On Uncertain Seismic Wave Propagation

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Stein Sture
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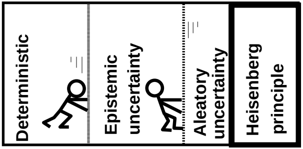
Outline

Personal Motivation

- Probabilistic fish counting
- Stein Sture's lectures, MGM project
- David Muir Wood's book
- John Williams' DEM sims., displacement vortexes
- Kaspar Willam's spec. topics course, comments at EM98
- Kenneth Runesson's explanations on bifurcation
- Lev Kavvas' probabilistic hydrology

On Uncertainties

- Epistemic uncertainty - due to lack of knowledge
  - Can be reduced by collecting more data
  - Mathematical tools not well developed, trade-off with aleatory uncertainty



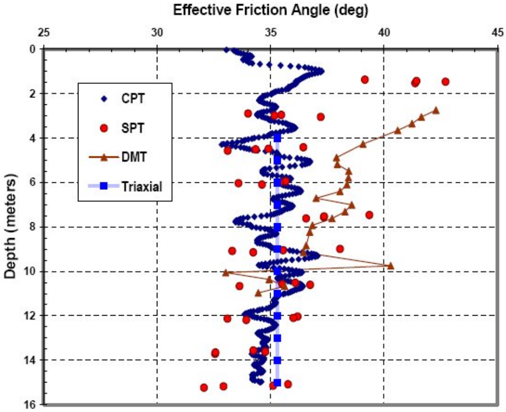
- Aleatory uncertainty - inherent variation of physical system
  - Can not be reduced
  - Has highly developed mathematical tools
- Ergodicity – exchange ensemble average for time average?
  - Possibly yes
  - Issues up for discussion (soil, concrete, rock, biomaterials...)

Historical Overview

- Brownian motion, Langevin equation $\rightarrow$ PDF governed by simple diffusion Eq. (Einstein 1905)
- With external forces $\rightarrow$ Fokker-Planck-Kolmogorov (FPK) for the PDF (Kolmogorov 1941)
- Approach for random forcing $\rightarrow$ relationship between the autocorrelation function and spectral density function (Wiener 1930)
- Approach for random coefficient $\rightarrow$ Functional integration approach (Hopf 1952), Averaged equation approach (Bharrucha-Reid 1968), Numerical approaches, Monte Carlo method

Material Behavior Inherently Uncertainties

- Spatial variability
- Point-wise uncertainty, testing error, transformation error



(Mayne et al. (2000)

Problem Setup

- Incr. 3D el–pl:
  $$d\sigma_{ij}/dt = \left\{ D^{el}_{ijkl} - \frac{D^{el}_{ijmn} m_{mn} n_{pq} D^{el}_{pqkl}}{n_{rs} D^{el}_{rstu} m_{tu} - \xi_* r_*} \right\} d\epsilon_{kl}/dt$$
- phase density ρ of $\sigma(x,t)$ varies in time according to a continuity Liouville equation (Kubo 1963)
- Continuity equation written in ensemble average form (eg. cumulant expansion method (Kavvas and Karakas 1996))
- van Kampen's Lemma (van Kampen 1976) $\rightarrow$ $< \rho(\sigma, t) >= P(\sigma, t)$, ensemble average of phase density is the probability density

Eulerian–Lagrangian FPK Equation

$$
\begin{aligned}
\frac{\partial P(\sigma(x_t,t),t)}{\partial t} &= -\frac{\partial}{\partial \sigma}\left[\left\{\left\langle \eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t))\right\rangle\right.\right. \\
&+ \int_0^t d\tau Cov_0\left[\frac{\partial \eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t))}{\partial \sigma};\right. \\
&\left.\left.\left.\eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau)\right]\right\} P(\sigma(x_t,t),t)\right] \\
&+ \frac{\partial^2}{\partial \sigma^2}\left[\left\{\int_0^t d\tau Cov_0\left[\eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t));\right.\right.\right. \\
&\left.\left.\left.\eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau))\right]\right\} P(\sigma(x_t,t),t)\right]
\end{aligned}
$$

B. Jeremić, K. Sett, and M. L. Kavvas, "Probabilistic Elasto–Plasticity: Formulation in 1–D", *Acta Geotechnica*, Vol. 2, No. 3, 2007, In press (published online in the *Online First* section)

Euler–Lagrange FPK Equation

- Advection-diffusion equation

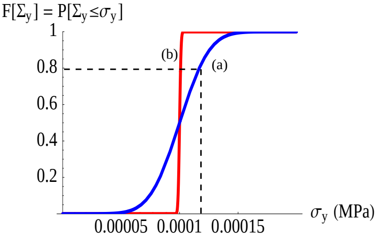
$$\frac{\partial P(\sigma,t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left[N_{(1)}P(\sigma,t) - \frac{\partial}{\partial \sigma}\left\{N_{(2)}P(\sigma,t)\right\}\right]$$

- Complete probabilistic description of response
- Solution PDF is second-order exact to covariance of time (exact mean and variance)
- It is deterministic equation in probability density space
- It is linear PDE in probability density space $\rightarrow$ Simplifies the numerical solution process
- Template FPK diffusion–advection equation is applicable to any material model $\rightarrow$ only the coefficients $N_{(1)}$ and $N_{(2)}$ are different for different material models

K. Sett, B. Jeremić and M.L. Kavvas, "The Role of Nonlinear Hardening/Softening in Probabilistic Elasto–Plasticity", *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 31, No. 7, pp. 953-975, 2007

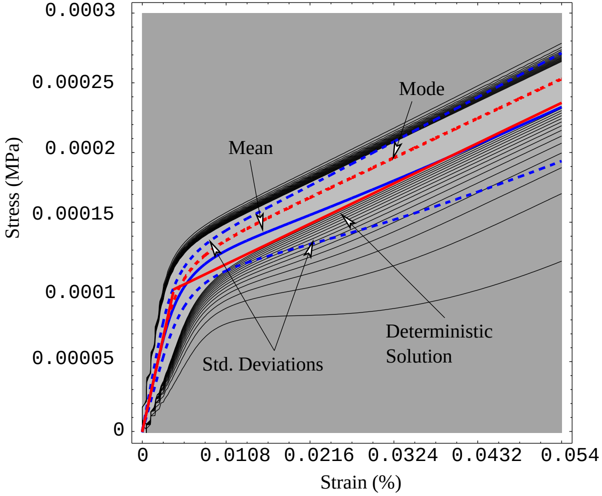
Probabilistic Yielding

- Weighted elastic and elastic–plastic Solution
  $\partial P(\sigma,t)/\partial t = -\partial \left(N^{w}_{(1)} P(\sigma,t) - \partial \left(N^{w}_{(2)} P(\sigma,t) \right) / \partial\sigma \right) / \partial\sigma$
- Weighted advection and diffusion coefficients are then
  $N^{w}_{(1,2)}(\sigma) = (1 - P[\Sigma_y \leq \sigma]) N^{el}_{(1)} + P[\Sigma_y \leq \sigma] N^{el-pl}_{(1)}$
  (Black–Scholes options pricing model '73, Nobel Economics Prize '97)
- Cumulative Probability Density function (CDF) of the yield function



B. Jeremić and K. Sett. On Probabilistic Yielding of Materials. in print in *Communications in Numerical Methods in Engineering*, 2007.

Transformation of a Bi–Linear (von Mises) Response



linear elastic – linear hardening plastic von Mises

Spectral Stochastic Elastic–Plastic FEM

- Minimizing norm of error of finite representation using Galerkin technique (Ghanem and Spanos 2003):

$$\sum_{n=1}^{N} K_{mn} d_{ni} + \sum_{n=1}^{N} \sum_{j=0}^{P} d_{nj} \sum_{k=1}^{M} C_{ijk} K'_{mnk} = \langle F_m \psi_i[\{\xi_r\}] \rangle$$

$$K_{mn} = \int_D B_n D B_m dV \qquad K'_{mnk} = \int_D B_n \sqrt{\lambda}_k h_k B_m dV$$

$$C_{ijk} = \left\langle \xi_k(\theta) \psi_i[\{\xi_r\}] \psi_j[\{\xi_r\}] \right\rangle \qquad F_m = \int_D \phi N_m dV$$

Inside SEPFEM

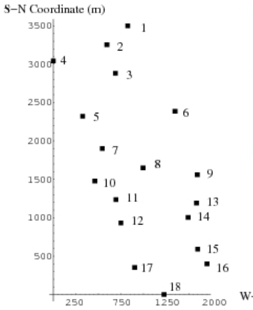
- Explicit stochastic elastic–plastic finite element computations
- FPK probabilistic constitutive integration at Gauss integration points
- Increase in (stochastic) dimensions (KL and PC) of the problem (parallelism)
- Development of the probabilistic elastic–plastic stiffness tensor

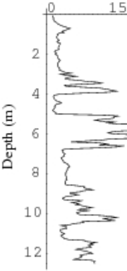
Seismic Wave Propagation through Stochastic Soil

- Soil as 12.5 m deep 1–D soil column (von Mises Material)
  - Properties (including testing uncertainty) obtained through random field modeling of CPT q_T
    $\langle q_T \rangle = 4.99\ MPa$; $Var[q_T] = 25.67\ MPa^2$;
    Cor. Length $[q_T] = 0.61\ m$; Testing Error $= 2.78\ MPa^2$
- q_T was transformed to obtain G: $G/(1-\nu) = 2.9 q_T$
  - Assumed transformation uncertainty = 5%
    $\langle G \rangle = 11.57 MPa$; $Var[G] = 142.32 MPa^2$
    Cor. Length $[G] = 0.61 m$
- Input motions: modified 1938 Imperial Valley

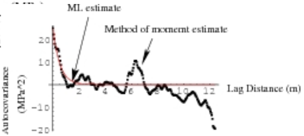
Random Field Parameters from Site Data

- Maximum likelihood estimates

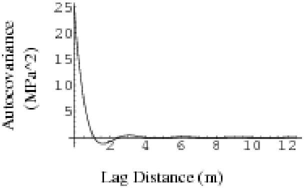




Typical CPT q_T

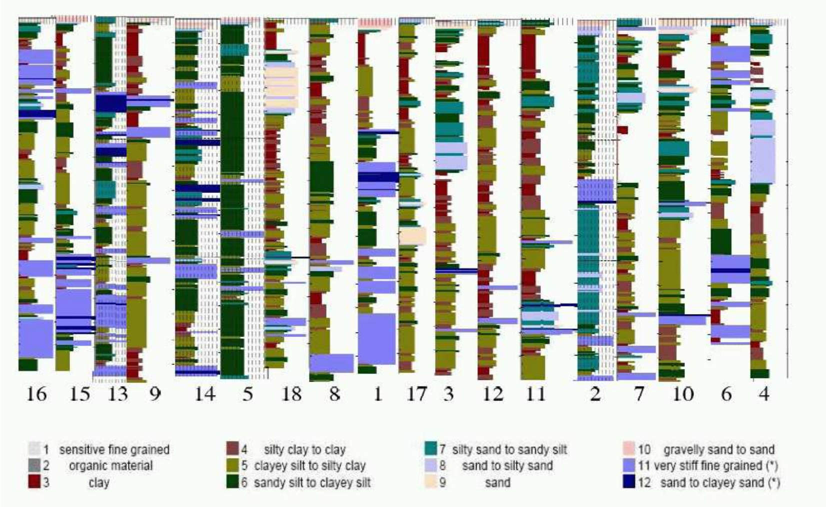


Finite Scale

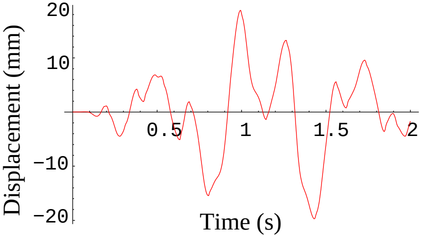


Fractal

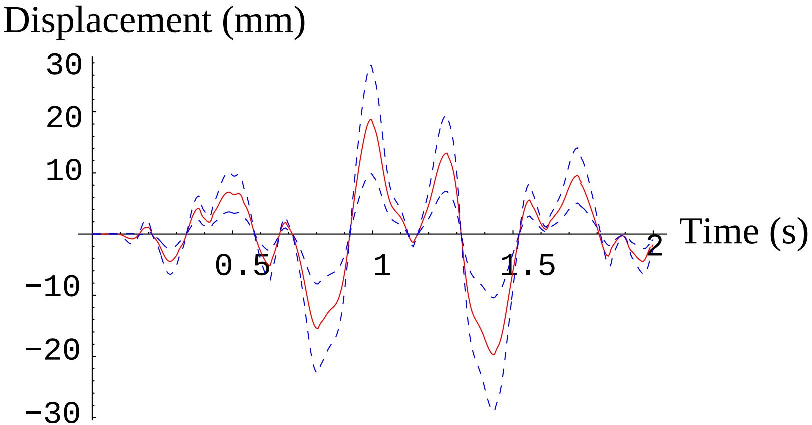
“Uniform” CPT Site Data



Surface Displacement Time History

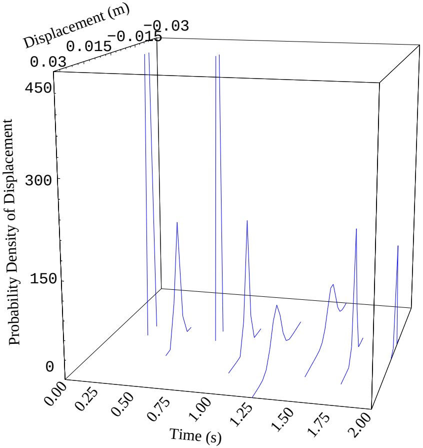


Mean $\pm$ Standard Deviation

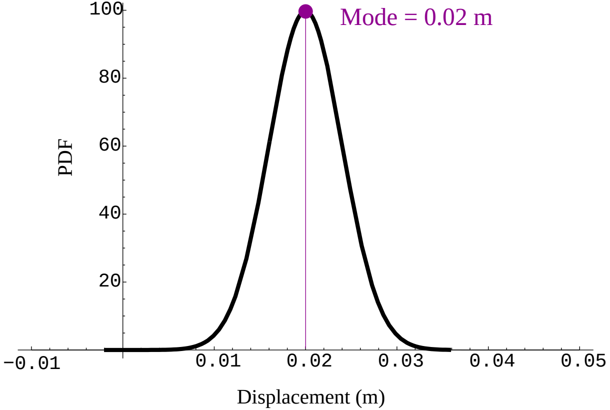


PDF of Surface Displacement Time History

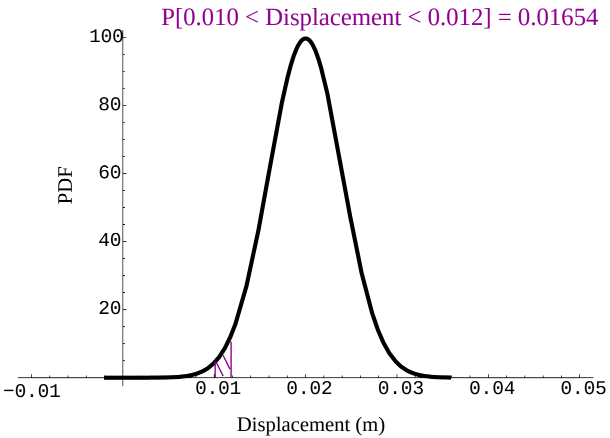
- PDF at the finite element nodes can be obtained using, e.g., Edgeworth expansion (Ghanem and Spanos 2003)
- Numerous applications, especially where extreme statistics are critical



Most Probable Solution

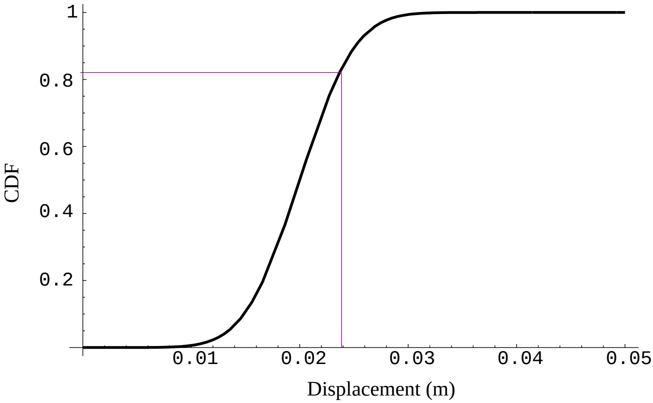


Tails of PDF


P[0.010 < Displacement < 0.012] = 0.01654
100
80
60
40
20
PDF
−0.01
0.01
0.02
0.03
0.04
0.05
Displacement (m)

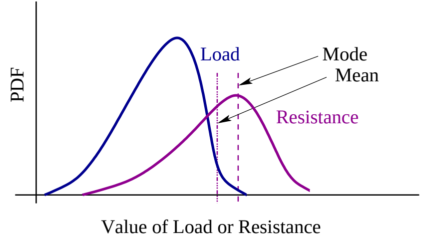
Probability of Exceedance

Probability that displacement exceeds 0.025 m = 1 − 0.825 = 0.175

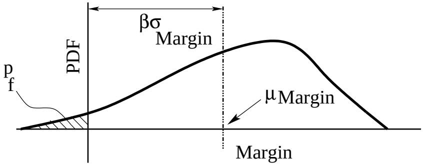


Derivative Applications

- Performance based engineering
  - Reliability index (β)
  - Probability of failure (p_f)



$$\text{Margin} = \text{Reistance} - \text{Load}$$



- Financial risk analysis
- Sensitivity analysis

Summary

- Work on probabilistic elasto–plasticity as continuation of work with Stein Sture at CU Bolder
- Getting older, supposed to have more answers, in fact I have more questions

Thank you Stein!