Aleatory Uncertainties in Computational Earthquake Engineering

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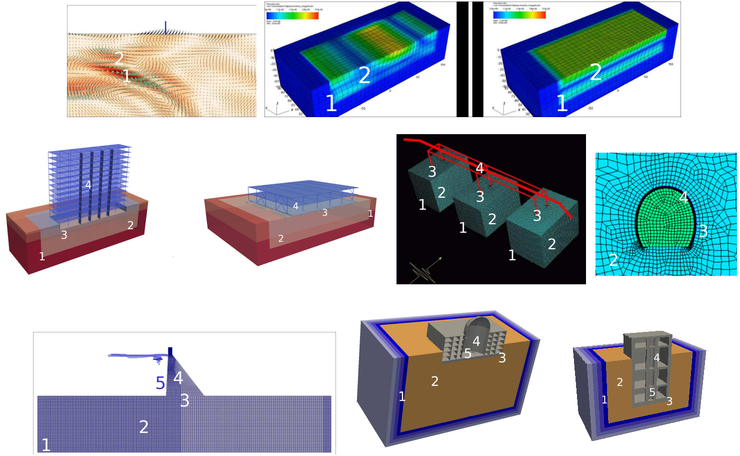
Outline

Motivation

- Safety and economy of infrastructure
- Design, build and maintain sustainable infrastructure
- Responsible Engineer, with Executive Powers
- Engineer with versatile, quality assured analysis tool to
  - Explore design concepts
  - Assess infrastructure performance
- Engineer needs to know!

Civil Engineering Analysis Challenges



Progress, Infrastructure

Nov1990

Jul2019

Infrastructure Digital Twin

- Infrastructure exists in three dimensions 3D
- Material behavior is nonlinear, inelastic
- Soil and Structure work together
- Modeling, epistemic uncertainty, analysis sophistication
- Parametric, aleatory uncertainty
  - Uncertain material parameters
  - Uncertain loads

Numerical Prediction under Uncertainty

- Modeling, Epistemic Uncertainty

  Modeling Simplifications

  Modeling sophistication for confidence in results

- Parametric, Aleatory Uncertainty

  $M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$,

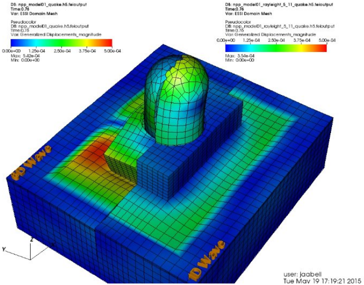
  Uncertain: mass M, viscous damping C and stiffness K^{ep}

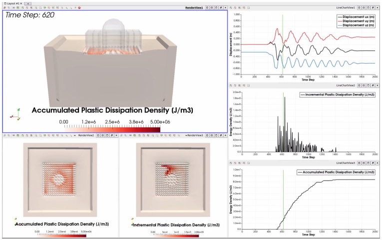
  Uncertain loads, $F(t)$

  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

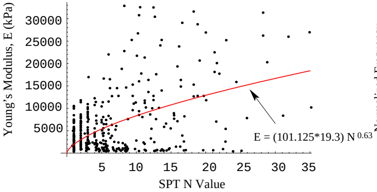
Modeling, Epistemic Uncertainty

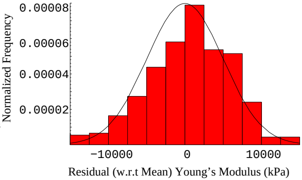
- Simplified modeling, 3D/2D/1D, 1C/2C/3C/6C. damping, viscous, elastic/el-pl, algorithmic
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that features being simplified out are not important (?!)



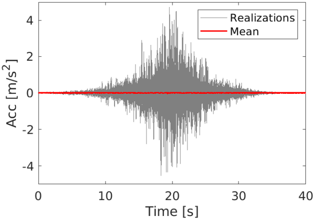


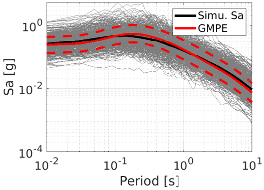
Parametric, Aleatory Uncertainty





(cf. Phoon and Kulhawy (1999B))





(cf. Wang et al. (2019))

Engineer Needs to Know!

- Forward propagation of uncertainty, full probabilistic, nonlinear/inelastic Earthquake-Soil-Structure-Interaction, ESSI response in time domain
  (Jeremic et al 2011, Wang et al 2019)

- Backward propagation, sensitivity analysis, quantifies the relative importance of input uncertain parameters on the variance of the probabilistic system response
  (Sobol 2001, Sudret 2008, Jeremic et al 2021)

Forward Uncertain Inelasticity

- Incremental el–pl constitutive equation

$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

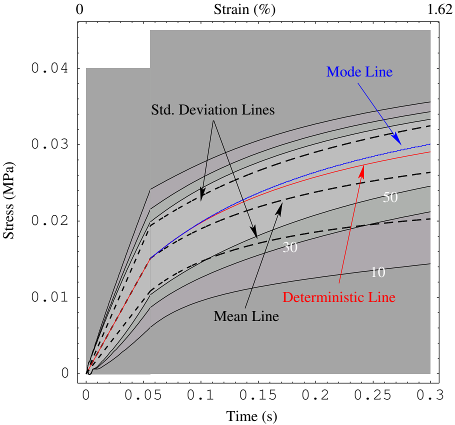
- Dynamic Finite Elements

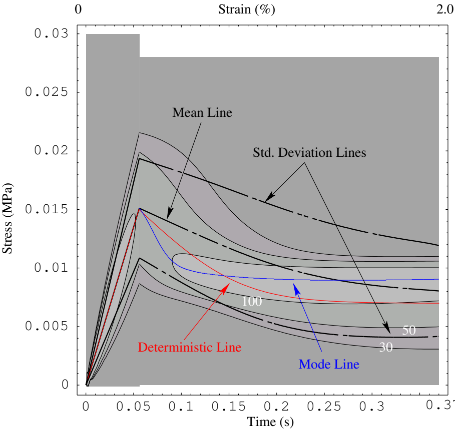
$$M\ddot{u}_i + C\dot{u}_i + K^{ep} u_i = F(t)$$

- Material behavior (LHS) is uncertain

- Loads (RHS) are uncertain

Cam Clay with Random G, M and p_0





Stochastic Elastic-Plastic FEM

Dynamic Finite Elements $M\ddot{u}_i + C\dot{u}_i + K^{ep}u_i = F(t)$

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary): Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients
- Output response process: Multi-dimensional Hermite PC with unknown coefficients
- Galerkin projection: minimize the error to compute unknown coefficients of response process
- SEPFEM eliminates Monte-Carlo inefficiency and inaccuracy

Stochastic Elastic-Plastic Finite Element Method

- Material uncertainty expanded into stochastic shape funcs.
- Loading uncertainty expanded into stochastic shape funcs.
- Displacement expanded into stochastic shape funcs.
- Jeremić et al. 2011

$$\begin{bmatrix} \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} \end{bmatrix} \begin{bmatrix} \Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\ \sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\ \vdots \\ \sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i > \end{bmatrix}$$

Sobol Sensitivity Analysis

- The ANalysis Of VAriance representation (Sobol 2001)
- Total variance of the probabilistic model response $y = f(\boldsymbol{X})$

$$D = \mathrm{Var}[f(\boldsymbol{X})] = \int_{I^n} f^2(\boldsymbol{x})\, d\boldsymbol{x} - f_0^2$$

- Sobol' indices $S_{i_1 \ldots i_s}$, fractional contributions from random inputs $\{X_{i_1}, ..., X_{i_s}\}$ to the total variance D: $S_{i_1 \ldots i_s} = D_{i_1 \ldots i_s}/D$
- Total sensitivity indices, influence of input parameter X_i

$$S_i^{\mathrm{total}} = \sum_{\mathscr{S}_i} D_{i_1 \ldots i_s}$$

Sobol-Sudret Sensitivity Analysis

- PC expansion of response in ANOVA form (Sudret 2008)
- Multi-dimensional PC bases $\{\Psi_j(\boldsymbol{\xi})\}$ decomposition

$$\Psi_j(\boldsymbol{\xi}) = \prod_{i=1}^{n} \phi_{\alpha_i}(\xi_i)$$

- ANOVA representation $\rightarrow$ PC-based Sobol' indices $S^{PC}_{i_1 \ldots i_s}$

$$S^{PC}_{i_1 \ldots i_s} = \sum_{\boldsymbol{\alpha} \in \mathscr{S}_{i_1,\ldots,i_s}} y^2_{\boldsymbol{\alpha}} \boldsymbol{E}\left[\Psi^2_{\boldsymbol{\alpha}}\right] / D^{PC}$$

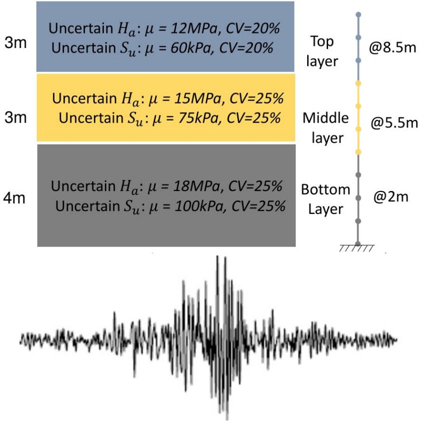
- Total Sobol' indices $S^{PC,\text{total}}_{j_1 \ldots j_t}$

$$S^{PC,\text{total}}_{j_1 \ldots j_t} = \sum_{(i_1,\ldots,i_s) \in \mathscr{S}_{j_1,\ldots,j_t}} S^{PC}_{i_1 \ldots i_s}$$

- Sobol-Sudret sensitivity indices within SEPFEM are analytic and inexpensive

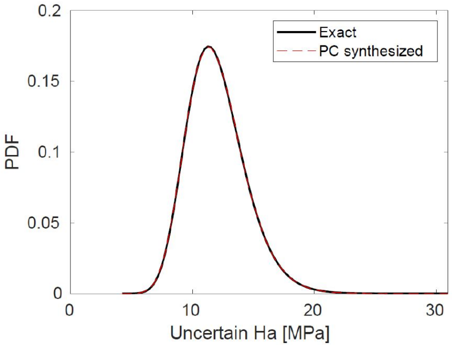
Example: Stochastic Site Response

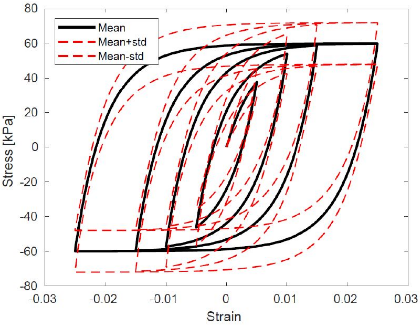
- Uncertain material: uncertain random field, marginally log-normal distribution, exponential correlation length 10m
- Uncertain seismic rock motions: seismic scenario M=7, R=50km



Stochastic Material Parameters

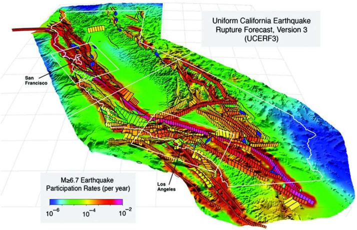
Log-normal distributed random field with PC Dim. 3 Order 2

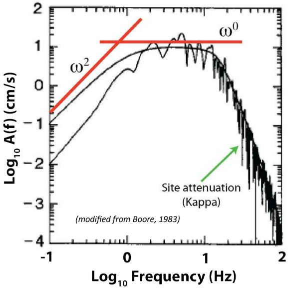




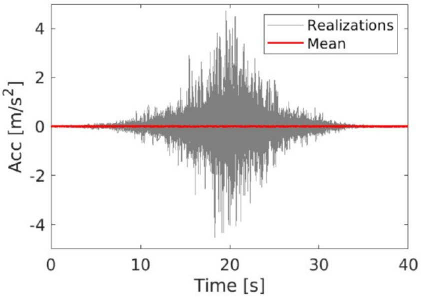
Stochastic Seismic Motion Development

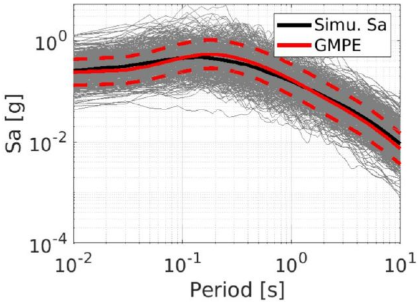
- UCERF3 (Field et al. 2014)
- Stochastic motions (Boore 2003)
- Polynomial Chaos Karhunen-Loève expansion
- Probabilistic DRM (Bielak et al. 2003, Wang et al. 2021)



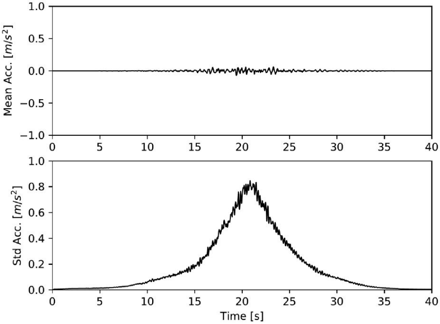


Stochastic Seismic Motions

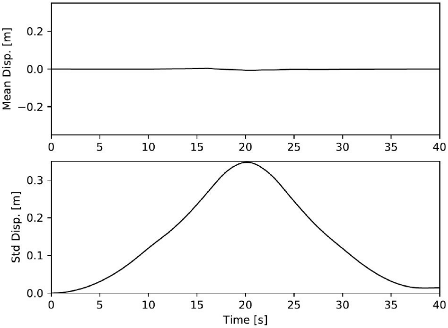




Stochastic Site Response, Mean and St.Dev.



Acceleration



Displacement

Sensitivity Analysis

Total variance in PGA, in this case (!), dominated by uncertain ground motions

- 49% from uncertain rock motions at depth
- 2% from uncertain soil
- 49% from interaction of uncertain rock motions and uncertain soil

Summary

- Numerical analysis to predict and inform
- Engineer needs to know!
- Real-ESSI Simulator System: `http://real-essi.info`
- Collaborators at UCDavis: Wang, Yang, Lacour, Staszewska
- Collaboration with and financial support from the US-NRC, US-NSF, US-DOE, UN-IAEA, is much appreciated
- "Le doute n'est pas une condition agréable, mais la certitude est absurde" (Voltaire)