Fully Coupled FEM Analysis

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Liquefaction Workshop

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Outline

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Engineering Analysis

- Improve analysis and design for infrastructure
- All ESSI problems are three dimensional, 3D
- Simplifying assumptions $\rightarrow$ epistemic uncertainty
- Design and assessment of infrastructure
- Predict and inform!
- Engineer needs to know!

Prediction under Uncertainty

- Modeling, Epistemic Uncertainties

  Modeling simplifications

  Low, medium, high sophistication modeling and simulation

  Modeling sophistication level for confidence in results

  Verification and Validation

- Parametric, Aleatory Uncertainties

  $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$

  Uncertain: mass M, viscous damping C and stiffness K^{ep}

  Uncertain loads, $F(t)$

  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

Engineer, Analyst

- Educated, Trained, Experienced
- Sound engineering judgement
- Assess various analysis sophistication levels
- Engineer is in full control of the model and the analysis
- Engineer uses models to assess design options
- Confidence in all modeling choices
- Confidence in all analysis results

Analysis Program

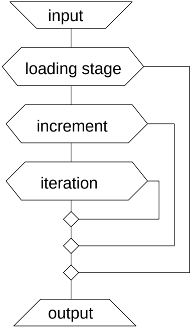
- Hierarchy of model sophistication capabilities
- Hierarchy of simulation/algorithmic capabilities
- Full Verification !
- Extensive Validation
- Confidence in analysis results

Modeling Features

- Solid elements: dry, (un-)saturated, elastic, inelastic
- Structural elements: beams, shells, elastic, inelastic
- Contact/interface/joint elements: Bonded, Shear/Frictional; Gap/Normal; linear, nonlinear, dry, coupled/saturated,
- Material models: soil, rock, concrete, steel...
- Seismic input: 1C and 3C, deterministic or probabilistic
- Energy dissipation: elastic-plastic, viscous, algorithmic
- Solid/Structure-Fluid interaction, Internal and External
- Intrusive probabilistic inelastic modeling, Forw. and Backw.

Simulation Features

- Static loading stages
- Dynamic loading stages
- Explicit and Implicit computations
- Restart, simulation tree
- Solution advancement methods/algorithms, on global and constitutive levels, with and without enforcing equilibrium
- High Performance Computing
  - Fine grained, template mataprograms, small matrix library
  - Coarse grained, distributed memory parallel



Outline

FEM Formulation Assumptions

- 3D Geometry
- Solids and Structures
- Material nonlinear, inelastic and/or elastic
- Small deformations:
  - small strain
  - small translations
  - small rotations
- Fully or Partially/Un- Saturated, two phase material
- Mixture of pore fluid and porous solid
- Compressible pore fluid and compressible porous solid

Two Phase, Fully Coupled Systems

- Based on:
  O. C. Zienkiewicz and T. Shiomi. Dynamic behaviour of saturated porous media; the generalized Biot formulation and its numerical solution. International Journal for Numerical and Analytical Methods in Geomechanics, 8:71-96, 1984.

- Full coupling of internal, pore fluid with porous solid
- Full saturation and partial, unsaturated soils
- Statics and dynamics of coupled systems

FEM Discretization, Approximation

- Displacement approximation: $u_i \approx \hat{u}_a = H_I \bar{u}_{Ia}$

- Strain: $\epsilon_{ab} \;\approx\; \hat{\epsilon}_{ab} = \frac{1}{2}\left(\left(H_{I,b}\;\bar{u}_{Ia}\right) + \left(H_{I,a}\;\bar{u}_{Ib}\right)\right)$

FEM Discretization, Stress-Strain

$$\Delta\hat{\sigma}_{ab} = E^{EP}_{abcd}\left(\Delta\hat{\epsilon}_{cd} - \Delta\epsilon^{0}_{cd}\right) + \Delta\sigma^{0}_{ab}$$

FEM Discretization, Stiffness Tensor Symmetries

- Minor symmetry of stiffness tensor:
  ${}^{EP}E_{abcd} = E^{EP}_{bacd} = E^{EP}_{abdc}$

- Major symmetry is not necessary, non-associated plasticity
  ${}^{EP}E_{abcd} \neq E^{EP}_{cdab}$

Notation

- Effective stress $\sigma_{ij}^{''} = \sigma_{ij} - \alpha\delta_{ij}p$
- σ_{ij} - the total Cauchy stress in the mixture
- $\alpha = 1 - K_T/K_S \approx 1$
- p - the pore fluid pressure
- u_i - the displacement of the solid skeleton
- w_i - disp. of fluid phase relative to the skeleton of solid
- $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ - strain increment of the solid phase,
- ρ, ρ_s, ρ_f - densities of mixture, solid phase and water
- $n = V_{voids}/V_{total}$ - porosity,
- $\dot{w}_{i,i}$ - rate of change of volume of fluid / volume of mixture

Governing Equations

- Equilibrium Equation of the Mixture:
  $\sigma_{ij,j} - \rho \ddot{u}_i - \rho_f [\ddot{w}_i + \overline{\dot{w}_j \dot{w}_{i,j}}] + \rho b_i = 0$
  $\rho = \frac{M_t}{V_t} = \frac{M_s + M_f}{V_t} = \frac{V_s \rho_s + V_f \rho_f}{V_t} = \frac{V_f}{V_t}\rho_f + \frac{V_t - V_f}{V_t}\rho_s = n\rho_f + (1-n)\rho_s$
- Equilibrium Equation of the Fluid
  $-p_{,i} - R_i - \rho_f \ddot{u}_i - \rho_f [\ddot{w}_i + \underline{\dot{w}_j \dot{w}_{i,j}}]/n + \rho_f b_i = 0$
  $R_i = k_{ij}^{-1} \dot{w}_j \qquad \text{or} \qquad R_i = k^{-1} \dot{w}_i$
- Flow Conservation Equation
  $\dot{w}_{i,i} + \alpha \dot{\varepsilon}_{ii} + \frac{\dot{p}}{Q} + \underline{n \frac{\dot{\rho}_f}{\rho_f}} + \dot{s}_0 = 0$
  $\frac{1}{Q} \equiv \frac{n}{K_f} + \frac{\alpha - n}{K_s} \cong \frac{n}{K_f} + \frac{1-n}{K_s}$

Simplified Governing Equations

- Equilibrium Equation of the Mixture:

$$\sigma_{ij,j} - \rho \ddot{u}_i - \rho_f \ddot{w}_i + \rho b_i = 0$$

- Equilibrium Equation of the Fluid

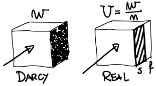
$$-p_{,i} - R_i - \rho_f \ddot{u}_i - \frac{\rho_f \ddot{w}_i}{n} + \rho_f b_i = 0$$

- Flow Conservation Equation

$$\dot{w}_{i,i} + \alpha \dot{\varepsilon}_{ii} + \frac{\dot{p}}{Q} = 0$$

Modified Governing Equations

$$U_i = u_i + \frac{w_i}{n}$$



$$\sigma''_{ij,j} - (\alpha - n)p_{,i} + (1-n)\rho_s b_i - (1-n)\rho_s \ddot{u}_i + nR_i = 0$$

$$-np_{,i} + n\rho_f b_i - n\rho_f \ddot{U}_i - nR_i = 0$$

$$-n\dot{U}_{i,i} = (\alpha - n)\dot{\varepsilon}_{ii} + \frac{1}{Q}\dot{p}$$

u_i: three solid displacement
p: pore pressure
U_i: three fluid displacement

FEM u-p-U Discretization

$$\begin{bmatrix} M_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_f \end{bmatrix} \begin{bmatrix} \ddot{\bar{u}} \\ \ddot{\bar{p}} \\ \ddot{\bar{U}} \end{bmatrix} + \begin{bmatrix} C_1 & 0 & -C_2 \\ 0 & 0 & 0 \\ -C_2^T & 0 & C_3 \end{bmatrix} \begin{bmatrix} \dot{\bar{u}} \\ \dot{\bar{p}} \\ \dot{\bar{U}} \end{bmatrix} +$$

$$\begin{bmatrix} K^{EP} & -G_1 & 0 \\ -G_1^T & -P & -G_2^T \\ 0 & -G_2 & 0 \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{p} \\ \bar{U} \end{bmatrix} = \begin{bmatrix} \bar{f}_s \\ 0 \\ \bar{f}_f \end{bmatrix}$$

FEM u-p-U Discretization, Index Form

$$
\begin{bmatrix} (M_s)_{KijL} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (M_f)_{KijL} \end{bmatrix}
\begin{bmatrix} \ddot{\bar{u}}_{Lj} \\ \ddot{\bar{p}}_{N} \\ \ddot{\bar{U}}_{Lj} \end{bmatrix}
+
\begin{bmatrix} (C_1)_{KijL} & 0 & -(C_2)_{KijL} \\ 0 & 0 & 0 \\ -(C_2)_{LjiK} & 0 & (C_3)_{KijL} \end{bmatrix}
\begin{bmatrix} \dot{\bar{u}}_{Lj} \\ \dot{\bar{p}}_{N} \\ \dot{\bar{U}}_{Lj} \end{bmatrix}
$$

$$
+
\begin{bmatrix} (K^{EP})_{KijL} & -(G_1)_{KiM} & 0 \\ -(G_1)_{LjM} & -P_{MN} & -(G_2)_{LjM} \\ 0 & -(G_2)_{KiL} & 0 \end{bmatrix}
\begin{bmatrix} \bar{u}_{Lj} \\ \bar{p}_{M} \\ \bar{U}_{Lj} \end{bmatrix}
=
\begin{bmatrix} \bar{f}^{solid}_{Ki} \\ 0 \\ \bar{f}^{fluid}_{Ki} \end{bmatrix}
$$

FEM u-p-U Matrices/Tensors

$$\boldsymbol{M_s} = (M_s)_{KijL} = \int_{\Omega} H_K^u (1-n)\rho_s \delta_{ij} H_L^u d\Omega \;\; ; \;\; \boldsymbol{M_f} = (M_f)_{KijL} = \int_{\Omega} H_K^U n \rho_f \delta_{ij} H_L^U d\Omega$$

$$\boldsymbol{C_1} = (C_1)_{KijL} = \int_{\Omega} H_K^u n^2 k_{ij}^{-1} H_L^u d\Omega \;\; ; \;\; \boldsymbol{C_2} = (C_2)_{KijL} = \int_{\Omega} H_K^u n^2 k_{ij}^{-1} H_L^U d\Omega$$

$$\boldsymbol{C_3} = (C_3)_{KijL} = \int_{\Omega} H_K^U n^2 k_{ij}^{-1} H_L^U d\Omega$$

$$\boldsymbol{K^{EP}} = (K^{EP})_{KijL} = \int_{\Omega} H_{K,m}^u D_{imjn} H_{L,n}^u d\Omega$$

$$\boldsymbol{G_1} = (G_1)_{KiM} = \int_{\Omega} H_{K,i}^u (\alpha - n) H_M^p d\Omega \;\; ; \;\; \boldsymbol{G_2} = (G_2)_{KiM} = \int_{\Omega} n H_{K,i}^U H_M^p d\Omega$$

$$\boldsymbol{P} = P_{NM} = \int_{\Omega} H_N^p \frac{1}{Q} H_M^p d\Omega$$

FEM u-p-U Loads

$$
\begin{aligned}
\bar{f}_{Ki}^{solid} &= (f_1^u)_{Ki} - (f_4^u)_{Ki} + (f_5^u)_{Ki} \\
\bar{f}_{Ki}^{fluid} &= -(f_1^U)_{Ki} + (f_2^U)_{Ki} \\
(f_1^u)_{Ki} &= \int_{\Gamma_t} H_K^u n_j \sigma_{ij}^{''} d\Gamma \\
(f_4^u)_{Ki} &= \int_{\Gamma_p} H_K^u (\alpha - n) n_i p d\Gamma \\
(f_5^u)_{Ki} &= \int_{\Omega} H_K^u (1 - n) \rho_s b_i d\Omega \\
(f_1^U)_{Ki} &= \int_{\Gamma_p} n H_K^U n_i p d\Gamma \\
(f_2^U)_{Ki} &= \int_{\Omega} n H_K^U \rho_f b_i d\Omega
\end{aligned}
$$

Coupled FEM Discretization, Matrix Form

- Coupled FEM, full matrix form:

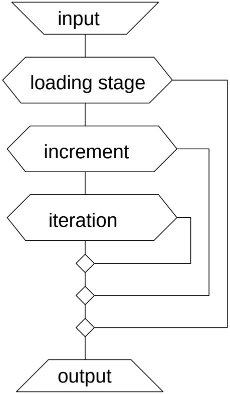
$$\begin{bmatrix} M_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_f \end{bmatrix} \begin{bmatrix} \ddot{\bar{u}} \\ \ddot{\bar{p}} \\ \ddot{\bar{U}} \end{bmatrix} + \begin{bmatrix} C_1 & 0 & -C_2 \\ 0 & 0 & 0 \\ -C_2^T & 0 & C_3 \end{bmatrix} \begin{bmatrix} \dot{\bar{u}} \\ \dot{\bar{p}} \\ \dot{\bar{U}} \end{bmatrix} +$$

$$+ \begin{bmatrix} K^{EP} & -G_1 & 0 \\ -G_1^T & -P & -G_2^T \\ 0 & -G_2 & 0 \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{p} \\ \bar{U} \end{bmatrix} = \begin{bmatrix} \bar{f}_s \\ 0 \\ \bar{f}_f \end{bmatrix}$$

- Coupled FEM, generalized matrix form:
$M_{PQ} \, \ddot{\bar{u}}_P + C_{PQ} \, \dot{\bar{u}}_P + K_{PQ} \, \bar{u}_P = F_Q$

Coupled FEM, Residuals, Equilibrium

- Nonlinear coupled FEM residuals, equilibrium:
  $R_Q =$
  $F_Q - \left(M_{PQ} \, \ddot{\bar{u}}_P + C_{PQ} \, \dot{\bar{u}}_P + K_{PQ} \, \bar{u}_P \right)$

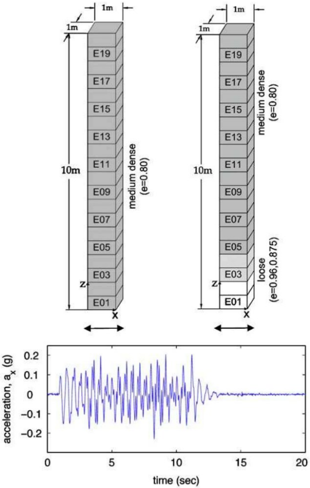


Outline

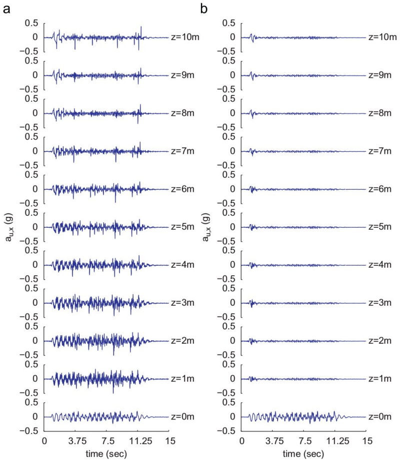
Coupled Analysis in Practice

- The coupled u-p-U formulation is very powerful
- The u-p-U formulation is useful for many problems
- ESSI: internal fluids problems
- ESSI: external fluid problems

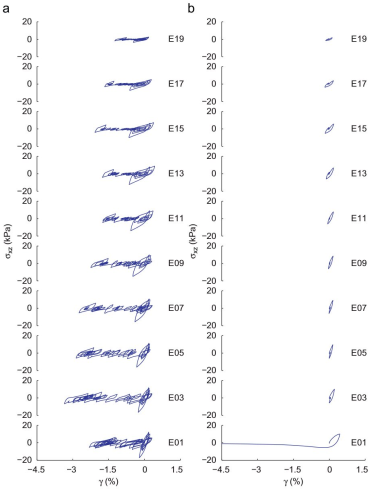
Liquefaction as Base Isolation



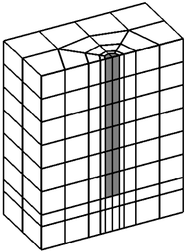
Liquefaction, Wave Propagation

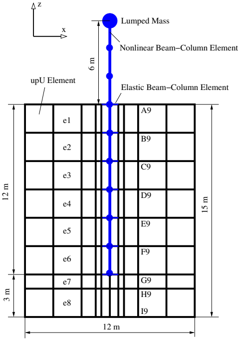


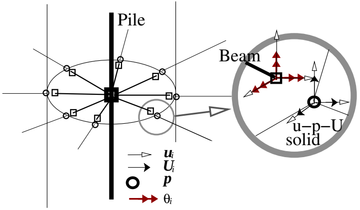
Liquefaction, Stress-Strain Response



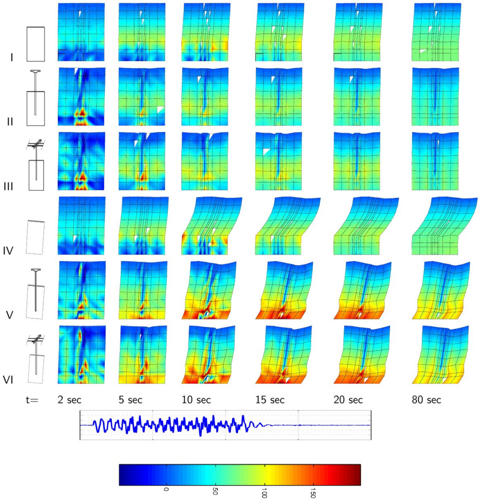
Pile in Liquefiable Soil



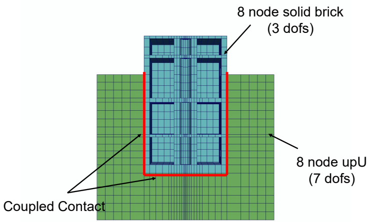


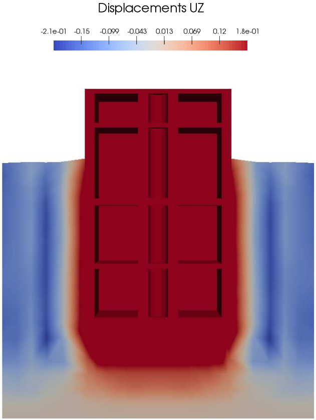


Pile in Liquefiable Soil



Buoyant Force Effects

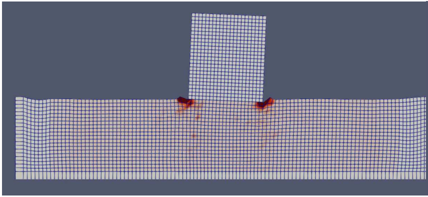


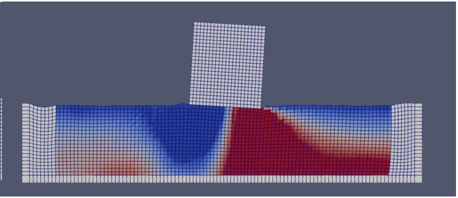


Building on Liquefiable Soil

Plastic Strain

Pore Fluid Pressures





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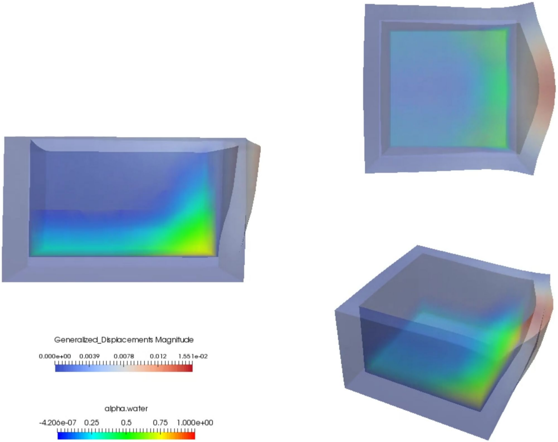
Solid/Structure-Fluid Interaction



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Outline

Analysis of ESSI Systems

- Soil, elastic, elastic-plastic/inelastic
  - Dry, single phase
  - Unsaturated/partially saturated
  - Fully saturated
- Contact/Interface/Joint, inelastic
  - Dry, single phase, Normal, hard and soft, gap open/close
  - Dry, single phase, Tangential, friction, nonlinear
  - Fully saturated, suction/excess pressure, buoyant force
- Structure inelasticity/damage
  - Nonlinear/inelastic 1D fiber beam
  - Nonlinear/inelastic 3D shell/wall/plate element
- Fluid-Solid interaction, open surface fluid

Summary

- Reduction of modeling uncertainty
- Choice of analysis level of sophistication
- Numerical analysis to predict and inform
- Engineer needs to know!
- The Real ESSI Simulator system

http://real-essi.info