Soil Uncertainty and Seismic Ground Motion

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Outline

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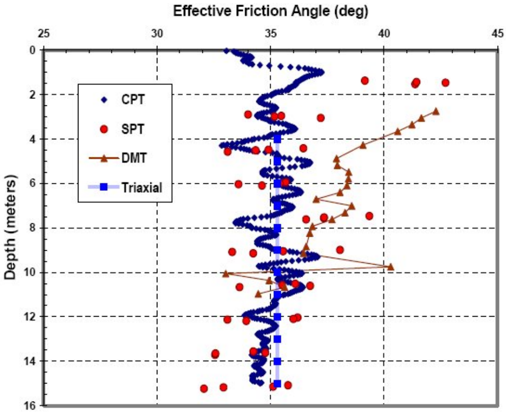
Soils are Inherently Uncertain

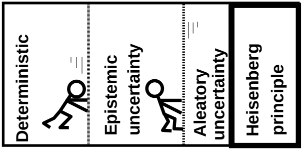
- Spatial variability
- Point-wise uncertainty,
- Testing error,
- Transformation error



(Mayne et al. (2000)

On Uncertainties

- Epistemic uncertainty - due to lack of knowledge
    - Can be reduced by collecting more data
    - Mathematical tools not well developed, trade-off with aleatory uncertainty



- Aleatory uncertainty - inherent variation of physical system
    - Can not be reduced
    - Has highly developed mathematical tools
- Ergodicity – exchange ensemble average for time average?
    - Possibly yes
    - Issues up for discussion (soil, concrete, rock, biomaterials...)

Soil Uncertainties and Quantification

- Natural variability of soil deposit (Fenton 1999)
  - Function of soil formation process

- Testing error (Stokoe et al. 2004)
  - Imperfection of instruments
  - Error in methods to register quantities

- Transformation error (Phoon and Kulhawy 1999)
  - Correlation by empirical data fitting (e.g. CPT data → friction angle etc.)

Probabilistic material (Soil Site) Characterization

- Ideal: complete probabilistic site characterization
- Large (physically large but not statistically) amount of data
  - Site specific mean and coefficient of variation (COV)
  - Covariance structure from similar sites (e.g. Fenton 1999)
- Moderate amount of data → Bayesian updating (e.g. Phoon and Kulhawy 1999, Baecher and Christian 2003)
- Minimal data: general guidelines for typical sites and test methods (Phoon and Kulhawy (1999))
  - COVs and covariance structures of inherent variability
  - COVs of testing errors and transformation uncertainties.
- Marosi and Hiltunen (2004) and Stokoe et al. (2004) extended the general guidelines for SASW method and G/G_{max} curve

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Problem Setup

- Incr. 3D el–pl:
  $$d\sigma_{ij}/dt = \left\{ D^{el}_{ijkl} - \frac{D^{el}_{ijmn} m_{mn} n_{pq} D^{el}_{pqkl}}{n_{rs} D^{el}_{rstu} m_{tu} - \xi_* r_*} \right\} d\epsilon_{kl}/dt$$
- phase density ρ of $\sigma(x,t)$ varies in time according to a continuity Liouville equation (Kubo 1963)
- Continuity equation written in ensemble average form (eg. cumulant expansion method (Kavvas and Karakas 1996))
- van Kampen's Lemma (van Kampen 1976) → $< \rho(\sigma,t) >= P(\sigma,t)$, ensemble average of phase density is the probability density

Eulerian–Lagrangian FPK Equation

$$
\begin{aligned}
\frac{\partial P(\sigma(x_t,t),t)}{\partial t} &= -\frac{\partial}{\partial \sigma}\Bigg[\Bigg\{\Bigg\langle \eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t))\Bigg\rangle \\
+ \quad & \int_0^t d\tau \, Cov_0 \Bigg[\frac{\partial \eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t))}{\partial \sigma}; \\
& \eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau)\Bigg]\Bigg\} P(\sigma(x_t,t),t)\Bigg] \\
+ \quad & \frac{\partial^2}{\partial \sigma^2}\Bigg[\Bigg\{\int_0^t d\tau \, Cov_0 \Bigg[\eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t)); \\
& \eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau))\Bigg]\Bigg\} P(\sigma(x_t,t),t)\Bigg]
\end{aligned}
$$

B. Jeremić, K. Sett, and M. L. Kavvas, "Probabilistic Elasto–Plasticity: Formulation in 1–D", *Acta Geotechnica*, Vol. 2, No. 3, 2007, In press (published online in the *Online First* section)

Euler–Lagrange FPK Equation

- Advection-diffusion equation

$$\frac{\partial P(\sigma,t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left[N_{(1)}P(\sigma,t) - \frac{\partial}{\partial \sigma}\left\{N_{(2)}P(\sigma,t)\right\}\right]$$

- Complete probabilistic description of response
- Solution PDF is second-order exact to covariance of time (exact mean and variance)
- It is deterministic equation in probability density space
- It is linear PDE in probability density space → Simplifies the numerical solution process
- Template FPK diffusion–advection equation is applicable to any material model → only the coefficients $N_{(1)}$ and $N_{(2)}$ are different for different material models

K. Sett, B. Jeremić and M.L. Kavvas, "The Role of Nonlinear Hardening/Softening in Probabilistic Elasto–Plasticity", *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 31, No. 7, pp. 953-975, 2007

Spectral Stochastic Elastic–Plastic FEM

$$\sum_{n=1}^{N} K_{mn} d_{ni} + \sum_{n=1}^{N} \sum_{j=0}^{P} d_{nj} \sum_{k=1}^{M} C_{ijk} K'_{mnk} = \langle F_m \psi_i[\{\xi_r\}] \rangle$$

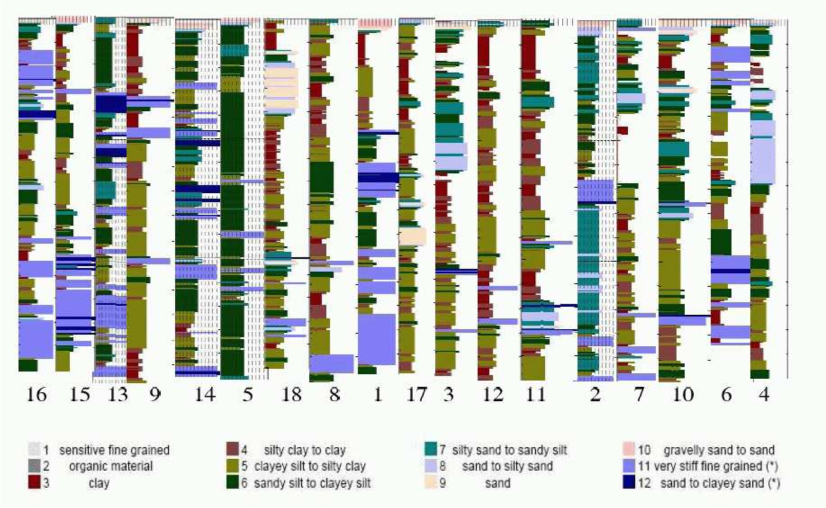
$$K_{mn} = \int_D B_n D B_m dV \qquad K'_{mnk} = \int_D B_n \sqrt{\lambda_k} h_k B_m dV$$

$$C_{ijk} = \langle \xi_k(\theta) \psi_i[\{\xi_r\}] \psi_j[\{\xi_r\}] \rangle \qquad F_m = \int_D \phi N_m dV$$

- SFEM: Ghanem and Spanos 2003
- Material variables random field represented through a finite number of random variables using KL-expansion
- Unknown solution random variables represented using polynomial chaos of (known) input random variables
- Fokker–Planck–Kolmogorov approach based probabilistic constitutive integration at Gauss integration points

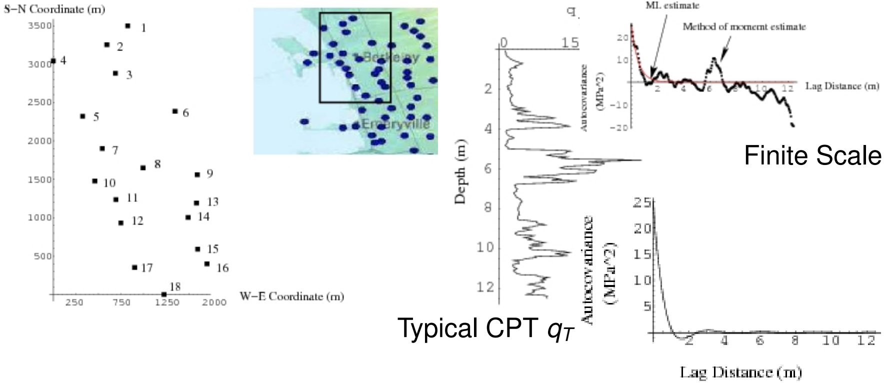
Outline

“Uniform” CPT Site Data



Random Field Parameters from Site Data

- Maximum likelihood estimates of correlation length



Finite Scale

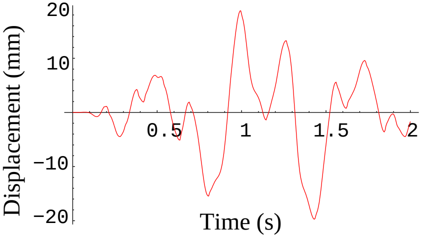
Typical CPT q_T

Fractal

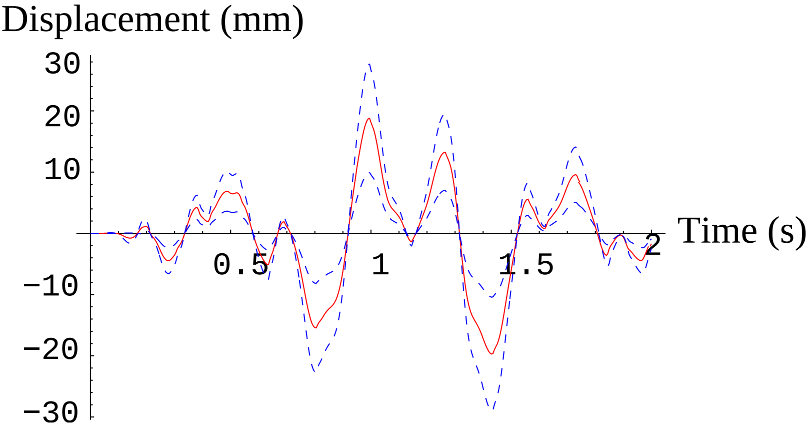
Seismic Wave Propagation through Stochastic Soil

- Soil as 12.5 m deep 1–D soil column (von Mises Material)
  - Properties (including testing uncertainty) obtained through random field modeling of CPT q_T
    $\langle q_T \rangle = 4.99\ MPa$; $Var[q_T] = 25.67\ MPa^2$;
    Cor. Length $[q_T] = 0.61\ m$; Testing Error $= 2.78\ MPa^2$
- q_T was transformed to obtain G: $G/(1-\nu) = 2.9 q_T$
  - Assumed transformation uncertainty = 5%
    $\langle G \rangle = 11.57 MPa$; $Var[G] = 142.32 MPa^2$
    Cor. Length $[G] = 0.61 m$
- Input motions: modified 1938 Imperial Valley

Surface Displacement Time History

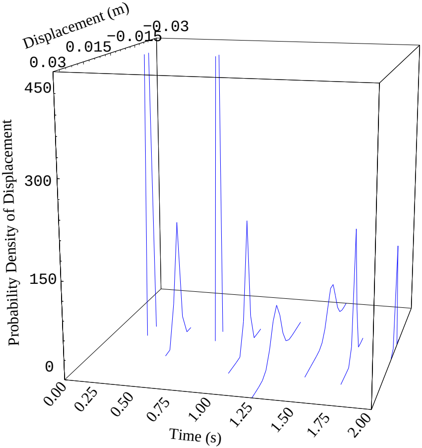


Mean ± Standard Deviation

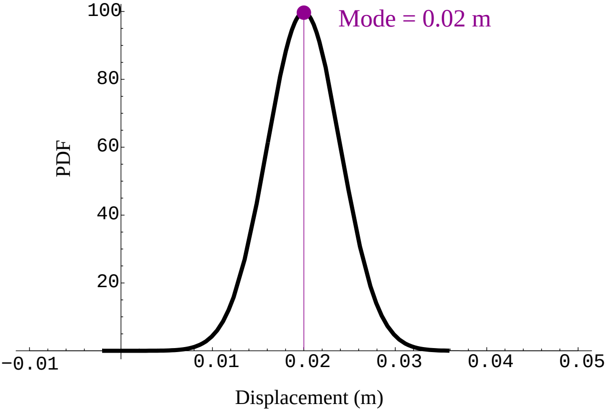


PDF of Surface Displacement Time History

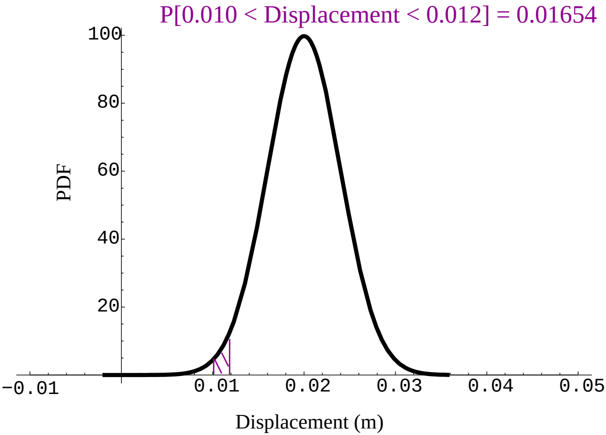
- PDF at the finite element nodes can be obtained using, e.g., Edgeworth expansion (Ghanem and Spanos 2003)
- Numerous applications, especially where extreme statistics are critical



Most Probable Solution

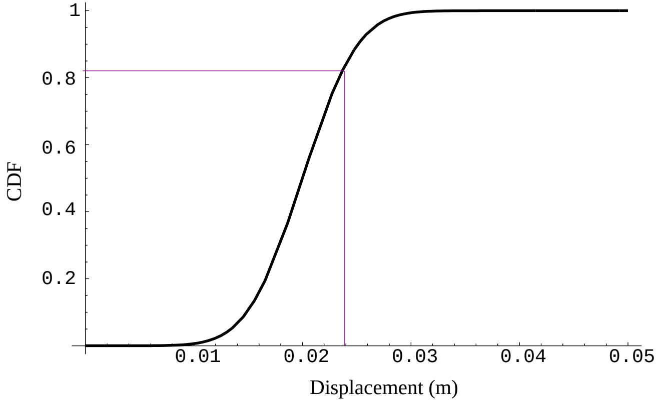


Tails of PDF


P[0.010 < Displacement < 0.012] = 0.01654
PDF
100
80
60
40
20
−0.01
0.01
0.02
0.03
0.04
0.05
Displacement (m)

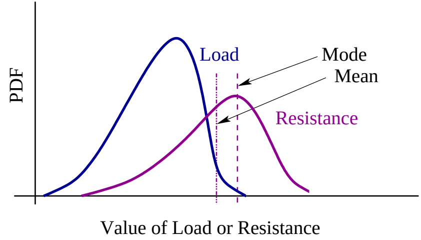
Probability of Exceedance

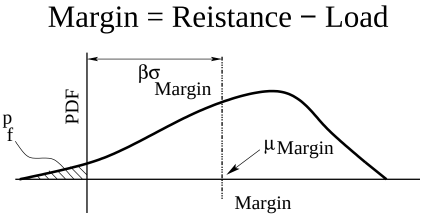
Probability that displacement exceeds 0.025 m = 1 − 0.825 = 0.175



Derivative Applications

- Performance based engineering
  - Reliability index (β)
  - Probability of failure (p_f)





- Sensitivity analysis
- Financial risk analysis

Summary

- Development of probabilistic elastic–plastic modeling and simulation methodology
- Takes into account
  - uncertainty of (measured) soil parameters and
  - soil spatial variability
- Possibly useful for applications where mean, mode and extreme statistics is important