Seismic Wave Propagation in Stochastic Soils

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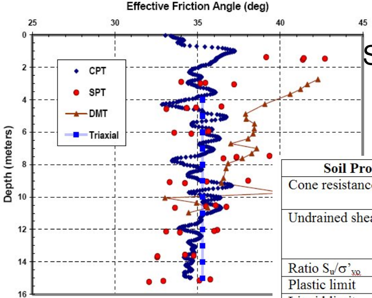
Outline

Soils are Inherently Uncertain



Spatial Variation of Friction Angle (Mayne et al. (2000))

Soil Property	Soil Type	PDF	Mean	COV (%)
Cone resistance	Sand Clay Clay	LN N/LN	*	*
Undrained shear strength	Clay (triaxial) Clay (index S_u) Clayey silt	LN LN N	*	5-20 10-35 5-15
Ratio S_u/σ'_{vo}	Clay	N/LN	*	5-15
Plastic limit	Clay	N	0.13-0.23	3-20
Liquid limit	Clay	N	0.30-0.80	3-20
Submerged unit weight	All soils	N	5-11 (kN/m^3)	0-10
Friction angle	Sand	N	*	2-5
Void ratio, porosity, initial void ratio	All soils	N	*	7-30
Over consolidation ratio	Clay	N/LN	*	10-35

Typical COVs of Different Soil Properties (Lacasse and Nadim 1996)

Characterization and Quantification

- Natural variability of soil deposit (Phoon and Kulhawy 1999, Fenton 1999) $\rightarrow$ function of soil formation process
  - Ergodic assumption might not strictly apply (Der Kiureghian 2005)
- Testing error (Phoon and Kulhawy 1999, Marosi and Hiltunen 2004, Stokoe et al. 2004)
  - Imperfection of instruments
  - Error in methods to register quantities
- Transformation error (Phoon and Kulhawy 1999)
  - Correlation by empirical data fitting (e.g. CPT data $\rightarrow$ friction angle etc.)

Our Original Developments

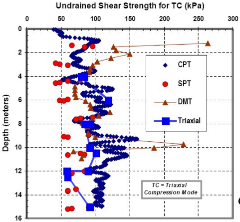
Constitutive : Second-order accurate (exact mean and variance) PDF of stress-strain response (Fokker – Planck – Kolmogorov Equation)

Spatial : Spectral Stochastic Elastic – Plastic Finite Element Method, to simulate uncertain spatial variability of elastic–plastic soils

- Obtain complete probabilistic description (PDF) for:
  - Stresses–Strain response
  - Displacements (velocities, accelerations)
- Use for:
  - Sensitivity analysis
  - Probability of failure (tails of PDF)
  - Probabilistic site characterization design

Input Soil Parameters Random Fields

- Truncated Karhunen–Loevé (KL) expansion
- Representation of input random fields in eigen-modes of covariance kernel



$$S_u(x,\theta) = \bar{S}_u(x) + \sum_{n=1}^{M} \sqrt{\lambda_n}\xi_n(\theta) f_n(x)$$

$$\int_D C(x_1, x_2) f(x_2) dx_2 = \lambda f(x_1)$$

$$\xi_i(\theta) = \frac{1}{\sqrt{\lambda_i}} \int_D [S_u(x,\theta) - \bar{S}_u(x)] f_i(x) dx$$

- Error minimizing property
- Optimal expansion $\rightarrow$ minimization of number of stochastic dimensions

Stochastic Elastic–Plastic Finite Elements

$$\sum_{n=1}^{N} K_{mn}^{e,ep} d_{ni} + \sum_{n=1}^{N} \sum_{j=0}^{P} d_{nj} \sum_{k=1}^{M} C_{ijk} K'^{e,ep}_{mnk} = \langle F_m \psi_i[\{\zeta_r\}] \rangle$$

$$K_{mn} = \int_D B_n D B_m dV \quad ; \quad K'_{mnk} = \int_D B_n \sqrt{\lambda}_k h_k B_m dV$$

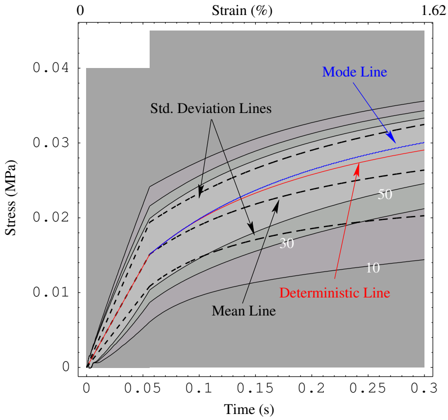
$$C_{ijk} = \langle \zeta_k(\theta) \psi_i[\{\zeta_r\}] \psi_j[\{\zeta_r\}] \rangle \quad ; \quad F_m = \int_D \phi N_m dV$$

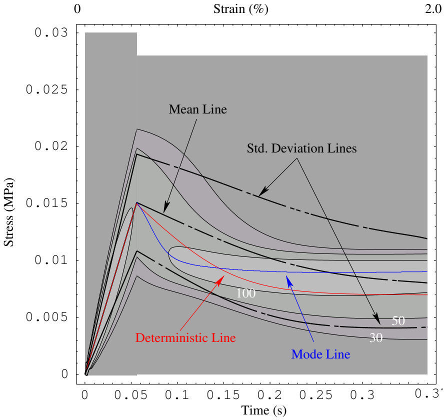
- Based on SSFEM (Ghanem and Spanos (2003))
- Random **material** and random **forcing**
- Efficient representation of input random fields into finite number of random variables using KL-expansion
- Representation of (unknown) solution random variables using polynomial chaos of (known) input random variables

Probabilistic Elasto–Plasticity

- Probabilistic elastic–plastic constitutive incremental equation $\Delta\sigma_{ij} = D^{e,ep}_{ijkl} \Delta\epsilon_{kl}$
  - Random stiffness $D^{e,ep}_{ijkl}$
  - Random strain increment $\Delta\epsilon_{kl}$
- Use of Euler Lagrange form of Fokker–Planck–Kolmogorov (FPK) equation (Kavvas 2003) to obtain
- Second-order accurate (exact mean and variance) stress–strain solution (PDF)
- Complete probabilistic description of response $\rightarrow$ PDF
- FPK equation is applicable to any elastic–plastic material model

1–D Low and High OCR Cam Clay

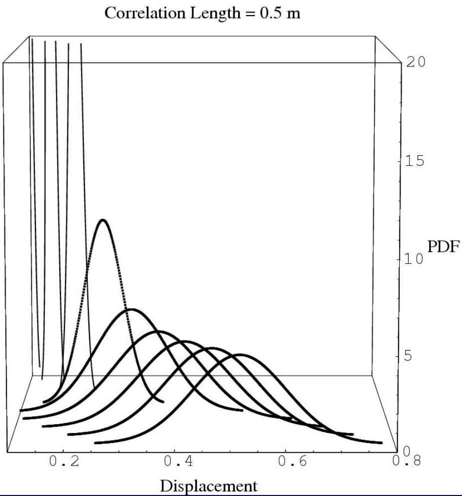




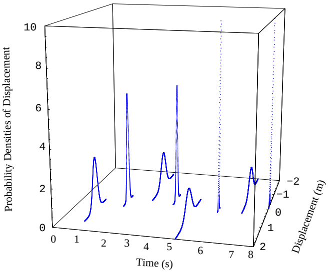
random G, M and p_0

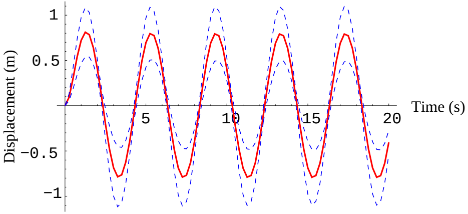
1–D Shear Column Example

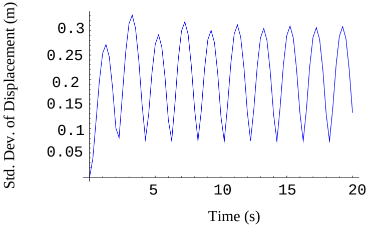
- Static pushover of a Stochastic shear column 10m high (deep)
- Small correlation length results in mean that tends to deterministic solution
- High correlation length increases mean and standard deviation



Stochastic Seismic Ground Motions







- Sinusoidal motions example
- Complete PDF of motions for each time step.
- In general, increase in system uncertainty

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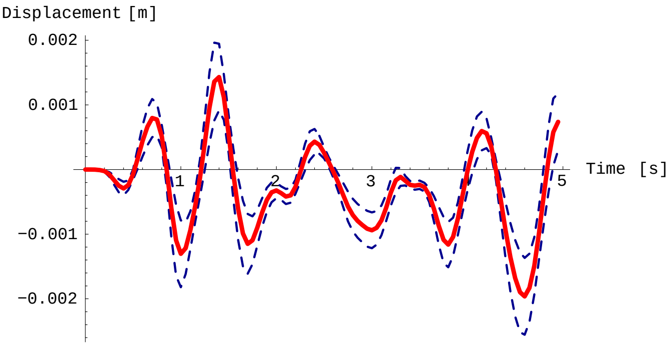

- Example motions: Imperial Valley
- Mean and SD of ground motions
- Large uncertainty at ground motion peaks

Summary

- A new, second-order accurate (exact mean and variance) formulation for probabilistic elastic–plastic soil simulation
- Analytic modeling and simulations of
  - spatial variability and
  - point-wise uncertainty

  of soil elastic of elastic–plastic properties for static and dynamic problems
- Application to:
  - Sensitivity analysis
  - Probability of failure (tails of PDF)
  - Site characterization design (probabilistic)
  - General stochastic modeling in elastic–plastic solid and structural mechanics