Epistemic and Aleatory Uncertainties in Numerical Analysis of Earthquake Soil Structure Interaction

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24Jun2025

Outline

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Introduction

ESSI Uncertainties
- Modeling, Epistemic Uncertainties
- Aleatory, Parametric Uncertainties

Summary

Motivation

Safety and economy of infrastructure objects

Improve analysis of infrastructure objects

Earthquakes, Soils, Structures and their Interaction

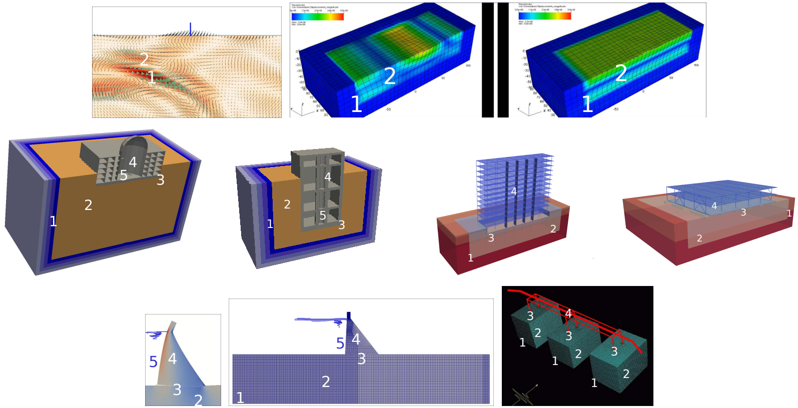
Modeling, Epistemic uncertainty

Parametric, Aleatory uncertainty

Goal is to predict and inform

Engineer needs to know!

ESSI Challenges



Numerical Prediction under Uncertainty

- Modeling, Epistemic Uncertainty

  Model simplifications

  Model sophistication level for confidence in results

- Parametric, Aleatory Uncertainty

  $M\Delta\ddot{u}_i + C\Delta\dot{u}_i + K^{ep}\Delta u_i = \Delta F(t)$,

  Uncertain: mass M, viscous damping C and stiffness K^{ep}

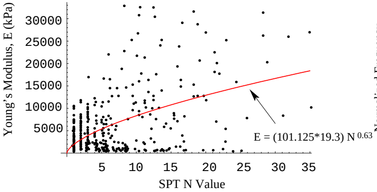
  Uncertain loads $F(t)$

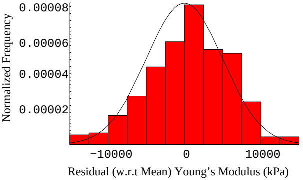
  Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

Modeling, Epistemic Uncertainty

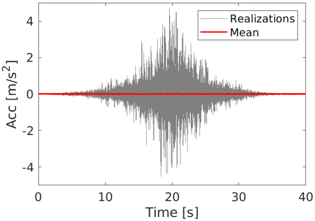
- Important (?!) features are simplified
  - 3C/6C vs 1C seismic motions
  - Elastic vs inelastic behavior
- Modeling simplifications are justifiable if one or two level higher sophistication model demonstrates that behavior being simplified out is not important

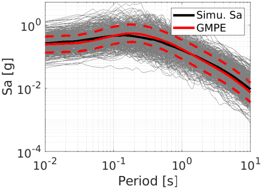
Parametric, Aleatory Uncertainty





(cf. Phoon and Kulhawy (1999B))





(cf. Wang et al. (2019))

Outline

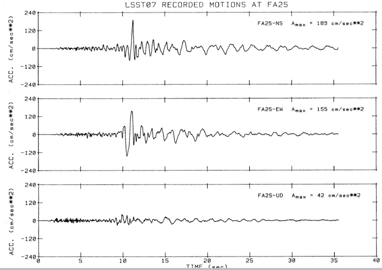
Modeling, Epistemic Uncertainties

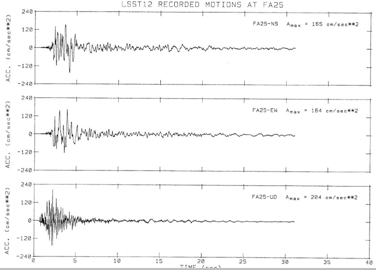
Modeling simplifications

- SSI vs nonSSI response
- Model geometry: 1D, 2D, 3D
- 1C vs 3C/6C seismic motions
- Elastic vs Inelastic behavior
- Energy dissipation, inelastic vs viscous

Real Earthquake Ground Motions

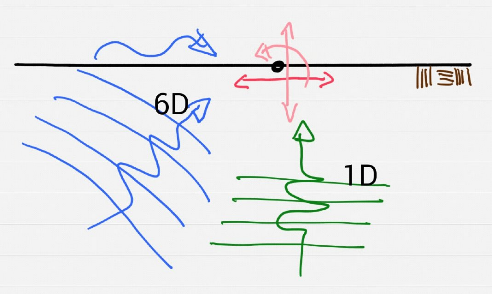
- Inclined body waves: P and S waves
- Surface waves: Rayleigh, Love waves
- Near surface waves: Stoneley waves...
- All, most measured motions are full 3C/6C (3t, 3r)
- Example EQ: 2C LSST07(L); 3C/6C LSST12(R)





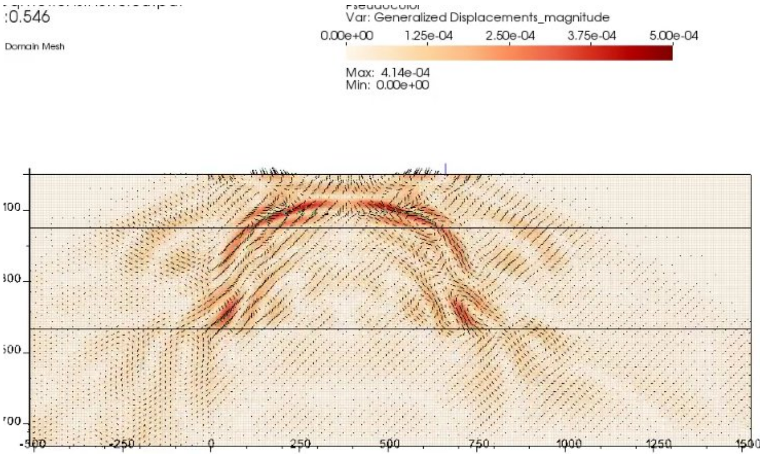
ESSI: 3C/6C vs 1C Seismic Motions

- Assume: full 3C/6C motions, recorded only 1C
- From 1C motions, de-convolute/convolute 1C
- Apply 6C and 1C to ESSI system



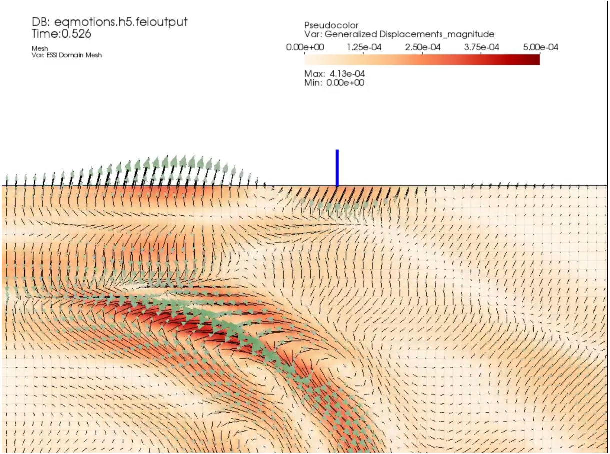
Realistic Ground Motions

- Free field, regional scale models



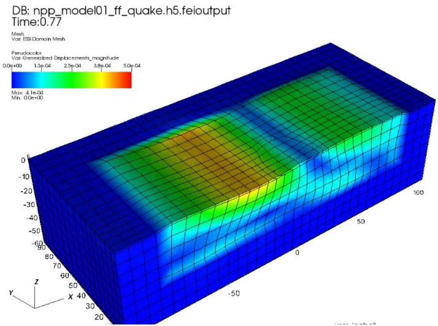
Development of Realistic Motions

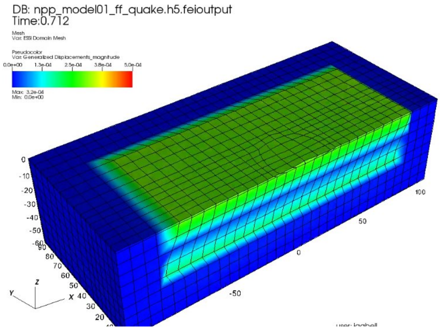
- Sources will send both P and S waves



1C vs 6C Free Field Motions

- One component of motions, 1C from 6C
- Excellent fit, wrong mechanics





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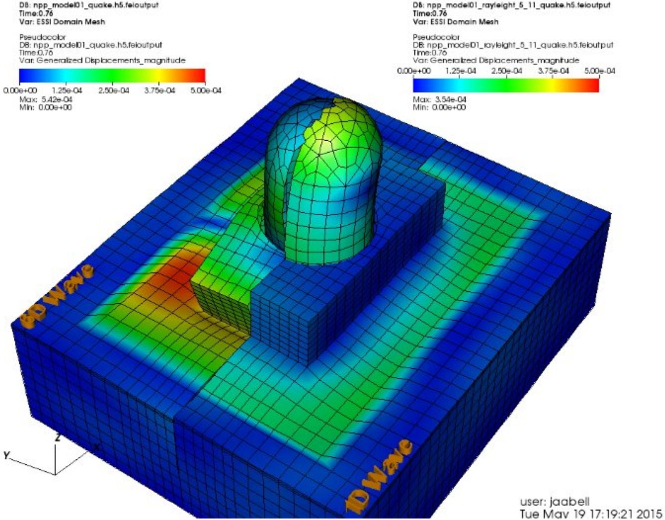
6C vs 1C NPP ESSI Response Comparison



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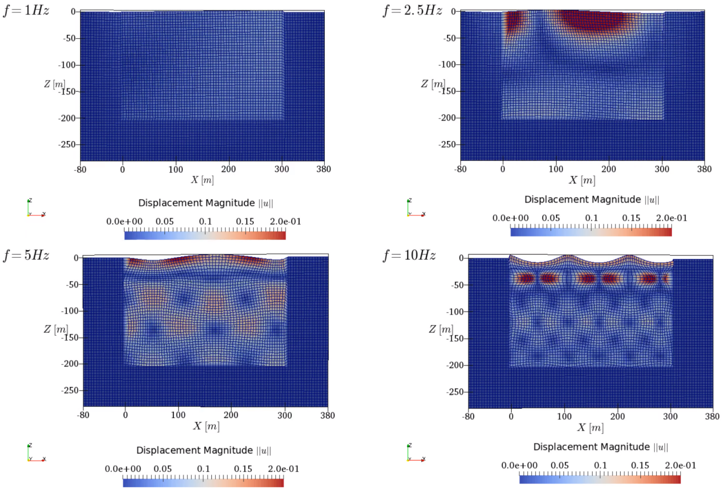
Realistic Seismic Wave Fields

- Stress test motions:
  - Variable wave length, frequency
  - Variable wave inclination
- Use surface and shallow motion measurements to develop full 6C wave field: 3D-deconvolution

Free Field, Variable Wave Length, $\theta = 60^o$



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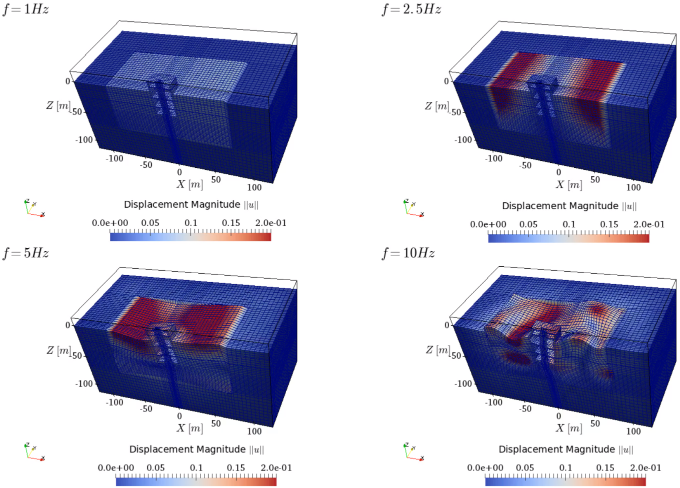
SMR ESSI, Variable Wave Length, $\theta = 60^o$



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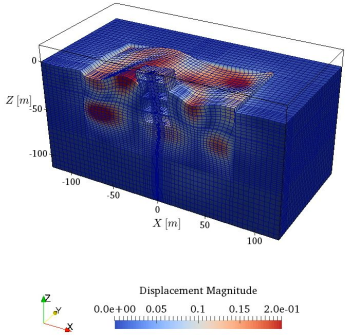
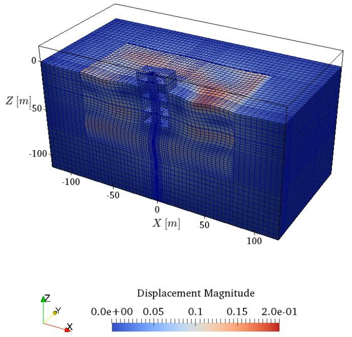
SMR ESSI, 3C vs 3×1C

$3C$



$3 \times 1C$



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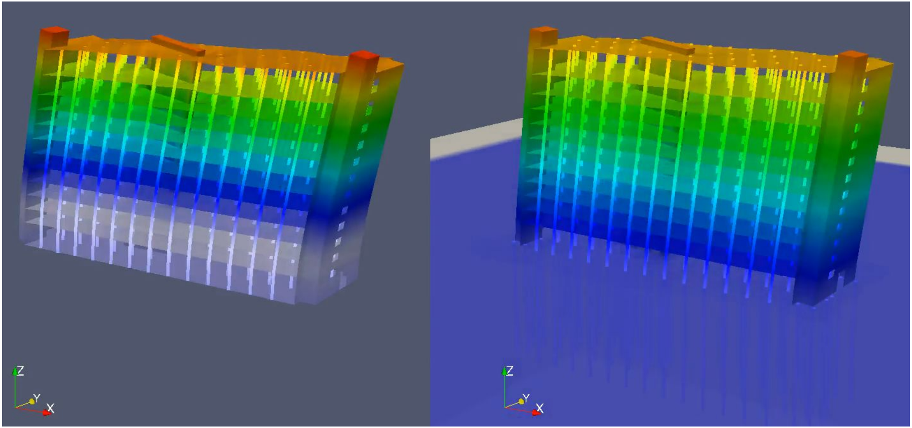
Ventura Hotel, Northridge Earthquake, nonSSI vs SSI



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Energy Input and Dissipation

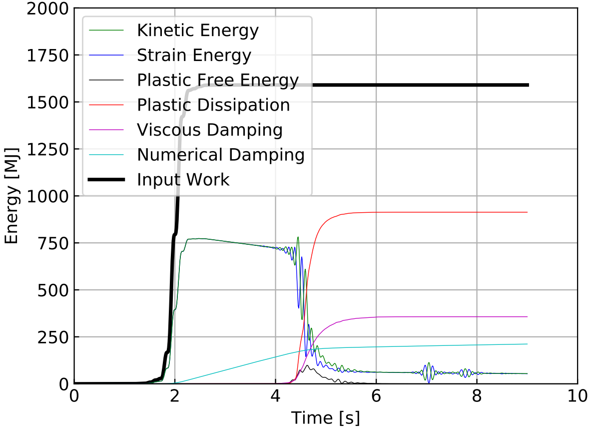
Energy input, dynamic forcing

Energy dissipation outside SSI domain:
- SSI system oscillation radiation
- Reflected wave radiation

Energy dissipation/conversion inside SSI domain:
- Inelasticity of soil, interfaces, structure, dissipators
- Viscous coupling with internal/pore and external fluids
- Energy deflectors, meta-materials

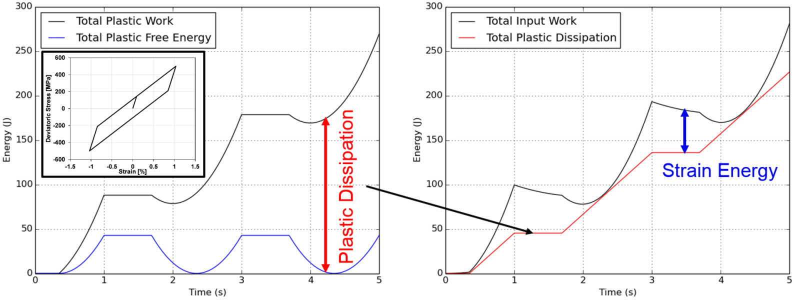
Numerical energy dissipation/production

Energy Dissipation Control

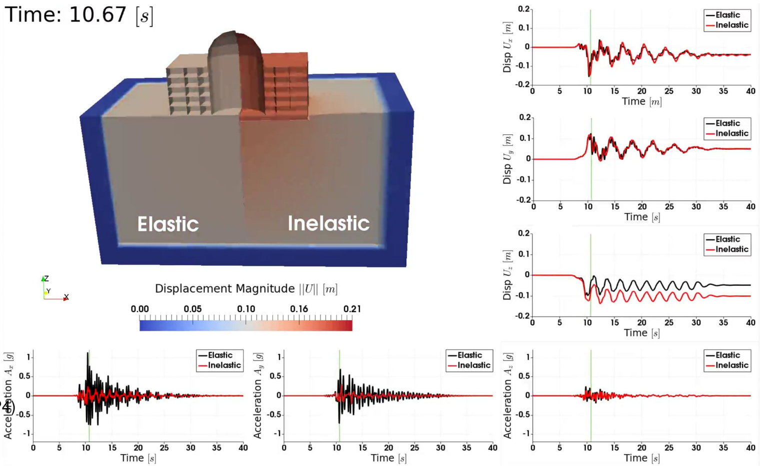


Plastic Energy Dissipation $\neq$ Plastic Work

Area of load-displacement loop is NOT plastic dissipation



Elastic vs Inelastic NPP Response



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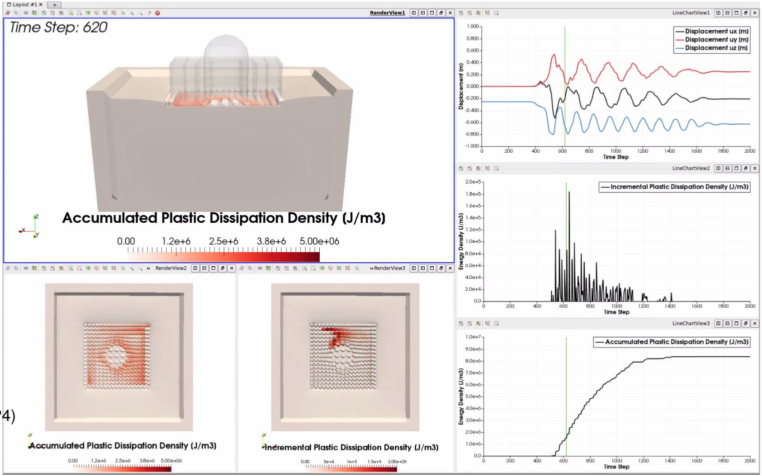
NPP Seismic Reponse, Energy Dissipation



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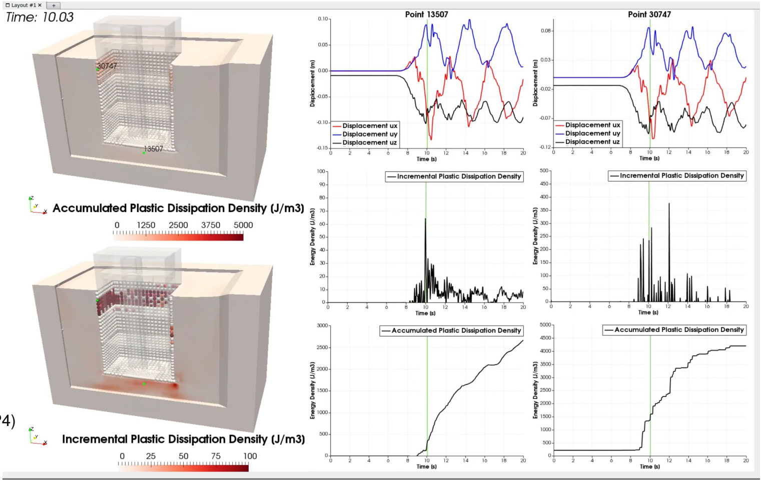
SMR Seismic Reponse, Energy Dissipation



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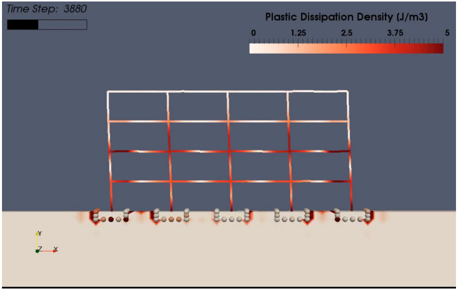
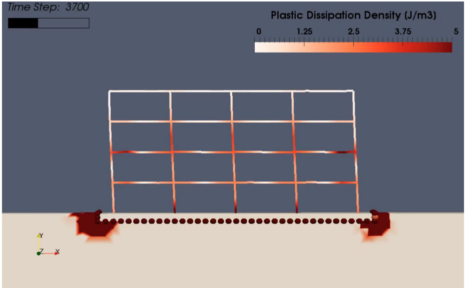
Design Alternatives





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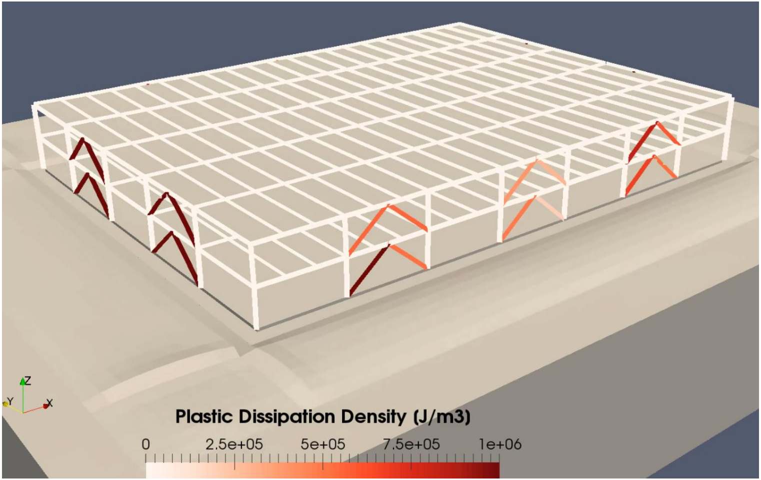
ASCE-7-21, Low Building: BRB Energy Dissipation



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Outline

Aleatory, Parametric Uncertainties

$$M\Delta\ddot{u}_i + C\Delta\dot{u}_i + K^{ep}\Delta u_i = \Delta F(t),$$

Uncertain: mass M, viscous damping C and stiffness K^{ep}

Uncertain loads $F(t)$

Results are PDFs and CDFs for σ_{ij}, ϵ_{ij}, u_i, $\dot{u}_i$, $\ddot{u}_i$

Forward Uncertainty Propagation

- Given uncertain material and uncertain loads
- Determine uncertain response, $u_i, \dot{u}_i, \ddot{u}_i, \epsilon_{ij}, \sigma_{ij}$, PDFs/CDFs
- Intrusive, analytic development, SEPFEM
- Avoid Monte Carlo inefficiencies

Forward Uncertain Inelasticity

- Incremental el–pl constitutive equation

$$\Delta\sigma_{ij} = E^{EP}_{ijkl}\ \Delta\epsilon_{kl} = \left[E^{el}_{ijkl} - \frac{E^{el}_{ijmn} m_{mn} n_{pq} E^{el}_{pqkl}}{n_{rs} E^{el}_{rstu} m_{tu} - \xi_* h_*} \right] \Delta\epsilon_{kl}$$

- Dynamic Finite Elements

$$M\Delta\ddot{u}_i + C\Delta\dot{u}_i + K^{ep}\Delta u_i = \Delta F(t)$$

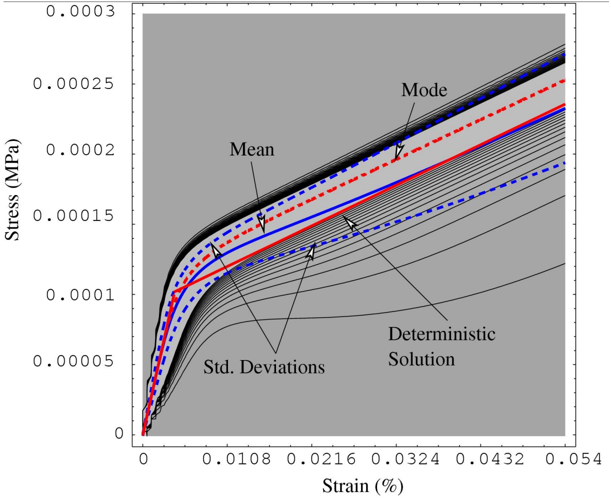
- Material and loads are uncertain

Stochastic Elastic-Plastic Finite Element Method

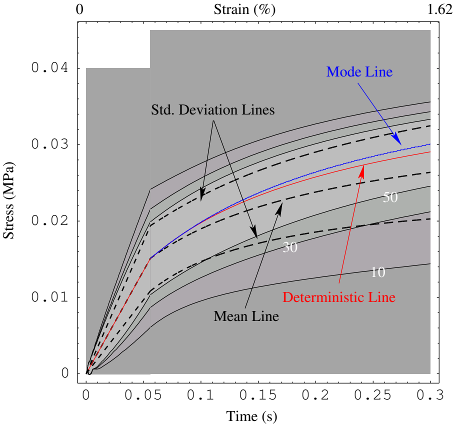
- Material uncertainty expanded into stochastic shape funcs.
- Loading uncertainty expanded into stochastic shape funcs.
- Displacement expanded into stochastic shape funcs.
- Jeremić et al. 2011

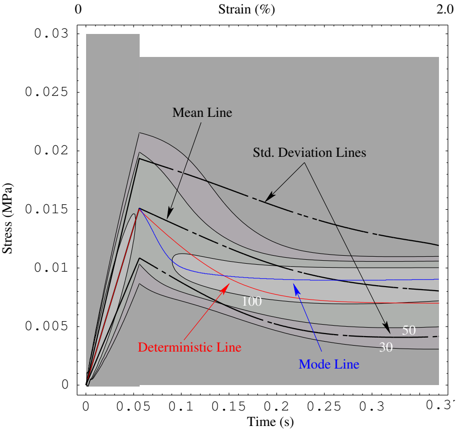
$$
\begin{bmatrix}
\sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_0 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_0 > K^{(k)} & \\
\sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_1 > K^{(k)} & \dots & \sum_{k=0}^{P_d} < \Phi_k \Psi_P \Psi_1 > K^{(k)} & \\
\vdots & \vdots & \vdots & \vdots \\
\sum_{k=0}^{P_d} < \Phi_k \Psi_0 \Psi_P > K^{(k)} & \dots & \sum_{k=0}^{M} < \Phi_k \Psi_P \Psi_P > K^{(k)} &
\end{bmatrix}
\begin{bmatrix}
\Delta u_{10} \\ \vdots \\ \Delta u_{N0} \\ \vdots \\ \Delta u_{1P_u} \\ \vdots \\ \Delta u_{NP_u}
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=0}^{P_f} f_i < \Psi_0 \zeta_i > \\
\sum_{i=0}^{P_f} f_i < \Psi_1 \zeta_i > \\
\sum_{i=0}^{P_f} f_i < \Psi_2 \zeta_i > \\
\vdots \\
\sum_{i=0}^{P_f} f_i < \Psi_{P_u} \zeta_i >
\end{bmatrix}
$$

Probabilistic Elastic-Plastic Response



Cam Clay with Random G, M and p_0



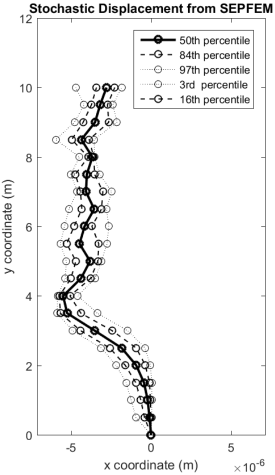


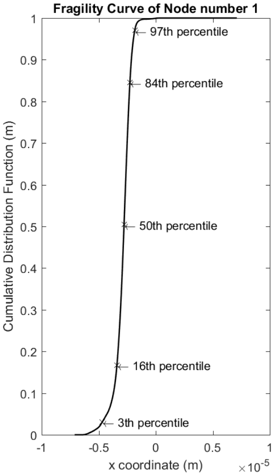
Stochastic Elastic-Plastic Finite Element Method

Dynamic Finite Elements $M\Delta\ddot{u}_i + C\Delta\dot{u}_i + K^{ep}\Delta u_i = \Delta F(t)$

- Input random field/process(non-Gaussian, heterogeneous/ non-stationary): Multi-dimensional Hermite Polynomial Chaos (PC) with known coefficients
- Output response process: Multi-dimensional Hermite PC with unknown coefficients
- Complete probabilistic response
- NO need to decide/define Intensity Measures (IMs) !

SEPFEM: Example in 1D





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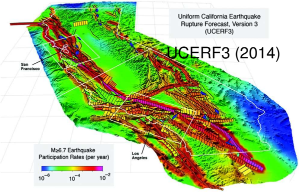
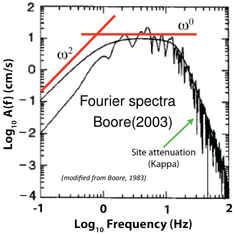
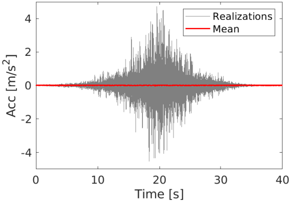
Application: Seismic Hazard

Seismic source characterization



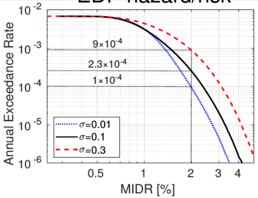
Stochastic ground motion



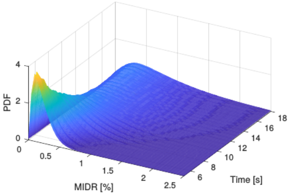


$$\lambda(EDP > z) = \sum N_i(M_i, R_i) P(EDP > z | M_i, R_i)$$

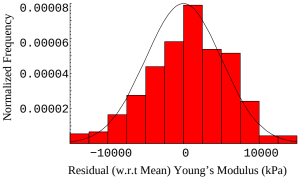
EDP hazard/risk



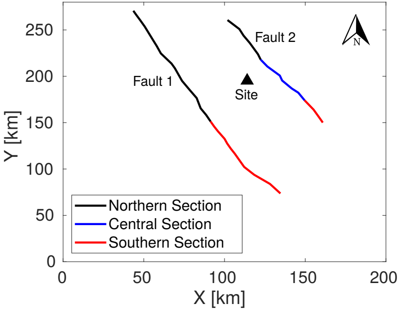
Uncertainty propagation
SEPFEM

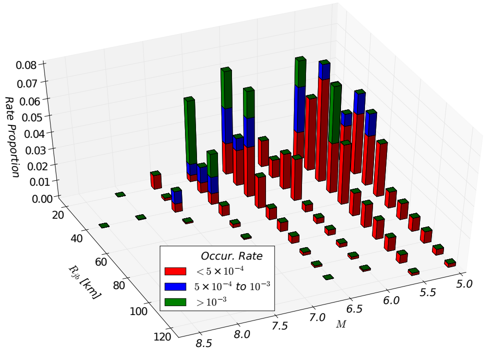


Uncertainty characterization
Hermite polynomial chaos



Example Object



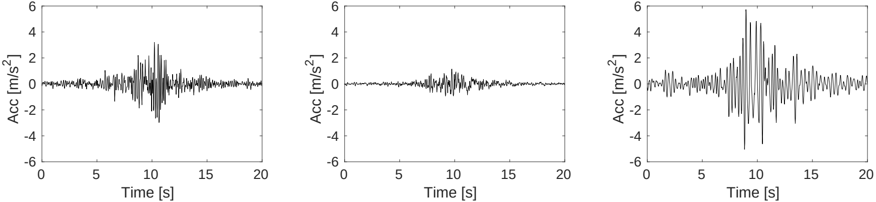


- Fault 1: San Gregorio fault
- Fault 2: Calaveras fault
- Uncertainty: Segmentation, slip rate, rupture geometry, etc.

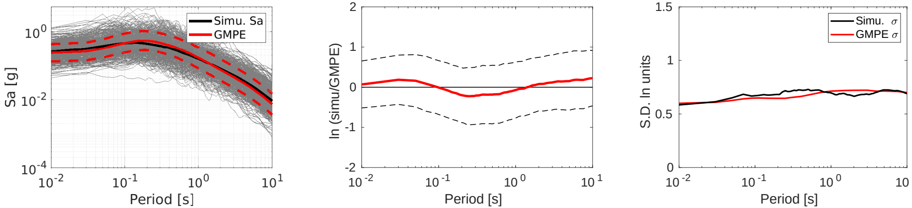
- 371 total seismic scenarios
- M 5 $\sim$ 5.5 and 6.5 $\sim$ 7.0
- R_{jb} 20km $\sim$ 40km

Stochastic Ground Motion Modeling

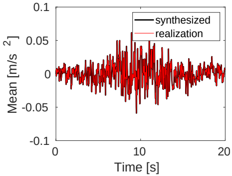
Realizations of simulated uncertain motions for scenario $M = 7$, $R = 15km$:



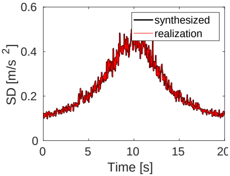
Verification with GMPE:



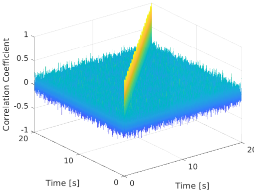
Stochastic Ground Motion Characterization



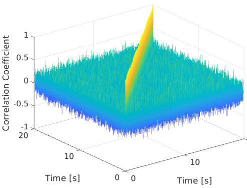
Acc. marginal mean



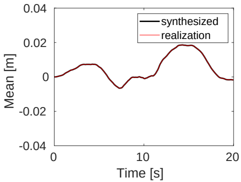
Acc. marginal S.D.



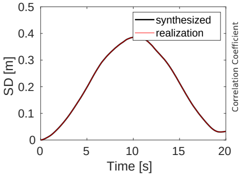
Acc. realization Cov.



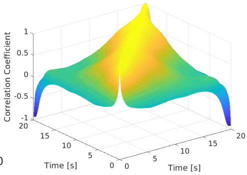
Acc. synthesized Cov.



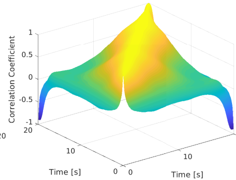
Dis. marginal mean



Dis. marginal S.D.

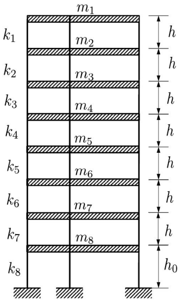


Dis. realization Cov.

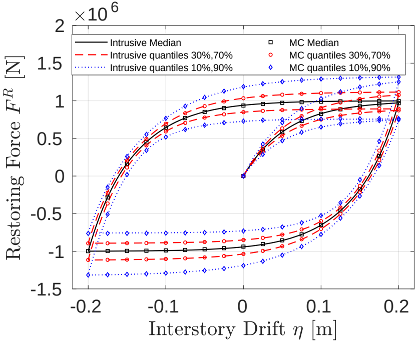


Dis. synthesized Cov.

Stochastic Soil, Structure Modeling



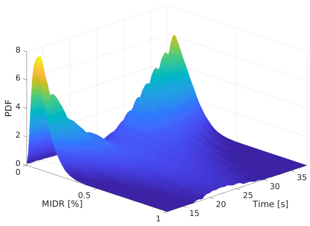
(a) Frame

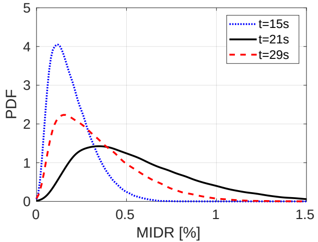


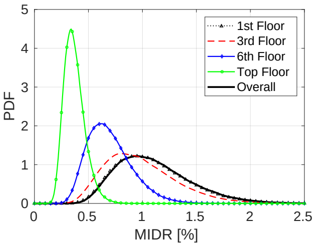
(b) Interstory response

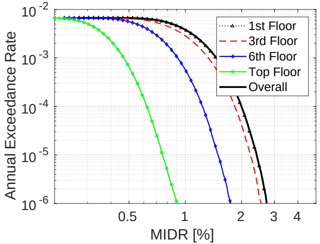
Seismic Risk Analysis

Engineering demand parameter (EDP): Maximum inter-story drift ratio (MIDR)



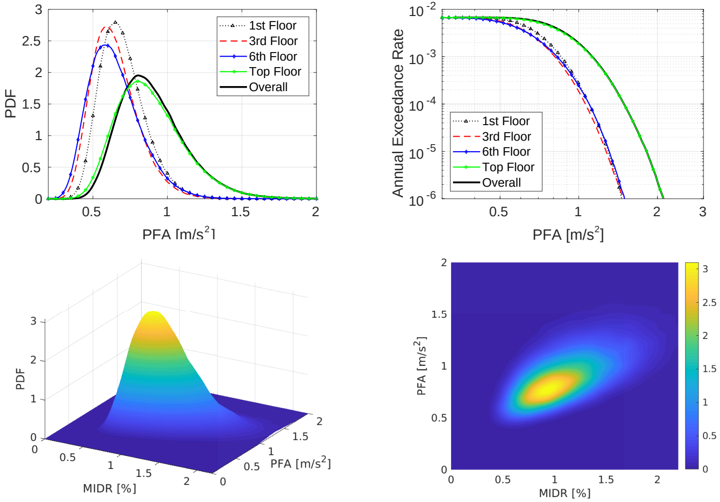






Seismic Risk Analysis

Engineering demand parameter (EDP): Peak floor acceleration (PFA)



Seismic Risk Analysis

- Damage measure defined on single EDP:

DM	MIDR>0.5%	**MIDR>1%**	MIDR>2%	PFA>0.5$\mathrm{m/s^2}$	**PFA>1$\mathbf{m/s^2}$**	PFA>1.5$\mathrm{m/s^2}$
Risk [/yr]	6.66×10^{-3}	$\mathbf{3.83\times10^{-3}}$	9.97×10^{-5}	6.65×10^{-3}	$\mathbf{1.92\times10^{-3}}$	9.45×10^{-5}

- Damage measure (DM) defined on multiple EDPs:

  DM : $\{\text{MIDR} > 1\% \cup \text{PFA} > 1\mathrm{m/s^2}\}$, seismic risk is $\mathbf{4.2\times10^{-3}}/yr$

  DM : $\{\text{MIDR} > 1\% \cap \text{PFA} > 1\mathrm{m/s^2}\}$, seismic risk is $\mathbf{1.71\times10^{-3}}/yr$

- Seismic risk for DM defined on multiple EDPs can be quite different from that defined on single EDP

Backward Uncertainty Propagation, Sensitivities

- Given forward uncertain response, PDFs, CDFs...
- Contributions of uncertain input to forward uncertainties
- Sensitivity of uncertain response to input uncertainties
- Sobol indices

Sobol Sensitivity Analysis

- The ANalysis Of VAriance representation (Sobol 2001)
- Total variance of the probabilistic model response $y = f(\boldsymbol{X})$

$$D = \mathrm{Var}[f(\boldsymbol{X})] = \int_{I^n} f^2(\boldsymbol{x})\, d\boldsymbol{x} - f_0^2$$

- Sobol' indices $S_{i_1 \dots i_s}$, fractional contributions from random inputs $\{X_{i_1}, ..., X_{i_s}\}$ to the total variance D: $S_{i_1 \dots i_s} = D_{i_1 \dots i_s}/D$
- Total sensitivity indices, influence of input parameter X_i

$$S_i^{\mathrm{total}} = \sum_{\mathscr{S}_i} D_{i_1 \dots i_s}$$

Sobol-Sudret Sensitivity Analysis

- PC expansion of response in ANOVA form (Sudret 2008)
- Multi-dimensional PC bases $\{\Psi_j(\boldsymbol{\xi})\}$ decomposition

$$\Psi_j(\boldsymbol{\xi}) = \prod_{i=1}^{n} \phi_{\alpha_i}(\xi_i)$$

- ANOVA representation $\rightarrow$ PC-based Sobol' indices $S^{PC}_{i_1 \ldots i_s}$

$$S^{PC}_{i_1 \ldots i_s} = \sum_{\boldsymbol{\alpha} \in \mathscr{S}_{i_1, \ldots, i_s}} y^2_{\boldsymbol{\alpha}} \boldsymbol{E}\left[\Psi^2_{\boldsymbol{\alpha}}\right] / D^{PC}$$

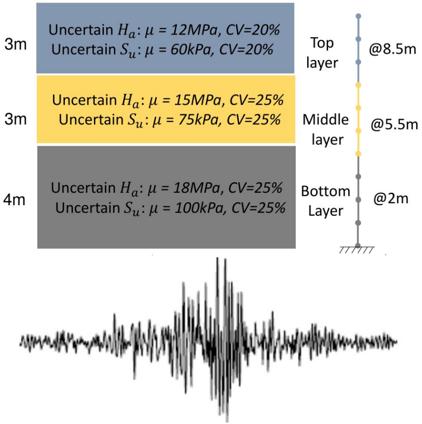
- Total Sobol' indices $S^{PC,\text{total}}_{j_1 \ldots j_t}$

$$S^{PC,\text{total}}_{j_1 \ldots j_t} = \sum_{(i_1, \ldots, i_s) \in \mathscr{S}_{j_1, \ldots, j_t}} S^{PC}_{i_1 \ldots i_s}$$

- Sobol-Sudret sensitivity indices within SEPFEM are analytic and inexpensive

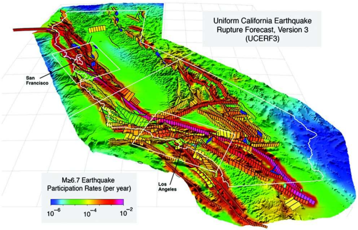
Application: Stochastic Site Response

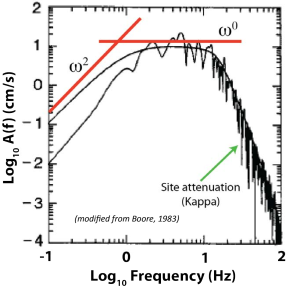
- Uncertain material: uncertain random field, marginally lognormal distribution, exponential correlation length 10m
- Uncertain seismic rock motions: seismic scenario M=7, R=50km



Stochastic Seismic Motion Development

- UCERF3 (Field et al. 2014)
- Stochastic motions (Boore 2003)
- Polynomial Chaos Karhunen-Loève expansion
- Probabilistic DRM (Bielak et al. 2003, Wang et al. 2021)





Sensitivity Analysis

Total variance in PGA, in this particular case (!), dominated by uncertain rock motions at depth

- 49% from uncertain rock motions at depth
- 2% from uncertain soil
- 49% from interaction of uncertain rock motions and uncertain soil

Outline

Appropriate Science and Engineering Quotes

- François-Marie Arouet, Voltaire: "Le doute n'est pas une condition agréable, mais la certitude est absurde"
- Max Planck: "A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it"
- Niklaus Wirth: "Software is getting slower more rapidly than hardware becomes faster."

Summary

- Engineering analysis uncertainties
  Modeling, Epistemic
  Parametric, Aleatory
- Engineering analysis to predict and inform
- Engineer needs to know!
- Real-ESSI Simulator $\rightarrow$ `http://real-essi.us/`
- Collaborators at UCD: Yang, Sinha, Wang, Lacour, Wang, Pisanó, Abell, Sett, Tafazzoli, Jie, Preisig, Tasiopoulou, Watanabe, Luo, Cheng, Yang, Kanellopoulos, Staszewska
- Funding from and collaboration with the ATC/US-FEMA, US-DOE, US-NRC, US-NSF, CNSC-CCSN, UN-IAEA, Shimizu, PEER, Caltrans, ENSI-CH, KEPCO, EDF, is greatly appreciated!