Fully Coupled, Two Phase Behavior of Geomaterials

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Outline

Motivation

- There is no limit to what problems one can address (can numerically simulate)
- Mechanics of coupled, elastic–plastic porous solid – elastic pore fluid
- Mechanics of infrustructure systems featuring coupled, elastic–plastic porous solid – elastic pore fluid
- Accurate modeling and simulation for infrustructure system design (safety and economy)

Outline

Dynamic Equilibrium for Coupled Systems

- Effective stress principle $\sigma'_{ij} = \sigma_{ij} + \alpha\delta_{ij}p$; $(p = -1/3\sigma_{kk})$
- Equilibrium of the mixture
$\sigma_{ij,j} - \rho\ddot{u}_i - \rho_f[\ddot{w}_i + \underline{\dot{w}_j\dot{w}_{i,j}}] + \rho b_i = 0$; $(\rho = n\rho_f + (1-n)\rho_s)$
- Equilibrium of the fluid
$-p_{,i} - R_i - \rho_f\ddot{u}_i - \rho_f[\ddot{w}_i + \underline{\dot{w}_j\dot{w}_{i,j}}]/n + \rho_f b_i = 0$; (Darcy: $n\dot{w}_j = Ki$; $i = h_{,j}$; $R_i = k_{ij}^{-1}\dot{w}_j$; $k_{ij} = K_{ij}/\rho_f g\ [m]^3[s]/[kg]$)
- Flow conservation $\dot{w}_{i,i} + \alpha\dot{\varepsilon}_{ii} + \dot{p}/Q + \underline{n\dot{\rho}_f/\rho_f + \dot{s}_0} = 0$;
$1/Q \equiv n/K_f + (1-n)/K_s$

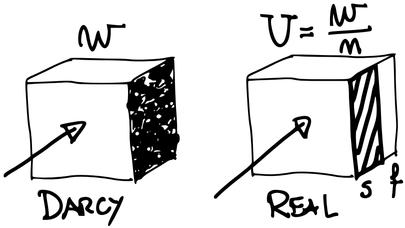
Dynamic Equilibrium for Coupled Systems (cont.)

After neglecting convective accelerations, density variations and assuming isothermal process (no volume expansion):

- Equilibrium of the mixture
  $\sigma_{ij,j} - \rho \ddot{u}_i - \rho_f \ddot{w}_i + \rho b_i = 0$
- Equilibrium of the fluid
  $-p_{,i} - R_i - \rho_f \ddot{u}_i - \rho_f \ddot{w}_i / n + \rho_f b_i = 0$
- Flow conservation
  $\dot{w}_{i,i} + \alpha \dot{\varepsilon}_{ii} + \dot{p}/Q = 0$

Dynamic Equilibrium for Coupled Systems (cont.)

Replace relative pseudo–displacement w_i with real displacement $U_i = u_i + U_i^R = u_i + w_i/n$



Dynamic Equilibrium for Coupled Systems (cont.)

After some manipulations we obtain

$$\sigma''_{ij,j} - (\alpha - n)p_{,i} + (1-n)\rho_s b_i - (1-n)\rho_s \ddot{u}_i + nR_i = 0$$

$$-np_{,i} + n\rho_f b_i - n\rho_f \ddot{U}_i - nR_i = 0$$

$$-n\dot{U}_{i,i} = (\alpha - n)\dot{\varepsilon}_{ii} + \dot{p}/Q$$

Fully Coupled $u - p - U$ Formulation

- Formulation: fully coupled by Zienkiewicz and Shiomi 1984), nonlinear dynamics by Argyris and Mlejnek (1991)
- Physical, velocity proportional damping from solid–fluid interaction (not using Rayleigh damping)
- Accelerations of pore fluid not neglected
  - important for SFSI
  - inertial forces of fluid allow liquefaction modeling
- Stable formulation for near incompressible pore fluid

Finite Element Discretization

$$
\begin{bmatrix} (M_s)_{KijL} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (M_f)_{KijL} \end{bmatrix}
\begin{bmatrix} \ddot{\bar{u}}_{Lj} \\ \ddot{\bar{p}}_N \\ \ddot{\bar{U}}_{Lj} \end{bmatrix} +
$$

$$
+ \begin{bmatrix} (C_1)_{KijL} & 0 & -(C_2)_{KijL} \\ 0 & 0 & 0 \\ -(C_2)_{LjiK} & 0 & (C_3)_{KijL} \end{bmatrix}
\begin{bmatrix} \dot{\bar{u}}_{Lj} \\ \dot{\bar{p}}_N \\ \dot{\bar{U}}_{Lj} \end{bmatrix} +
$$

$$
+ \begin{bmatrix} (K^{EP})_{KijL} & -(G_1)_{KiM} & 0 \\ -(G_1)_{LjM} & -P_{MN} & -(G_2)_{LjM} \\ 0 & -(G_2)_{KiL} & 0 \end{bmatrix}
\begin{bmatrix} \bar{u}_{Lj} \\ \bar{p}_M \\ \bar{U}_{Lj} \end{bmatrix} =
\begin{bmatrix} \bar{f}_{Ki}^{solid} \\ 0 \\ \bar{f}_{Ki}^{fluid} \end{bmatrix}
$$

Finite Element Discretization

$$(M_s)_{KijL} = \int_\Omega N_K^u (1-n)\rho_s \delta_{ij} N_L^u d\Omega \quad ; \quad (M_f)_{KijL} = \int_\Omega N_K^U n \rho_f \delta_{ij} N_L^U d\Omega$$

$$(C_1)_{KijL} = \int_\Omega N_K^u n^2 k_{ij}^{-1} N_L^u d\Omega \quad ; \quad (C_2)_{KijL} = \int_\Omega N_K^u n^2 k_{ij}^{-1} N_L^U d\Omega$$

$$(C_3)_{KijL} = \int_\Omega N_K^U n^2 k_{ij}^{-1} N_L^U d\Omega \quad ; \quad (K^{EP})_{KijL} = \int_\Omega N_{K,m}^u D_{imjn} N_{L,n}^u d\Omega$$

$$(G_1)_{KiM} = \int_\Omega N_{K,i}^u (\alpha - n) N_M^p d\Omega \quad ; \quad (G_2)_{KiM} = \int_\Omega n N_{K,i}^U N_M^p d\Omega$$

$$P_{NM} = \int_\Omega N_N^p \frac{1}{Q} N_M^p d\Omega$$

Finite Element Discretization

$$\bar{f}_{Ki}^{solid} = \int_{\Gamma_t} N_K^u n_j \sigma_{ij}'' d\Gamma - \int_{\Gamma_p} N_K^u (\alpha - n) n_i p d\Gamma + \int_{\Omega} N_K^u (1-n) \rho_s b_i d\Omega$$

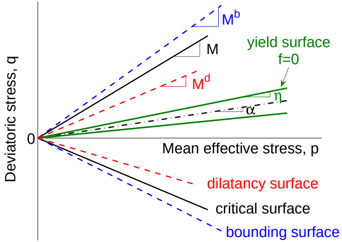
$$\bar{f}_{Ki}^{fluid} = -\int_{\Gamma_p} n N_K^U n_i p d\Gamma + \int_{\Omega} n N_K^U \rho_f b_i d\Omega$$

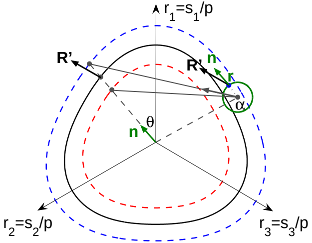
Outline

Dafalias Manzari Material Model

- Dafalias & Manzari (2004): critical state compatible elasto-plastic constitutive model for sands.
- Systematic and relatively simple calibration process.
- Capable of simulating different feature of sand response such as
  - hardening
  - softening
  - consolidation
  - dilation
- Single set of parameters for all stages of loading (self weight, cycling...)

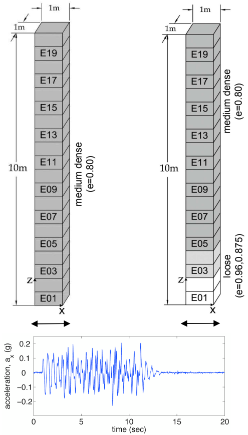
Multiaxial Representation



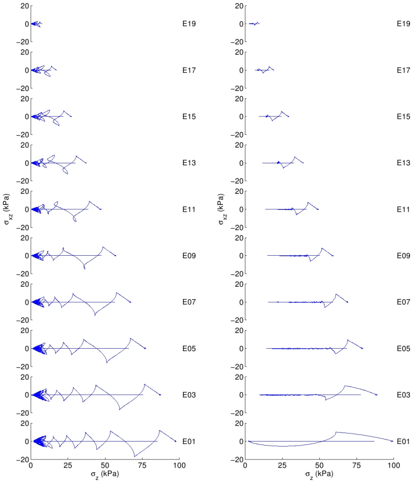


Outline

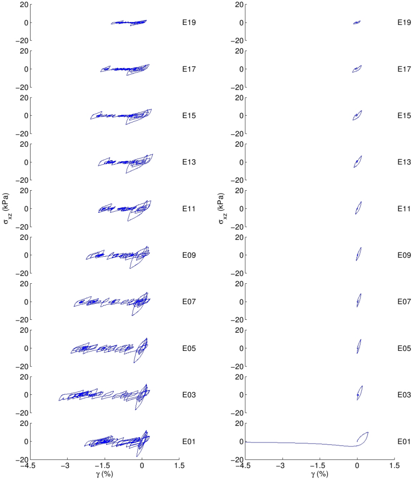
Model


1m
1m
10m
E19
E17
E15
E13
E11
E09
E07
E05
E03
E01
medium dense
(e=0.80)
loose
(e=0.96,0.875)
z
x
acceleration, a_x (g)
time (sec)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
20

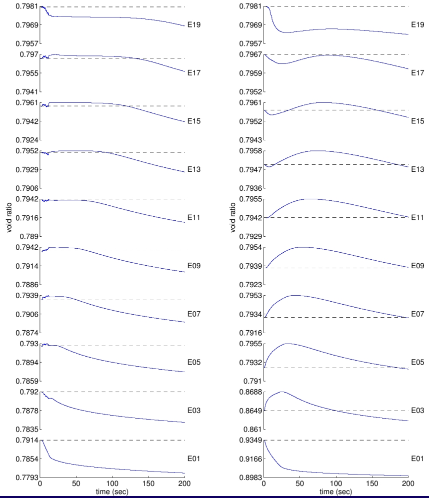
Stress Variation



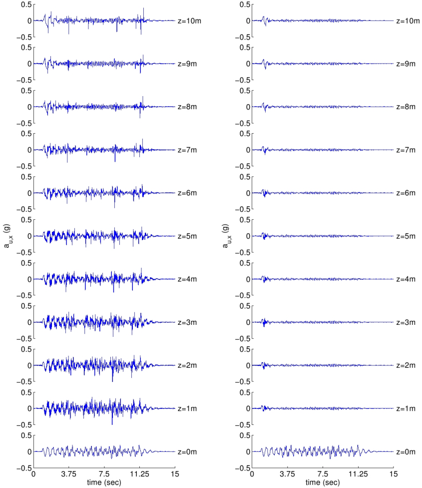
Stress Strain Response



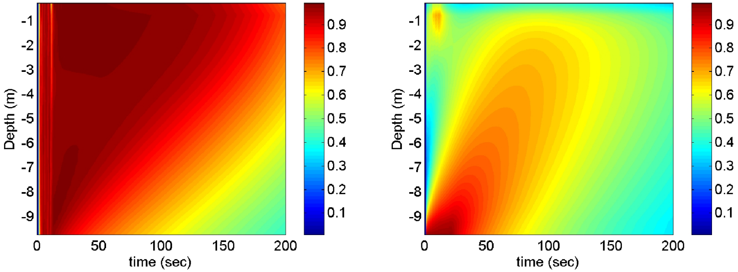
Void Ratio Variation

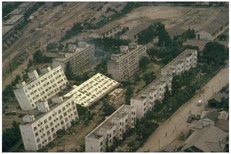


Acceleration Time History

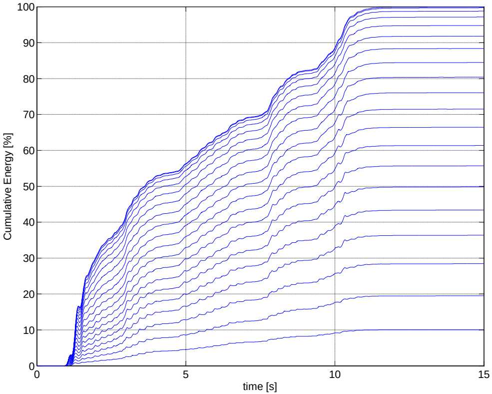


Excess Pore Pressure Ratio

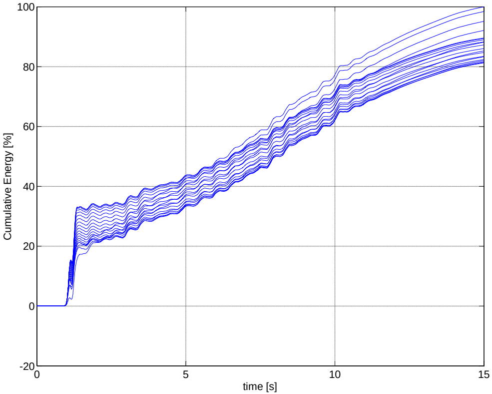




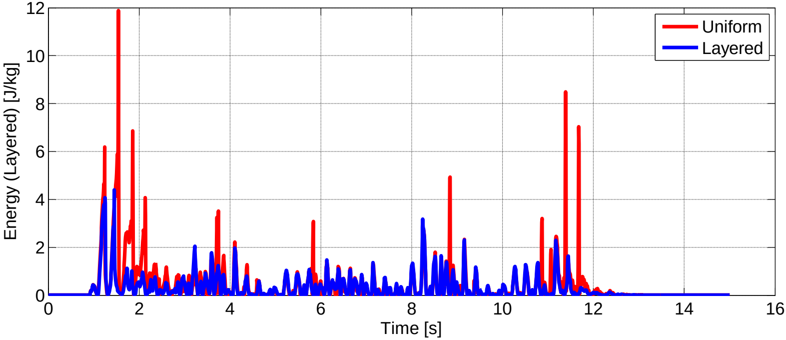
Elastic–Plastic Energy Dissipation: Uniform Soil



Elastic–Plastic Energy Dissipation: Layered Soil

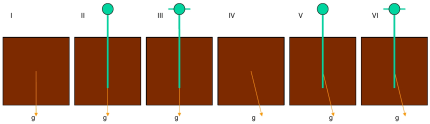


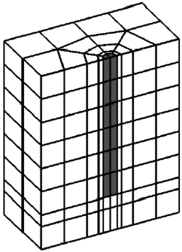
Kinetic Energy at the Top

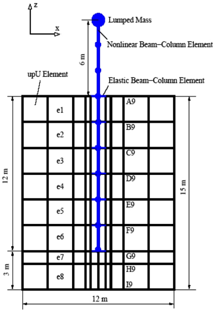


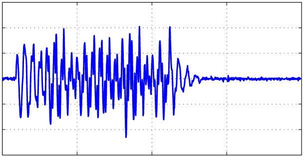
Outline

Bridge Pier–Pile Model





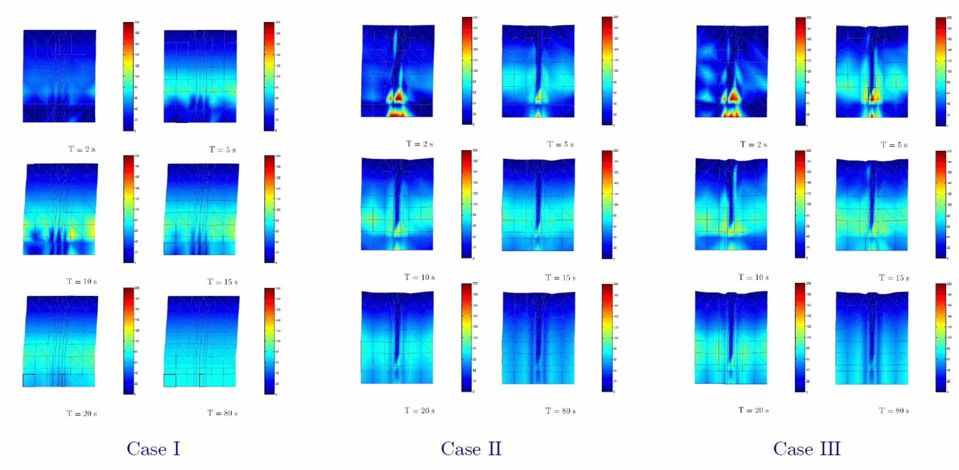




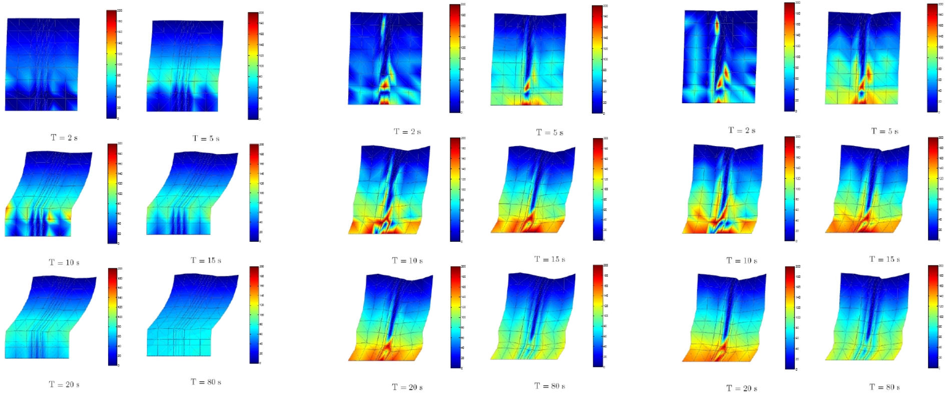
Bridge Pier–Pile Staged Construction

- Soil self weight (no pile)
- Excavations for pile
- Pile installation
  - impermeable filler material,
  - connecting solids and structure,
- Pile self weight,
- Construction of pier structure and self weight
- Seismic shaking
- Excess pore pressure dissipation

Bridge Pier in Level Ground



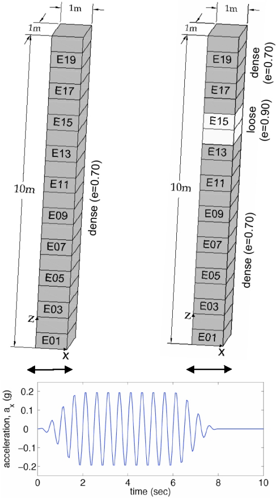
Bridge Pier in Sloping Ground



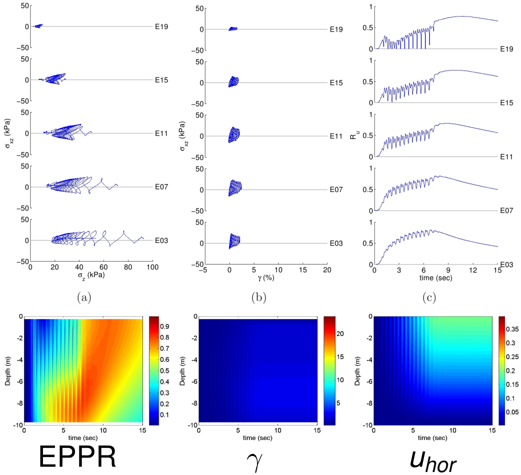
Case IV Case V Case VI

Outline

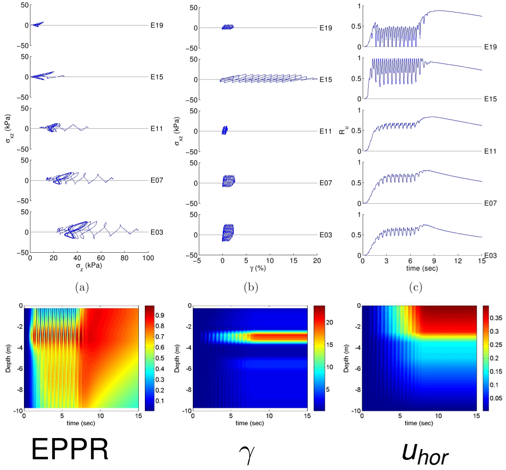
Slope Models



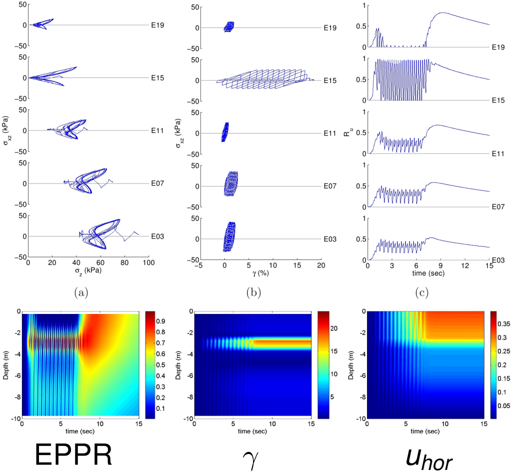
Uniform Slope with $a_{max} = 0.2g$



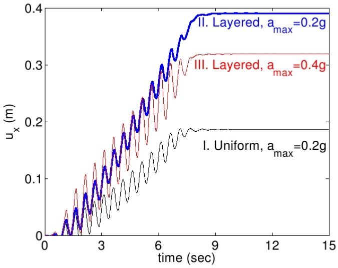
Layered Slope with $a_{max} = 0.2g$

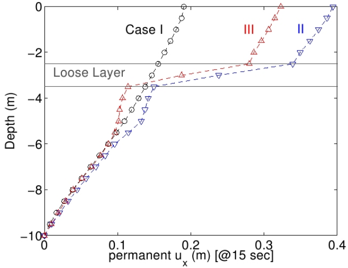


Layered Slope with $a_{max} = 0.4g$



Surface Displacements





Summary

- High fidelity numerical models (verified and validated) of Earthquake–Soil–(Structure) systems
- Space and time distribution of the matching triad: Earthquake, Soil and Structure (**ESS**) and its interaction determines possible benefits or detriments