

# Practical Methods for Verifying Computational Models

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# Outline

Introduction

Verification and Validation

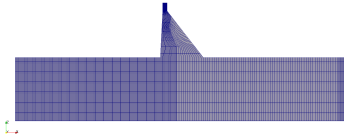
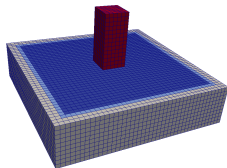
Summary

# Verification

- Source code management
- Source code verification
- Constitutive integration
- Static and dynamic behavior of single phase solids
- Static and dynamic behavior of fully and partially saturated, fully coupled, porous solid-pore fluid problems
- Static and dynamic behavior of structural elements
- Static and dynamic behavior of special elements (contacts-interface/gap-frictional/dry-saturated, isolators/dissipators)
- Static and dynamic FEM solution advancement
- Seismic wave propagation problems
- FEM Model verification, hierarchy of models

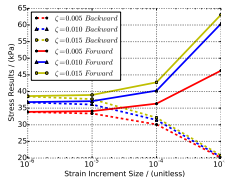
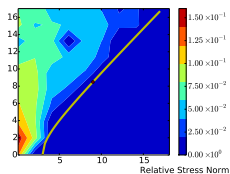
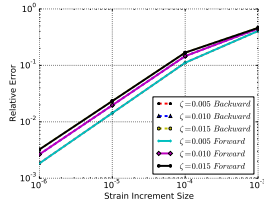
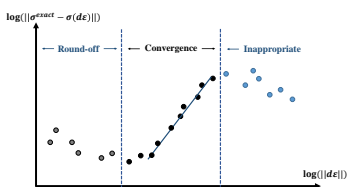
# Numerical Analysis Components

- Model physical components, ALL
- Solids and fluids
- Materials
- Static loads
- Dynamic loads
- Response quantities

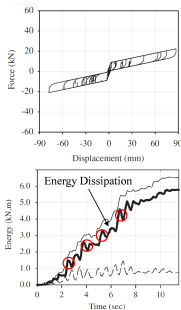
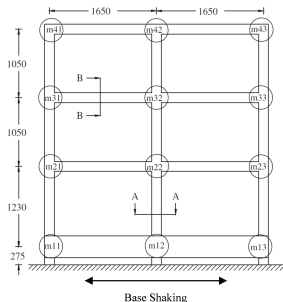


# Constitutive Integration Verification

- Asymptotic regime of convergence
- Richardson extrapolation
- Grid convergence index



# Energy Dissipation Verification: Plastic Work $\neq$ Plastic Dissipation



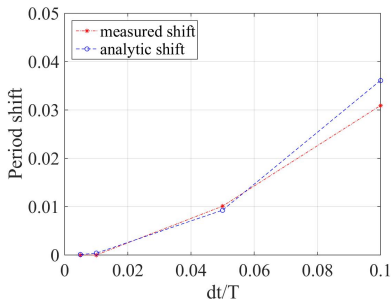
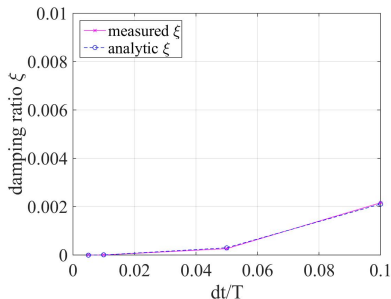
From a paper on *Soil Dynamics and Earthquake Engineering* (2011)

Direct violation of the second law of thermodynamics  
600 papers since 1990 (!?!) repeat this error

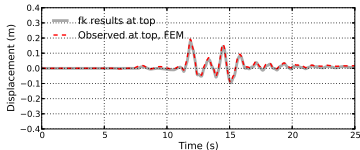
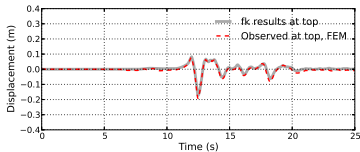
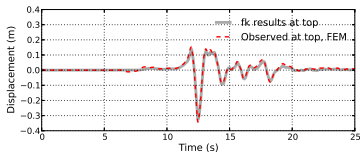
# Dynamic Time Stepping Verification

Based on the amplification matrix  $\mathbf{A}$ , to calculate the analytical solution of damping ratios and period shift.

Example: Hilber-Hughes-Taylor  $\alpha = -0.1$

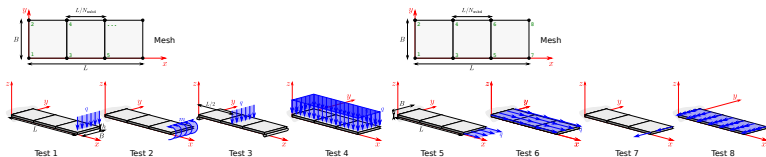


# Seismic Input Verification, DRM



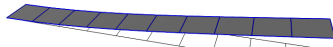


# Verification: ANDES Shell

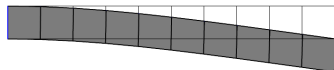


$N_{\text{subd}}$	$u_z$
2	96.2118
7	100.096
101	100.002

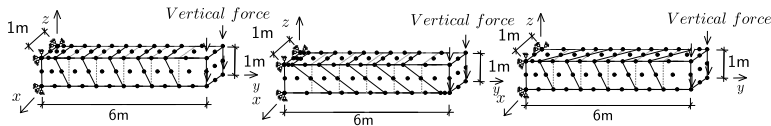
Mode 1, T = 0.999959s



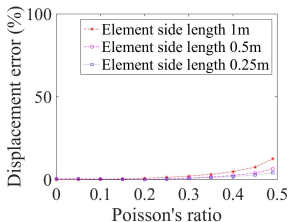
Mode 1, T = 0.998022s



# Verification: Irregular Solids and Poisson's Ratio

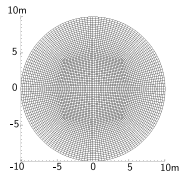
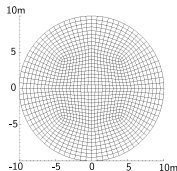
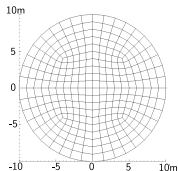
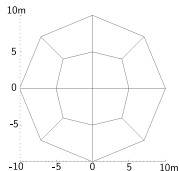


Force direction	Shape 1	Shape 2	Shape 3
Vertical ( $z$ )	0.40%	0.85%	0.60%
Transverse ( $y$ )	0.54%	3.67%	0.46%

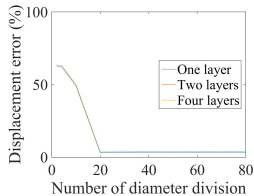
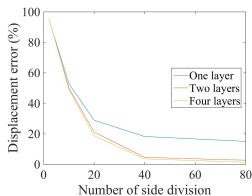


Poisson's ratio	27NodeBrick displacement	Theory displacement	Error
0.00	8.797E-04 m	8.784E-04 m	0.15%
0.05	8.801E-04 m	8.791E-04 m	0.11%
0.10	8.799E-04 m	8.799E-04 m	0.01%
0.15	8.792E-04 m	8.806E-04 m	0.16%
0.20	8.778E-04 m	8.813E-04 m	0.40%
0.25	8.758E-04 m	8.821E-04 m	0.71%
0.30	8.730E-04 m	8.828E-04 m	1.12%
0.35	8.692E-04 m	8.836E-04 m	1.63%
0.40	8.641E-04 m	8.844E-04 m	2.29%
0.45	8.567E-04 m	8.851E-04 m	3.21%
0.49	8.452E-04 m	8.857E-04 m	4.58%

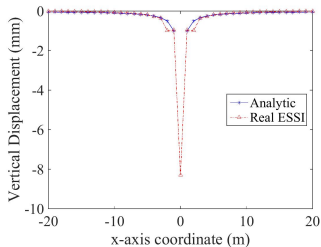
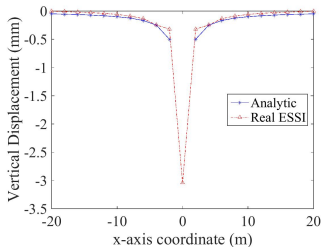
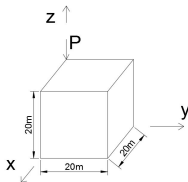
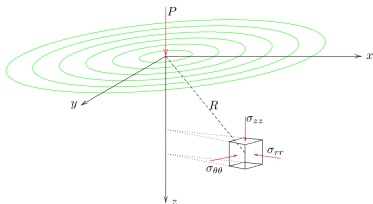
# Verification of Solid Shell/Plate



- Simply supported and clamped ends
- Timoshenko's analytic solutions

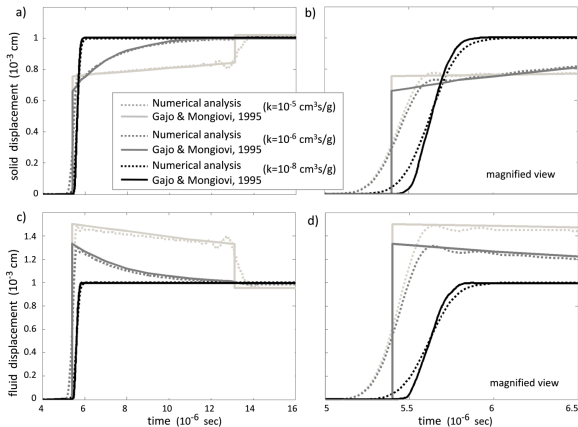


# Verification of Boussinesq Problem

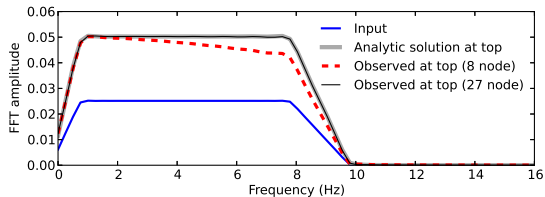
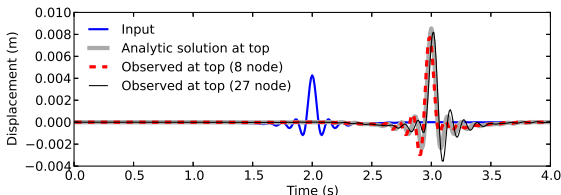




# Verification for Fully Coupled Problems

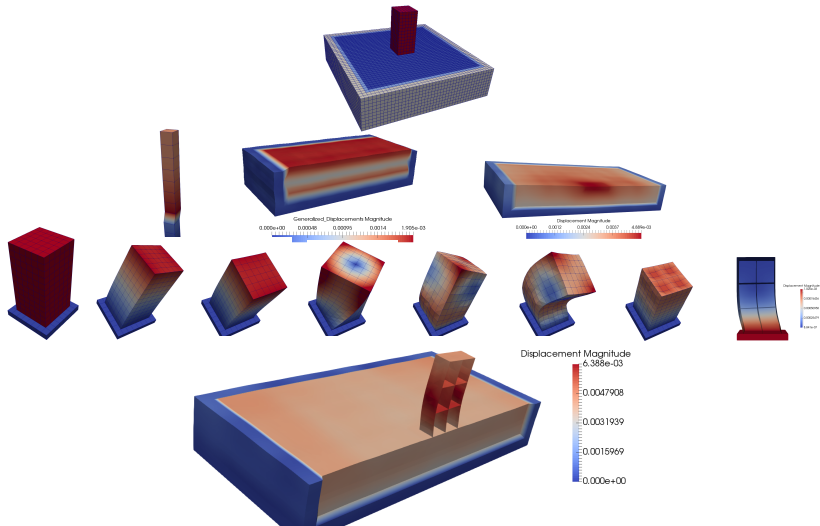


# Wave Propagation, Mesh Size Effects



(Case 1,  $V_s = 1000$  m/s, Cutoff Fq. = 8 Hz, E. Size = 20 m)

# ESSI Modeling Phases



# Summary

- Numerical analysis to predict and inform
- V&V most important for providing confidence in results
- Analysis of quality of numerical results
- Numerical analysis should not be used without V&V
- Engineer needs to know!
- Analysis governance
- Education and Training is the key!