMPE:



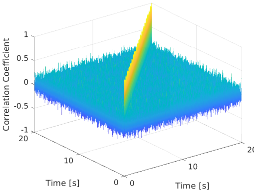
Stochastic Ground Motion Characterization



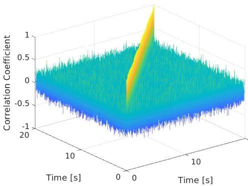
Acc. marginal mean



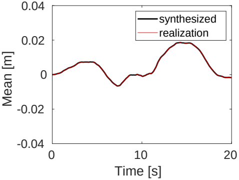
Acc. marginal S.D.



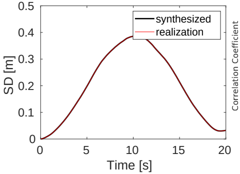
Acc. realization Cov.



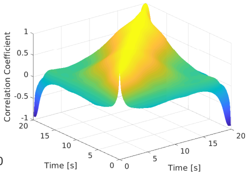
Acc. synthesized Cov.



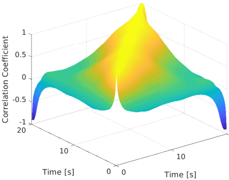
Dis. marginal mean



Dis. marginal S.D.

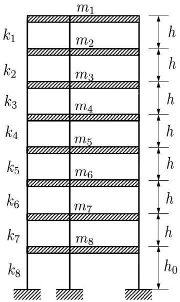


Dis. realization Cov.

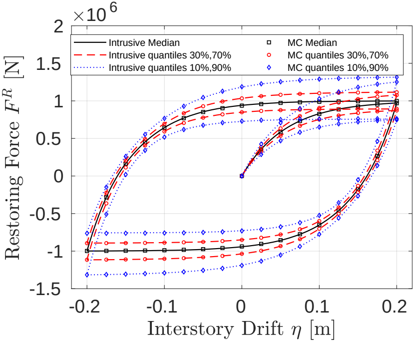


Dis. synthesized Cov.

Stochastic Material Modeling

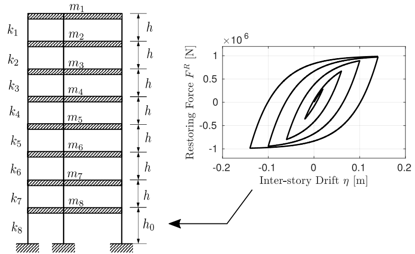


(a) Frame

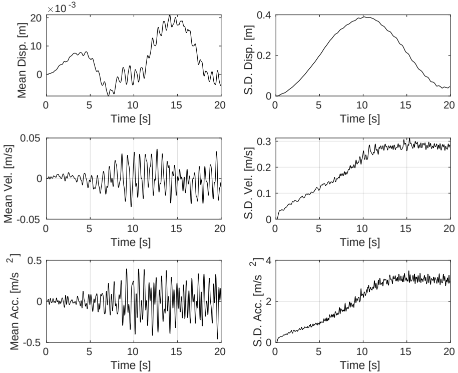


(b) Interstory response

Probabilistic Dynamic Structural Response



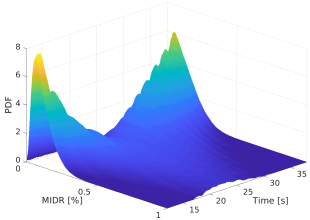
Probabilistic response of top floor from SFEM

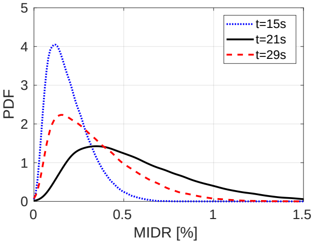


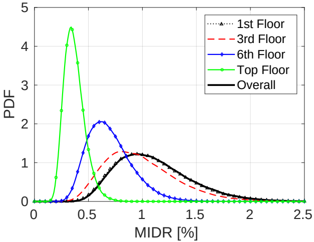
- Coefficient of variation 15% for H_a and C_r
- Time domain stochastic El-Pl FEM analysis (SEPFEM)

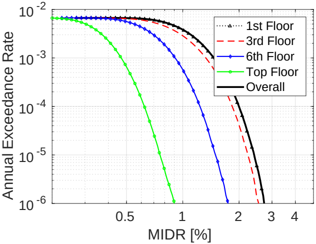
Seismic Risk Analysis

Engineering demand parameter (EDP): Maximum inter-story drift ratio (MIDR)



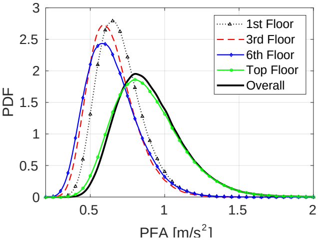


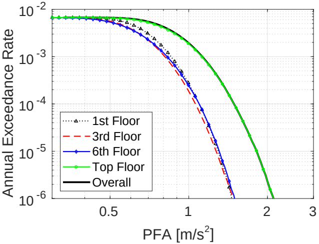


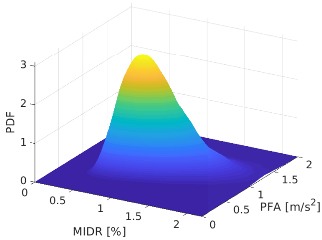


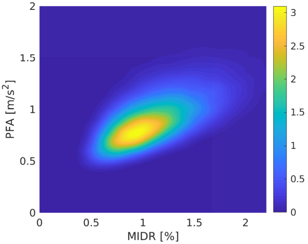
Seismic Risk Analysis

Engineering demand parameter (EDP): Peak floor acceleration (PFA)









Seismic Risk Analysis

- Damage measure defined on single EDP:

DM	MIDR>0.5%	**MIDR>1%**	MIDR>2%	PFA>0.5$\mathrm{m/s^2}$	**PFA>1$\mathbf{m/s^2}$**	PFA>1.5$\mathrm{m/s^2}$
Risk [/yr]	6.66×10^{-3}	$\mathbf{3.83\times10^{-3}}$	9.97×10^{-5}	6.65×10^{-3}	$\mathbf{1.92\times10^{-3}}$	9.45×10^{-5}

- Damage measure (DM) defined on multiple EDPs:

  DM : $\{\text{MIDR} > 1\% \cup \text{PFA} > 1\mathrm{m/s^2}\}$, seismic risk is $\mathbf{4.2 \times 10^{-3}}/\boldsymbol{yr}$

  DM : $\{\text{MIDR} > 1\% \cap \text{PFA} > 1\mathrm{m/s^2}\}$, seismic risk is $\mathbf{1.71 \times 10^{-3}}/\boldsymbol{yr}$

- Seismic risk for DM defined on multiple EDPs can be quite different from that defined on single EDP

Outline

Backward Uncertainty Propagation, Sensitivities

- Given forward uncertain response, PDFs, CDFs...
- Contributions of uncertain input to forward uncertainties
- Sensitivity of forward uncertain response to input uncertainties

ANOVA Representation

Model with n uncertain inputs ($\boldsymbol{x}$) and scalar output y:

$$y = f(\boldsymbol{x});\;\; \boldsymbol{x} \in I^n$$

The ANalysis Of VAriance representation (Sobol 2001):

$$f(x_1, ...x_n) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{1 \le i < j \le n} f_{ij}(x_i, x_j) + ...f_{1,...n}(x_1, ...x_n)$$

Sensitivity Analysis, ANOVA Representation

Total of 2^n summands

Mean value $f_0 = \int_{I^n} f(\boldsymbol{x}) d\boldsymbol{x}$

Integral of each summand $\int_0^1 f_{i_1,\dots i_s}(x_{i_1},\dots x_{i_s}) dx_{i_k} = 0$; $1 \leq k \leq s$

Summands are orthogonal to each other

$$\int_{I^n} f_{i_1,\dots i_s}(x_{i_1}, x_{i_2}, \dots x_{i_s}) f_{j_1,\dots j_t}(x_{j_1}, x_{j_2}, \dots x_{j_t}) d\boldsymbol{x} = 0;$$

$$\{i_1, \dots i_s\} \neq \{j_1, \dots j_t\}$$

Sensitivity Analysis, ANOVA Representation

ANOVA representation is unique!

Univariate terms:

$$f_i(x_i) = \int_{I^{n-1}} f(\boldsymbol{x}) \, d\boldsymbol{x}_{\sim i} - f_0$$

Bivariate terms:

$$f_{ij}(x_i, x_j) = \int_{I^{n-2}} f(\boldsymbol{x}) \, d\boldsymbol{x}_{\sim [ij]} - f_i(x_i) - f_j(x_j) - f_0$$

Sensitivity Analysis, Variance

Total variance of the probabilistic model response $y = f(\boldsymbol{X})$ is

$$D = Var[f(\boldsymbol{X})] = \int_{I^n} f^2(\boldsymbol{x})\, d\boldsymbol{x} - f_0^2$$

Total variance D, decomposed

$$D = \sum_{i=1}^{n} D_i + \sum_{1 \le i < j \le n} D_{ij} + ... + D_{1,2,...,n} = \sum_{s=1}^{n} \sum_{i_1 < ... < i_s} D_{i_1 ... i_s}$$

Variance contribution from individual summand

$$D_{i_1 .. i_s} = \int_{I^s} f^2_{i_1 ... i_s}(x_{i_1}, ..., x_{i_s})\, dx_{i_1}, ...dx_{i_s},$$
$$1 \le i_1 < ... < i_s \le n;\ s = 1, ...n$$

Sobol Indices

- Sobol' indices $S_{i_1 \dots i_s}$, fractional contributions from random inputs $\{X_{i_1}, ..., X_{i_s}\}$ to the total variance D: $S_{i_1 \dots i_s} = D_{i_1 \dots i_s}/D$
- First order indices $S_i \rightarrow$ individual influence of each uncertain input parameter
- Higher order indices $S_{i_1 \dots i_s} \rightarrow$ mixed influence from groups of uncertain input parameters
- Total sensitivity indices, influence of input parameter X_i

$$S_i^{total} = \sum_{\mathscr{S}_i} D_{i_1 \dots i_s}$$

Sobol Indices and Polynomial Chaos

PC expansion of response, ANOVA form (Sudret 2008)
Multi-dimensional PC bases $\{\Psi_j(\boldsymbol{\xi})\}$ decomposed into products of single dimension PC chaos bases of different orders

$$\Psi_j(\boldsymbol{\xi}) = \prod_{i=1}^{n} \phi_{\alpha_i}(\xi_i)$$

$\phi_{\alpha_i}(\xi_i)$ is the single dimensional, order α_i, polynomial function of underlying basic random variable ξ_i.
From ANOVA representation of probabilistic model response, the PC-based Sobol' indices $S^{PC}_{i_1 \dots i_s}$ are

$$S^{PC}_{i_1 \dots i_s} = \sum_{\boldsymbol{\alpha} \in S_{i_1,\dots,i_s}} y^2_{\boldsymbol{\alpha}} \boldsymbol{E}\left[\Psi^2_{\boldsymbol{\alpha}}\right] / D^{PC}$$

Sobol Sensitivity Analysis

Total Sobol' indices $S^{PC,total}_{j_1 \ldots j_t}$

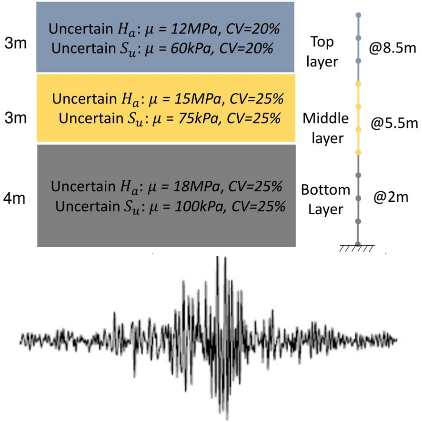
$$S^{PC,total}_{j_1 \ldots j_t} = \sum_{(i_1,\ldots,i_s) \in S_{j_1,\ldots,j_t}} S^{PC}_{i_1 \ldots i_s}$$

where $S_{j_1,\ldots,j_t} = \{(i_1, \ldots, i_s) : (j_1, \ldots, j_t) \subset (i_1, \ldots, i_s)\}$

Using PC representation of probabilistic model response, Sobol' sensitivity indices are analytic and inexpensive

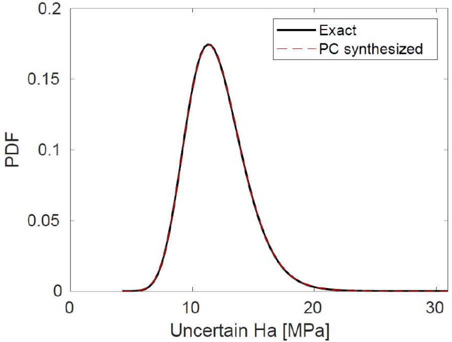
Application: Stochastic Site Response

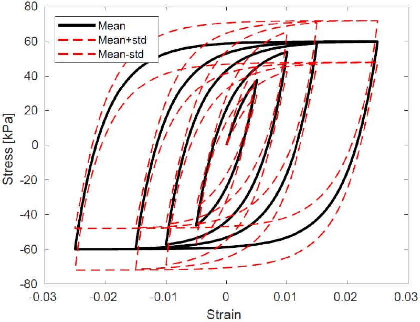
- Uncertain material: uncertain random field, marginally lognormal distribution, exponential correlation length 10m
- Uncertain seismic rock motions: seismic scenario M=7, R=50km



Stochastic Material Parameters

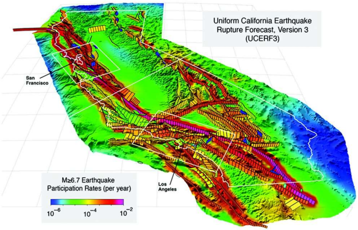
Lognormal distributed random field with PC Dim. 3 Order 2

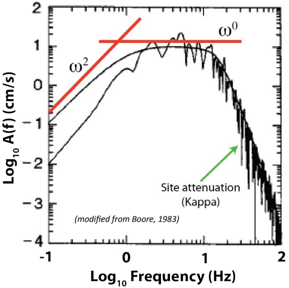




Stochastic Seismic Motion Development

- UCERF3 (Field et al. 2014)
- Stochastic motions (Boore 2003)
- Polynomial Chaos Karhunen-Loève expansion
- Probabilistic DRM (Bielak et al. 2003, Wang et al. 2021)



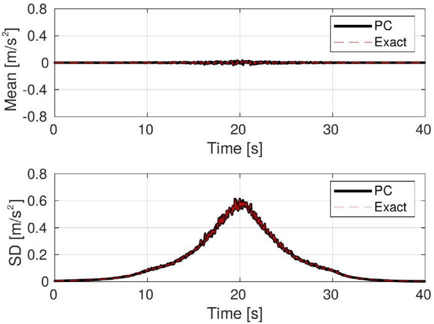


Stochastic Ground Motion Modeling

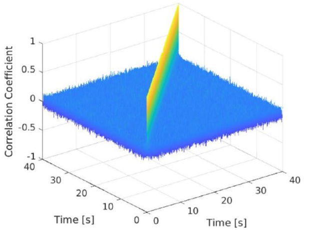
- Modeling fundamental characteristics of uncertain ground motions, Stochastic Fourier amplitude spectra (FAS). and Stochastic Fourier phase spectra (FPS) and not specific IM
- Mean behavior of stochastic FAS, w^2 source radiation spectrum by Brune(1970), and Boore(1983, 2003, 2015).
- Variability models for stochastic FAS, FAS GMPEs by Bora et al. (2015, 2018), Bayless & Abrahamson (2019), Stafford(2017) and Bayless & Abrahamson (2018).
- Stochastic FPS by phase derivative (Boore,2005), Logistic phase derivative model by Baglio & Abrahamson (2017)

Stochastic Seismic Motions, Accelerations

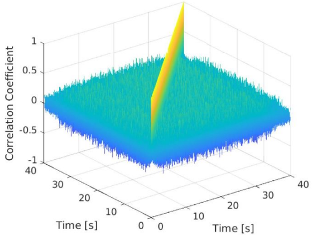
- Acceleration: **PC Dim. 150 Order 1**



Marginal behavior



Exact



PC Synthesized

Sensitivity of PGA from Uncertain Soil

- First 10 terms from soil uncertainty
- Total Sobol sensitivity index $S_{1-3}^{PC,total} = 0.51$

Sobol Index	Value
$S_{1,123}^{PC}$	0,04389
$S_{1,118}^{PC}$	0,02605
$S_{1,127}^{PC}$	0,02370
$S_{1,100}^{PC}$	0,01759
$S_{1,103}^{PC}$	0,01700
$S_{1,134}^{PC}$	0,01680
$S_{1,141}^{PC}$	0,01611
$S_{1,110}^{PC}$	0,01358
$S_{1,130}^{PC}$	0,01303
$S_{1,132}^{PC}$	0,01068
...	...

Sensitivity of PGA from Uncertain Rock Motions

- First 10 terms from motions uncertainty
- Total Sobol sensitivity index $S^{PC,total}_{1-153} = 0.98$

Sobol Index	Value
$S^{PC}_{1,123}$	0,04389
S^{PC}_{137}	0,03459
S^{PC}_{110}	0,03061
S^{PC}_{118}	0,02698
$S^{PC}_{1,118}$	0,02605
S^{PC}_{108}	0,02482
S^{PC}_{141}	0,02373
$S^{PC}_{1,127}$	0,02370
$S^{PC}_{1,100}$	0,01759
$S^{PC}_{1,103}$	0,01700
...	...

Sensitivity Analysis

Total variance in PGA, in this particular case (!), dominated by uncertain ground motions

49% from uncertain rock motions at depth

2% from uncertain soil

49% from interaction of uncertain rock motions and uncertain soil

Outline

Appropriate Science and Engineering Quotes

François-Marie Arouet, Voltaire:
"Le doute n'est pas une condition agréable, mais la certitude est absurde"

Max Planck:
"A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it"

Theodore Von Kármán:
"The Scientist studies what is, the engineer creates what has never been"

Summary

- Analysis of uncertainties and sensitivities
- Predict and Inform
- Engineer Needs to Know!
- Real-ESSI Simulator Systems
- `http://real-essi.info/`