The Case for Probabilistic Elasto–Plasticity

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GheoMat
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Outline

Motivation
  Stochastic Systems: Historical Perspectives
  Uncertainties in Material

Probabilistic Elasto–Plasticity
  PEP Formulations
  Probabilistic Elastic–Plastic Response

Stochastic Elastic–Plastic Finite Element Method
  SEPFEM Formulations
  SEPFEM Verification Example

Applications
  Seismic Wave Propagation Through Uncertain Soils
  Probabilistic Analysis for Decision Making

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The Case for Probabilistic Elasto–Plasticity
Failure Mechanisms for Geomaterials

Computed tomography (CT) images of resin-impregnated MGM specimens (above) are assembled to provide 3-D volume renderings (below) of density patterns formed by diffused bifurcation under the external loading stress profile applied during the experiments.

Soil: Inside Failure of "Uniform" MGM Specimen
Personal Motivation

- Probabilistic fish counting
- Williams’ DEM simulations, differential displacement vortices
- SFEM round table
- Kavvas’ probabilistic hydrology
Types of Uncertainties

- **Epistemic uncertainty** - due to lack of knowledge
  - Can be reduced by collecting more data
  - Mathematical tools are not well developed
  - Trade-off with aleatory uncertainty

- **Aleatory uncertainty** - inherent variation of physical system
  - Can not be reduced
  - Has highly developed mathematical tools
Ergodicity

- Exchange ensemble averages for time averages
- Is soil elasto-plasticity ergodic?
  - Can soil elastic–plastic statistical properties be obtained by temporal averaging?
  - Will soil elastic–plastic statistical properties "renew" at each occurrence?
  - Are soil elastic–plastic statistical properties statistically independent?
- Claim in literature that structural nonlinear behavior is non–ergodic while earthquake characteristics are (?!)
- However, earthquake characteristics is representing mechanics (fault slip) on a different scale...
Historical Overview

- Brownian motion, Langevin equation → PDF governed by simple diffusion Eq. (Einstein 1905)

- With external forces → Fokker-Planck-Kolmogorov (FPK) for the PDF (Kolmogorov 1941)

- Approach for random forcing → relationship between the autocorrelation function and spectral density function (Wiener 1930)

- Approach for random coefficient → Functional integration approach (Hopf 1952), Averaged equation approach (Bharrucha-Reid 1968), Numerical approaches, Monte Carlo method
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Material Behavior Inherently Uncertain

- Spatial variability
- Point-wise uncertainty, testing error, transformation error

(Mayne et al. (2000))
## Motivation

### Typical Coefficients of Variation of Different Soil Properties

<table>
<thead>
<tr>
<th>Soil Property</th>
<th>Soil Type</th>
<th>PDF</th>
<th>Mean</th>
<th>COV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone resistance</td>
<td>Sand Clay, Clay</td>
<td>LN/N/LN</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Undrained shear strength</td>
<td>Clay (triaxial), Clay (index $S_u$), Clayey silt</td>
<td>LN/N</td>
<td>*</td>
<td>5-20</td>
</tr>
<tr>
<td>Ratio $S_u/\sigma_{\infty}$</td>
<td>Clay</td>
<td>N/LN</td>
<td>*</td>
<td>5-15</td>
</tr>
<tr>
<td>Plastic limit</td>
<td>Clay</td>
<td>N</td>
<td>0.13-0.23</td>
<td>3-20</td>
</tr>
<tr>
<td>Liquid limit</td>
<td>Clay</td>
<td>N</td>
<td>0.30-0.80</td>
<td>3-20</td>
</tr>
<tr>
<td>Submerged unit weight</td>
<td>All soils</td>
<td>N</td>
<td>5-11 (kN/m$^3$)</td>
<td>0-10</td>
</tr>
<tr>
<td>Friction angle</td>
<td>Sand</td>
<td>N</td>
<td>*</td>
<td>2-5</td>
</tr>
<tr>
<td>Void ratio, porosity, initial void ratio</td>
<td>All soils</td>
<td>N</td>
<td>*</td>
<td>7-30</td>
</tr>
<tr>
<td>Over consolidation ratio</td>
<td>Clay</td>
<td>N/LN</td>
<td>*</td>
<td>10-35</td>
</tr>
</tbody>
</table>

(After Lacasse and Nadim 1996)
Soil Uncertainties and Quantification

- Natural variability of soil deposit (Fenton 1999)
  - Function of soil formation process

- Testing error (Stokoe et al. 2004)
  - Imperfection of instruments
  - Error in methods to register quantities

- Transformation error (Phoon and Kulhawy 1999)
  - Correlation by empirical data fitting (e.g. CPT data → friction angle etc.)
Probabilistic Material (Soil Site) Characterization

- Ideal: complete probabilistic site characterization
- Large (physically large but not statistically) amount of data
  - Site specific mean and coefficient of variation (COV)
  - Covariance structure from similar sites (e.g. Fenton 1999)
- Moderate amount of data → Bayesian updating (e.g. Phoon and Kulhawy 1999, Baecher and Christian 2003)
- Minimal data: general guidelines for typical sites and test methods (Phoon and Kulhawy (1999))
  - COVs and covariance structures of inherent variability
  - COVs of testing errors and transformation uncertainties.
Recent State-of-the-Art

- **Governing equation**
  - Dynamic problems \( \ddot{M}u + C\ddot{u} + Ku = \phi \)
  - Static problems \( Ku = \phi \)

- **Existing solution methods**
  - **Random r.h.s** (external force random)
    - FPK equation approach
    - Use of fragility curves with deterministic FEM (DFEM)
  - **Random l.h.s** (material properties random)
    - Monte Carlo approach with DFEM \( \rightarrow \) CPU expensive
    - Perturbation method \( \rightarrow \) a linearized expansion! Error increases as a function of COV
    - Spectral method \( \rightarrow \) developed for elastic materials so far

- **New developments for elasto–plastic applications**
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Uncertainty Propagation through Constitutive Eq.

 Incremental el–pl constitutive equation

\[
\frac{d\sigma_{ij}}{dt} = D_{ijkl} \frac{d\epsilon_{kl}}{dt}
\]

\[
D_{ijkl} = \begin{cases} 
D_{ijkl}^e & \text{for elastic} \\
D_{ijkl}^e - \frac{D_{ijmn}^e m_{mn} n_{pq} D_{pqkl}^e}{n_{rs} D_{rstu}^e m_{tu} - \xi^* r^*} & \text{for elastic–plastic}
\end{cases}
\]
Previous Work

- Linear algebraic or differential equations → Analytical solution:
  - Cumulant Expansion Method (Gardiner 2004)

- Nonlinear differential equations (elasto-plastic/viscoelastic-viscoplastic):
    → accurate, very costly
    → first and second order Taylor series expansion about mean - limited to problems with small C.O.V. and inherits "closure problem"
Problem Statement

- Incremental 3D elastic-plastic stress–strain:

\[ \frac{d\sigma_{ij}}{dt} = \left\{ D_{ijkl}^{el} - \frac{D_{ijm}^{el} m_{mn} n_{pq} D_{pqkl}^{el}}{n_{rs} D_{rstu}^{el} m_{tu} - \xi_r^* r_*} \right\} \frac{d\epsilon_{kl}}{dt} \]

- Focus on 1D → a nonlinear ODE with random coefficient (material) and random forcing (\(\epsilon\))

\[ \frac{d\sigma(x, t)}{dt} = \beta(\sigma(x, t), D_{el}(x), q(x), r(x); x, t) \frac{d\epsilon(x, t)}{dt} \]

\[ = \eta(\sigma, D_{el}, q, r, \epsilon; x, t) \]

with initial condition \(\sigma(0) = \sigma_0\)
Evolution of the Density $\rho(\sigma, t)$

- From each initial point in $\sigma$-space a trajectory starts out describing the corresponding solution of the stochastic process.
- Movement of a cloud of initial points described by density $\rho(\sigma, 0)$ in $\sigma$-space, is governed by the constitutive equation.
Stochastic Continuity (Liouville) Equation

- phase density $\rho$ of $\sigma(x, t)$ varies in time according to a continuity Liouville equation (Kubo 1963):

$$\frac{\partial \rho(\sigma(x, t), t)}{\partial t} = \frac{\partial \eta(\sigma(x, t), D^e(x), q(x), r(x), \epsilon(x, t))}{\partial \sigma} \rho[\sigma(x, t), t]$$

- with initial conditions $\rho(\sigma, 0) = \delta(\sigma - \sigma_0)$. 

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Ensemble Average form of Liouville Equation

Continuity equation written in ensemble average form (eg. cumulant expansion method (Kavvas and Karakas 1996)):

\[
\frac{\partial \langle \rho(\sigma(x_t, t), t) \rangle}{\partial t} = - \frac{\partial}{\partial \sigma} \left\{ \left[ \int_0^t d\tau \text{Cov}_0 \left[ \frac{\partial}{\partial \sigma} \eta(\sigma(x_t, t), D^e(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) \right] \right] \right\} \langle \rho(\sigma(x_t, t), t) \rangle
\]
\[
+ \int_0^t d\tau \text{Cov}_0 \left[ \frac{\partial}{\partial \sigma} \eta(\sigma(x_t, t), D^e(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) \right] \left\{ \int_0^t d\tau' \text{Cov}_0 \left[ \eta(\sigma(x_t, t), D^e(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) \right] \right\} \langle \rho(\sigma(x_t, t), t) \rangle
\]
\[
+ \frac{\partial^2}{\partial \sigma^2} \left\{ \int_0^t d\tau \text{Cov}_0 \left[ \eta(\sigma(x_t, t), D^e(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) \right] \right\} \left\{ \int_0^t d\tau' \text{Cov}_0 \left[ \eta(\sigma(x_t, t), D^e(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) \right] \right\} \langle \rho(\sigma(x_t, t), t) \rangle
\]
van Kampen’s Lemma (van Kampen 1976) \( \rightarrow < \rho(\sigma, t) > = P(\sigma, t) \), ensemble average of phase density is the probability density:

\[
\frac{\partial P(\sigma(x_t, t), t)}{\partial t} = - \frac{\partial}{\partial \sigma} \left[ \left\{ \eta(\sigma(x_t, t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) \right\} P(\sigma(x_t, t), t) \right]
\]

\[
+ \int_0^t d\tau \text{Cov}_0 \left[ \frac{\partial \eta(\sigma(x_t, t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t, t))}{\partial \sigma} ; \eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau)) \right] P(\sigma(x_t, t), t) \]

\[
+ \frac{\partial^2}{\partial \sigma^2} \left[ \left\{ \int_0^t d\tau \text{Cov}_0 \left[ \eta(\sigma(x_t, t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t, t)) ; \eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau)) \right] \right\} P(\sigma(x_t, t), t) \right]
\]
E–L FPK Equation

- Advection-diffusion equation

\[
\frac{\partial P(\sigma, t)}{\partial t} = -\frac{\partial}{\partial \sigma} \left[ N(1) P(\sigma, t) - \frac{\partial}{\partial \sigma} \{ N(2) P(\sigma, t) \} \right]
\]

- Complete probabilistic description of response
- Solution PDF is second-order exact to covariance of time (exact mean and variance)
- It is deterministic equation in probability density space
- It is linear PDE in probability density space → simplifies the numerical solution process
Template Solution of FPK Equation

- FPK diffusion–advection equation is applicable to any material model → only the coefficients $N(1)$ and $N(2)$ are different for different material models

\[
\frac{\partial P(\sigma, t)}{\partial t} = -\frac{\partial}{\partial \sigma} \left[ N(1) P(\sigma, t) - \frac{\partial}{\partial \sigma} \left\{ N(2) P(\sigma, t) \right\} \right] = -\frac{\partial \zeta}{\partial \sigma}
\]

- Initial condition
  - Deterministic → Dirac delta function → $P(\sigma, 0) = \delta(\sigma)$
  - Random → Any given distribution

- Boundary condition: Reflecting BC → conserves probability mass $\zeta(\sigma, t)\big|_{\text{At Boundaries}} = 0$

- Finite Differences used for solution (among many others)
Application of FPK equation to Material Models

- FPK equation is applicable to any incremental elastic–plastic material model
- Solution in terms of PDF, not a single value of stress
- Influence of initial condition on the PDF of stress
- Mean stress yielding or
- Probabilistic yielding
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The Case for Probabilistic Elasto–Plasticity
Elastic Response with Random $G$

- General form of elastic constitutive rate equation

\[
\frac{d\sigma_{12}}{dt} = 2G \frac{d\epsilon_{12}}{dt} = \eta(G, \epsilon_{12}; t)
\]

- Advection and diffusion coefficients of FPK equation

\[
N_{(1)} = 2 \frac{d\epsilon_{12}}{dt} < G >
\]

\[
N_{(2)} = 4t \left( \frac{d\epsilon_{12}}{dt} \right)^2 \text{Var}[G]
\]
Elastic Response with Random $G$

\[ \langle G \rangle = 2.5 \text{ MPa}; \text{ Std. Deviation}[G] = 0.5 \text{ MPa} \]
Verification – Variable Transformation Method
Modified Cam Clay Constitutive Model

\[
\frac{d\sigma_{12}}{dt} = G^{ep} \frac{d\epsilon_{12}}{dt} = \eta(\sigma_{12}, G, M, e_0, p_0, \lambda, \kappa, \epsilon_{12}; t)
\]

\[
\eta = \left[ 2G - \frac{\left(36 \frac{G^2}{M^4}\right) \sigma_{12}^2}{(1 + e_0)p(2p - p_0)^2 + \left(18 \frac{G}{M^4}\right) \sigma_{12}^2 + \frac{1 + e_0}{\lambda - \kappa} pp_0(2p - p_0)} \right]
\]

Advection and diffusion coefficients of FPK equation

\[
N_{(1)}^{(i)} = \left\langle \eta^{(i)}(t) \right\rangle + \int_0^t d\tau \text{cov} \left[ \frac{\partial \eta^{(i)}(t)}{\partial t}; \eta^{(i)}(t - \tau) \right]
\]

\[
N_{(2)}^{(i)} = \int_0^t d\tau \text{cov} \left[ \eta^{(i)}(t); \eta^{(i)}(t - \tau) \right]
\]
Low OCR Cam Clay with Random $G$, $M$ and $p_0$

- Non-symmetry in probability distribution
- Difference between mean, mode and deterministic
- Divergence at critical state because $M$ is uncertain
Comparison of Low OCR Cam Clay at $\epsilon = 1.62 \%$

- None coincides with deterministic
- Some very uncertain, some very certain
- Either on safe or unsafe side
High OCR Cam Clay with Random $G$ and $M$

- Large non-symmetry in probability distribution
- Significant differences in mean, mode, and deterministic
- Divergence at critical state, $M$ is uncertain
Probabilistic Yielding

- Weighted elastic and elastic-plastic Solution
  \[
  \frac{\partial P(\sigma, t)}{\partial t} = -\frac{\partial}{\partial \sigma} \left( N^w_{(1)} P(\sigma, t) - \frac{\partial}{\partial \sigma} \left( N^w_{(2)} P(\sigma, t) \right) \right) / \partial \sigma
  \]

- Weighted advection and diffusion coefficients are then
  \[
  N^w_{(1,2)}(\sigma) = (1 - P[\Sigma_y \leq \sigma]) N^{el}_{(1)} + P[\Sigma_y \leq \sigma] N^{el-pl}_{(1)}
  \]

- Cumulative Probability Density function (CDF) of the yield function

\[ F[\Sigma_y] = P[\Sigma_y \leq \sigma_y] \]
Transformation of a Bi–Linear (von Mises) Response

linear elastic – linear hardening plastic von Mises
SPT Based Determination of Shear Strength

Transformation relationship between SPT $N$-value and undrained shear strength, $s_u$ (cf. Phoon and Kulhawy (1999B))

Histogram of the residual (w.r.t the deterministic transformation equation) undrained strength, along with fitted probability density function.
SPT Based Determination of Young’s Modulus

Transformation relationship between SPT N-value and pressure-meter Young’s modulus, $E$ (cf. Phoon and Kulhawy (1999B))

Histogram of the residual (w.r.t the deterministic transformation equation) Young’s modulus, along with fitted probability density function
Cyclic Response of Such Uncertain Material
**G/G_{max} Response**

- **PI=200%** (Vucetic and Dobry 1991)
- **PI=100%** (Vucetic and Dobry 1991)
- **PI=100%** (Stokoe et al. 2004)

Deterministic

Mean ± Standard Deviation

Shear Strain (%)

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Damping Response

Corresponding to hysteresis loop of Mean of shear stress

Corresponding to hysteresis loop of Mean±Standard Deviation of shear stress

PI=100% (Vucetic and Dobry 1991)

PI=200% (Vucetic and Dobry 1991)

PI=100% (Stokoe et al. 2004)
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Governing Equations & Discretization Scheme

- Governing equations in geomechanics:

\[ A\sigma = \phi(t); \quad Bu = \epsilon; \quad \sigma = D\epsilon \]

- Discretization (spatial and stochastic) schemes

  - Input random field material properties \((D)\) → Karhunen–Loève (KL) expansion, optimal expansion, error minimizing property
  - Unknown solution random field \((u)\) → Polynomial Chaos (PC) expansion
  - Deterministic spatial differential operators \((A & B)\) → Regular shape function method with Galerkin scheme
Minimizing norm of error of finite representation using Galerkin technique (Ghanem and Spanos 2003):

\[
\sum_{n=1}^{N} K_{mn} d_{ni} + \sum_{n=1}^{N} \sum_{j=0}^{P} d_{nj} \sum_{k=1}^{M} C_{ijk} K'_{mnk} = \langle F_m \psi_i[\{\xi_r\}] \rangle
\]

\[
K_{mn} = \int_D B_n DB_m dV \quad K'_{mnk} = \int_D B_n \sqrt{\lambda_k} h_k B_m dV
\]

\[
C_{ijk} = \langle \xi_k(\theta) \psi_i[\{\xi_r\}] \psi_j[\{\xi_r\}] \rangle \quad F_m = \int_D \phi N_m dV
\]
Inside SEPFEM

- Explicit stochastic elastic–plastic finite element computations
- FPK probabilistic constitutive integration at Gauss integration points
- Increase in (stochastic) dimensions (KL and PC) of the problem
- Development of the probabilistic elastic–plastic stiffness tensor
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1-D Static Pushover Test Example

- Linear elastic model:
  \[ \langle G \rangle = 2.5 \text{ kPa}, \]
  \[ \text{Var}[G] = 0.15 \text{ kPa}^2, \]
  correlation length for \( G = 0.3 \text{ m}. \)

- Elastic-plastic material model,
  von Mises, linear hardening,
  \[ \langle G \rangle = 2.5 \text{ kPa}, \]
  \[ \text{Var}[G] = 0.15 \text{ kPa}^2, \]
  correlation length for \( G = 0.3 \text{ m}, \)
  \( C_u = 5 \text{ kPa}, \)
  \( C_u' = 2 \text{ kPa}. \)
**Linear Elastic FEM Verification**

Mean and standard deviations of displacement at the top node, linear elastic material model, KL-dimension=2, order of PC=2.
Mean and standard deviations of displacement at the top node, von Mises elastic-plastic linear hardening material model, KL-dimension=2, order of PC=2.
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- Stochastic elastic–plastic simulations of soils and structures
- Probabilistic inverse problems
- Geotechnical site characterization design
- Optimal material design
Seismic Wave Propagation through Stochastic Soil

- Soil as 12.5 m deep 1-D soil column (von Mises Material)
  - Properties (including testing uncertainty) obtained through random field modeling of CPT $q_T$
    \[ \langle q_T \rangle = 4.99 \text{ MPa}; \quad \text{Var} [q_T] = 25.67 \text{ MPa}^2; \]
    Cor. Length $[q_T] = 0.61\text{ m}$; Testing Error $= 2.78 \text{ MPa}^2$

- $q_T$ was transformed to obtain $G$: \[ G/ (1 - \nu) = 2.9 q_T \]
  - Assumed transformation uncertainty $= 5\%$
    \[ \langle G \rangle = 11.57 \text{ MPa}; \quad \text{Var}[G] = 142.32 \text{ MPa}^2 \]
    Cor. Length $[G] = 0.61\text{ m}$

- Input motions: modified 1938 Imperial Valley
Random Field Parameters from Site Data

- Maximum likelihood estimates

Finite Scale

Typical CPT $q_T$

Fractal
"Uniform" CPT Site Data
Seismic Wave Propagation through Stochastic Soil

Displacement (mm)

Mean ± Standard Deviation

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Three Approaches to Modeling

- Do nothing about site characterization (rely on experience): conservative *guess* of soil data, $COV = 225\%$, correlation length $= 12\text{m}$.

- Do better than standard site characterization: $COV = 103\%$, correlation length $= 0.61\text{m}$)

- Improve site characterization if probabilities of exceedance are unacceptable!
Evolution of Mean ± SD for Guess Case

Displacement (mm) vs. Time (sec)

- Mean ± Standard Deviation
  - Mean
  - Mean ± Standard Deviation

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UCDAVIS
Evolution of Mean ± SD for Real Data Case

Displacement (mm)

Time (sec)

mean ± standard deviation

mean
Full PDFs for Real Data Case
Example: PDF at 6 s

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- Real Soil Data
- Conservative Guess

Displacement (mm)

PDF
Example: CDF at 6 s
Probability of Exceedance of 20cm

With Conservative Guess

With Real Soil Data
Probability of Exceedance of 50cm

With Conservative Guess

With Real Soil Data

Displacement (cm)

Probability of Exceedance of 50 cm (%)

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Probabilities of Exceedance vs. Displacements

- **Real Soil Data**
- **Conservative Guess**

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Summary

- Behavior of materials is probably probabilistic!
- Technical developments are available and are being refined
- Human nature: how much do you want to know about potential problem?