Simulations in Geomechanics: The Issue of Uncertainty

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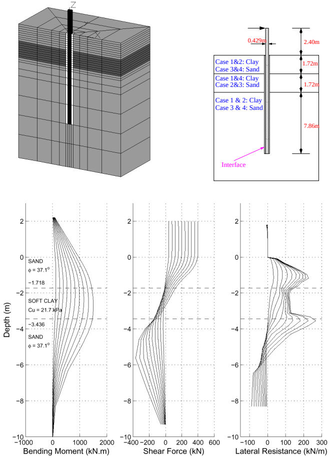
Outline

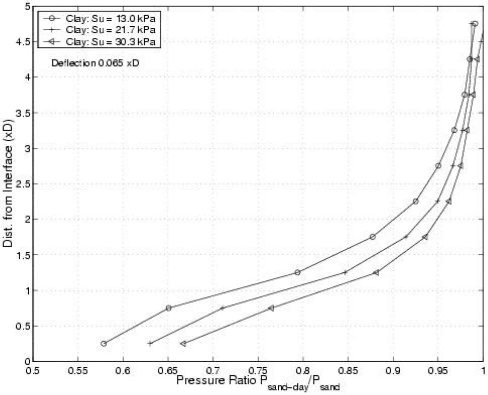
Outline

Numerical Simulation in Support of Design!

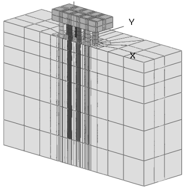
- Practical design experience
  - Design of concrete and rock dams, bridges (YU, IR, USA)
  - Design of residential and industrial buildings (SUI, SA)
  - Design of buildings, tunnels, oil exploration equipment (USA)
- Verified, validated predictions
- Proper modeling of (multi-) physics
- Flexible, usable, user friendly tools
- Detailed models that **reduce**
  - Kolmogorov Complexity
  - Modeling uncertainty

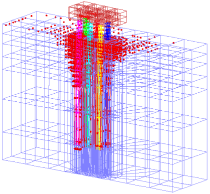
Pile in Layered Soil: Pressure Ratio Reduction

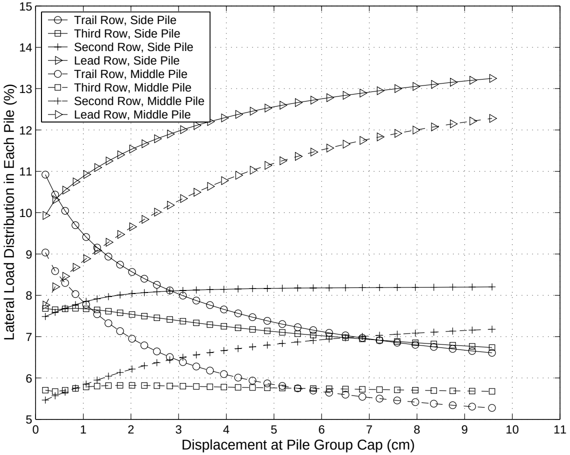




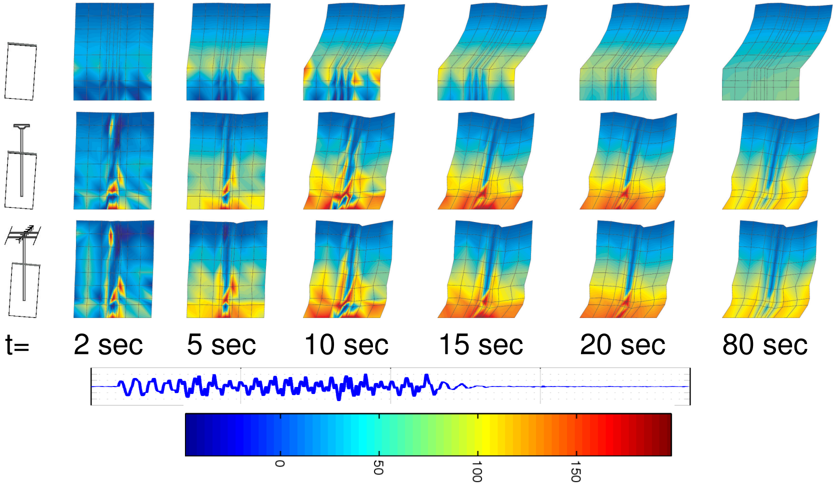
Pile Group Interaction



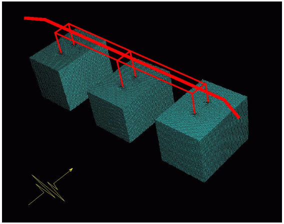


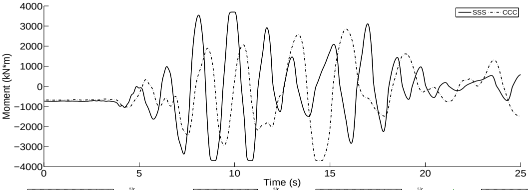


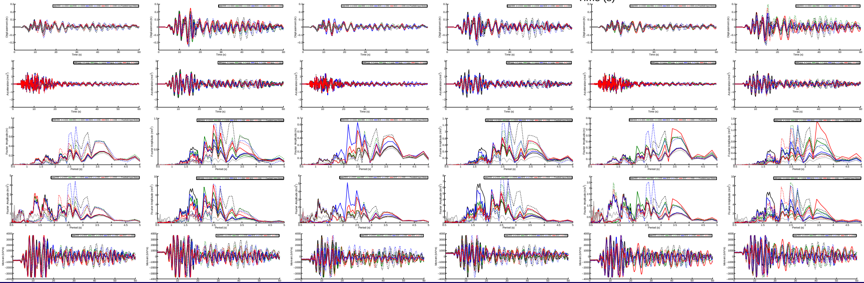
Pile in Liquefiable Sloping Ground



Earthquake-Soil-Structure Interaction

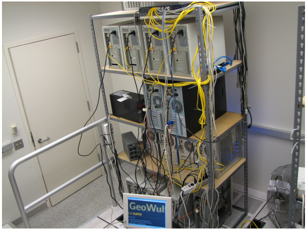


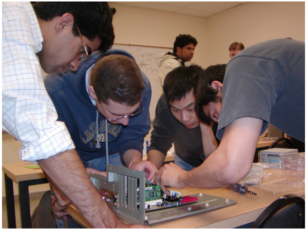




PDD and Parallel Computer GeoWulf

- **P**lastic **D**omain **D**ecomposition Elastic-Plastic Parallel Finite Element Method
- Distributed memory parallel computer
- Multiple generation compute nodes and networks
- Very cost effective!
- Same architecture as large parallel supercomputers (SDSC, TACC, EarthSimulator...)
- Local design, construction, available at all times!





Collaboratory

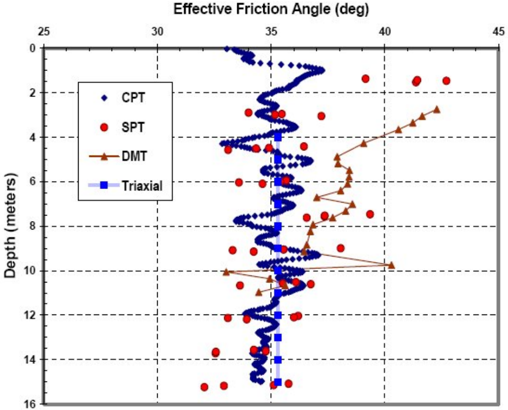
- Prof. Zhaohui Yang (U. of Alaska)
- Prof. Mahdi Taiebat (U. of British Columbia)
- Dr. Zhao Cheng (EarthMechanics Inc.)
- Dr. Guanzhou Jie (Wells Fargo Securities)
- Dr. Matthias Preisig (EPF de Lausanne)
- Prof. Kallol Sett (U. of Akron)

Outline

Motivation

Material Behavior Inherently Uncertain

- Spatial variability
- Point-wise uncertainty, testing error, transformation error



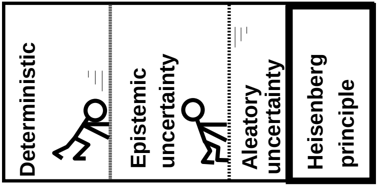
(Mayne et al. (2000)

Soil Uncertainties and Quantification

- Natural variability of soil deposit (Fenton 1999)
  - Function of soil formation process
- Testing error (Stokoe et al. 2004)
  - Imperfection of instruments
  - Error in methods to register quantities
- Transformation error (Phoon and Kulhawy 1999)
  - Correlation by empirical data fitting (e.g. CPT data $\rightarrow$ friction angle etc.)

Types of Uncertainties

- Aleatory uncertainty - inherent variation of physical system
  - Can not be reduced
  - Has highly developed mathematical tools
- Epistemic uncertainty - due to lack of knowledge
  - Can be reduced by collecting more data
  - Mathematical tools are not well developed
  - trade-off with aleatory uncertainty



- Ergodicity (exchanging ensemble averages for time average) assumed to hold

Historical Overview

- Brownian motion, Langevin equation → PDF governed by simple diffusion Eq. (Einstein 1905)
- Approach for random forcing → relationship between the autocorrelation function and spectral density function (Wiener 1930)
- With external forces → Fokker-Planck-Kolmogorov (FPK) for the PDF (Kolmogorov 1941)
- Approach for random coefficient → Functional integration approach (Hopf 1952), Averaged equation approach (Bharrucha-Reid 1968), Numerical approaches, Monte Carlo method

Recent State-of-the-Art

- Governing equation
  - Dynamic problems → $M\ddot{u} + C\dot{u} + Ku = F$
  - Static problems → $Ku = F$
- Existing solution methods
  - **Random r.h.s** (external force random)
    - FPK equation approach
    - Use of fragility curves with deterministic FEM (DFEM)
  - **Random l.h.s** (material properties random)
    - Monte Carlo approach with DFEM → CPU expensive
    - Perturbation method → a linearized expansion! Error increases as a function of COV
    - Spectral method → developed for elastic materials so far
- New developments for **Probabilistic Elasto–Plasticity**

Outline

Uncertainty Propagation through Constitutive Eq.

- Incremental el–pl constitutive equation $\dfrac{d\sigma_{ij}}{dt} = D_{ijkl}\dfrac{d\epsilon_{kl}}{dt}$

$$D_{ijkl} = \begin{cases} D^{el}_{ijkl} & \text{for elastic} \\ D^{el}_{ijkl} - \dfrac{D^{el}_{ijmn} m_{mn} n_{pq} D^{el}_{pqkl}}{n_{rs} D^{el}_{rstu} m_{tu} - \xi_* r_*} & \text{for elastic–plastic} \end{cases}$$

Previous Work

- Linear algebraic or differential equations → Analytical solution:
  - Variable Transf. Method (Montgomery and Runger 2003)
  - Cumulant Expansion Method (Gardiner 2004)
- Nonlinear differential equations (elasto-plastic/viscoelastic-viscoplastic):
  - Monte Carlo Simulation (Schueller 1997, De Lima et al 2001, Mellah et al. 2000, Griffiths et al. 2005...)
    → accurate, very costly
  - Perturbation Method (Anders and Hori 2000, Kleiber and Hien 1992, Matthies et al. 1997)
    → first and second order Taylor series expansion about mean - limited to problems with small C.O.V. and inherits "closure problem"

Problem Statement and Solution

- Incremental 3D elastic-plastic stress–strain:
  $d\sigma_{ij} = \left[D^{el}_{ijkl} - (D^{el}_{ijmn} m_{mn} n_{pq} D^{el}_{pqkl})/(n_{rs} D^{el}_{rstu} m_{tu} - \xi_* r_*)\right] d\epsilon_{kl}$
- Define stress density $\rho(\sigma, t)$ evolves in probabilistic space according to the constitutive equation
- Stress density $\rho(\sigma, t)$ varies in pseudo-time according to a continuity Liouville equation (Kubo 1963)
  $\partial\rho(\sigma(x,t),t)/\partial t =$
  $-\partial\eta(\sigma(x,t), D^{el}(x), q(x), r(x), \epsilon(x,t))\, \partial\sigma\rho[\sigma(x,t),t]$
- Continuity equation can be written in ensemble average form (Kavvas and Karakas 1996)
- van Kampen's Lemma (van Kampen 1976): ensemble average of phase density is the probability density

Eulerian–Lagrangian FPK Equation

$$\frac{\partial P(\sigma(x_t,t),t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left[\left\{\left\langle \eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t))\right\rangle\right.\right.$$

$$+ \quad \int_0^t d\tau Cov_0\left[\frac{\partial \eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t))}{\partial \sigma};\right.$$

$$\left.\left.\left.\eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau)\right]\right\} P(\sigma(x_t,t),t)\right]$$

$$+ \quad \frac{\partial^2}{\partial \sigma^2}\left[\left\{\int_0^t d\tau Cov_0\left[\eta(\sigma(x_t,t), D^{el}(x_t), q(x_t), r(x_t), \epsilon(x_t,t));\right.\right.\right.$$

$$\left.\left.\left.\eta(\sigma(x_{t-\tau}, t-\tau), D^{el}(x_{t-\tau}), q(x_{t-\tau}), r(x_{t-\tau}), \epsilon(x_{t-\tau}, t-\tau))\right]\right\} P(\sigma(x_t,t),t)\right]$$

Eulerian–Lagrangian FPK Equation

- Advection-diffusion equation

$$\frac{\partial P(\sigma,t)}{\partial t} = -\frac{\partial}{\partial \sigma}\left[N_{(1)}P(\sigma,t) - \frac{\partial}{\partial \sigma}\left\{N_{(2)}P(\sigma,t)\right\}\right]$$

- Complete probabilistic description of response
- Solution PDF is second-order exact to covariance of time (exact mean and variance)
- It is deterministic equation in probability density space
- It is linear PDE in probability density space
- FPK diffusion–advection equation is applicable to any material model → only the coefficients $N_{(1)}$ and $N_{(2)}$ are different for different material models

Outline

Elastic Material with Uncertain Shear Modulus G

- General form of elastic constitutive rate equation

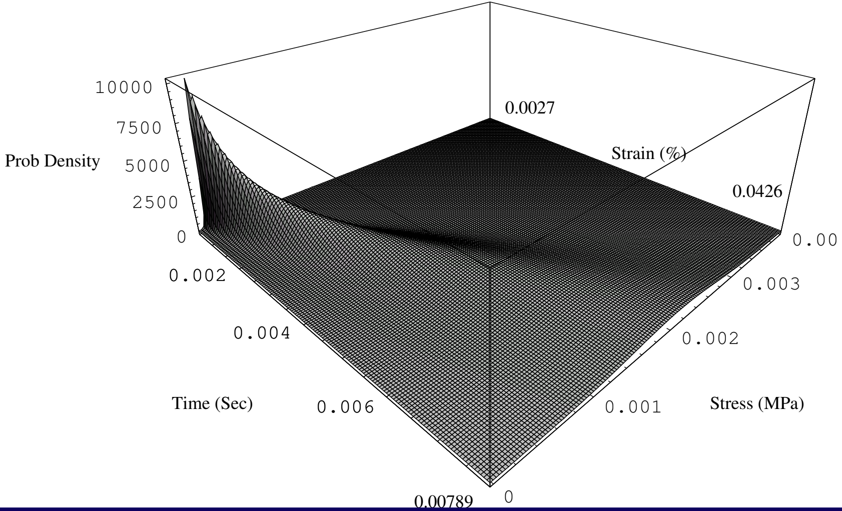
$$\frac{d\sigma_{12}}{dt} = 2G\frac{d\epsilon_{12}}{dt} = \eta(G, \epsilon_{12}; t)$$

- Advection and diffusion coefficients of FPK equation

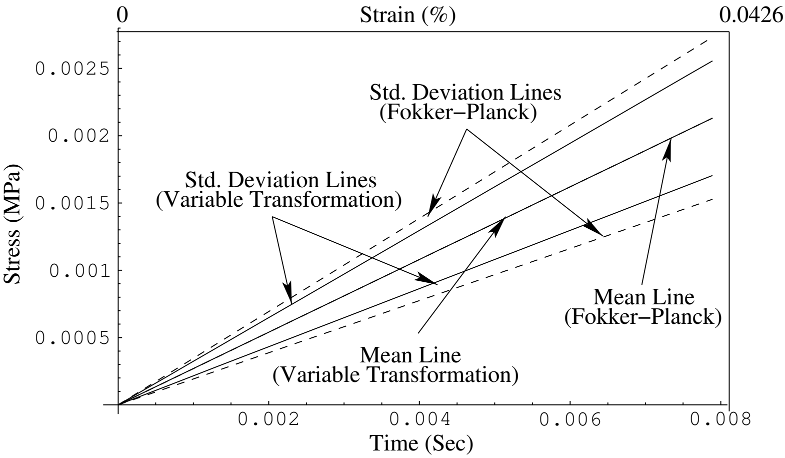
$$N_{(1)} = 2\frac{d\epsilon_{12}}{dt} < G > \quad ; \quad N_{(2)} = 4t\left(\frac{d\epsilon_{12}}{dt}\right)^2 Var[G]$$

- Example: $< G >$ = 2.5 MPa; Std. Deviation[G] = 0.5 MPa

Probabilistic Elastic Response



Verification – Variable Transformation Method



Modified Cam Clay Constitutive Model

$$\frac{d\sigma_{12}}{dt} = G^{ep}\frac{d\epsilon_{12}}{dt} = \eta(\sigma_{12}, G, M, e_0, p_0, \lambda, \kappa, \epsilon_{12}; t)$$

$$\eta = \left[2G - \frac{\left(36\frac{G^2}{M^4}\right)\sigma_{12}^2}{\frac{(1+e_0)p(2p-p_0)^2}{\kappa} + \left(18\frac{G}{M^4}\right)\sigma_{12}^2 + \frac{1+e_0}{\lambda-\kappa}pp_0(2p-p_0)}\right]$$

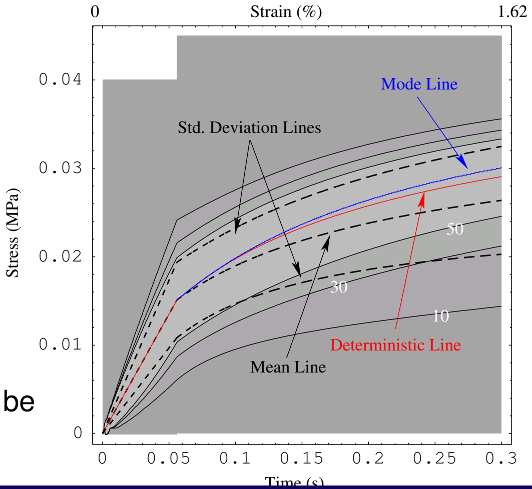
Advection and diffusion coefficients of FPK equation

$$N_{(1)}^{(i)} = \left\langle \eta^{(i)}(t) \right\rangle + \int_0^t d\tau cov\left[\frac{\partial \eta^{(i)}(t)}{\partial t}; \eta^{(i)}(t-\tau)\right]$$

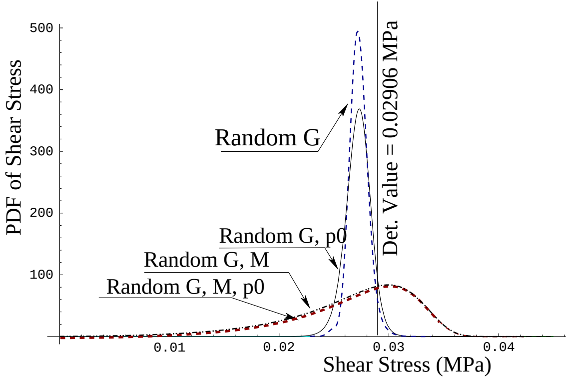
$$N_{(2)}^{(i)} = \int_0^t d\tau cov\left[\eta^{(i)}(t); \eta^{(i)}(t-\tau)\right]$$

Low OCR Cam Clay with Random G, M and p_0

- Non-symmetry in probability distribution
- Difference between mean, mode and deterministic
- Divergence at critical state because M might be (is) uncertain?



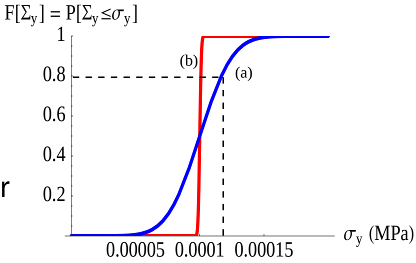
Comparison of Low OCR Cam Clay at ϵ = 1.62 %



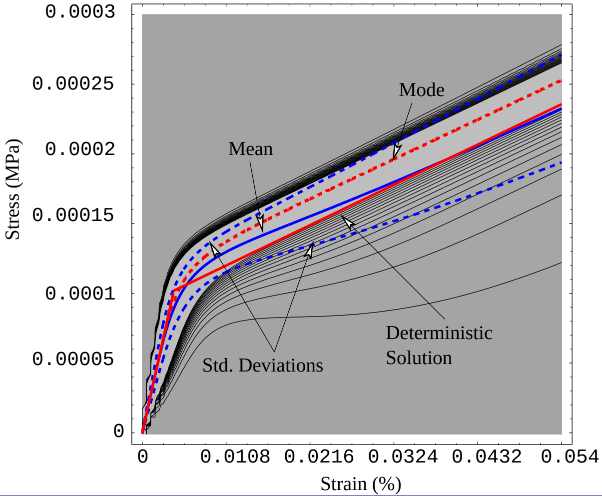
- None coincides with deterministic
- Some very uncertain, some very certain
- Either on safe or unsafe side

Probabilistic Yielding

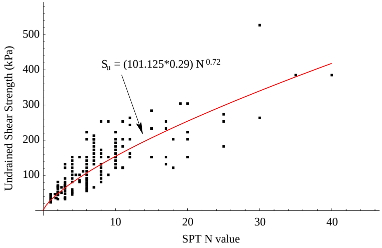
- Weighted elastic and elastic–plastic solution
  $\partial P(\sigma,t)/\partial t = -\partial\left(N^{w}_{(1)}P(\sigma,t) - \partial\left(N^{w}_{(2)}P(\sigma,t)\right)/\partial\sigma\right)/\partial\sigma$
- Weighted advection and diffusion coefficients are then
  $N^{w}_{(1,2)}(\sigma) = (1 - P[\Sigma_y \le \sigma])N^{el}_{(1)} + P[\Sigma_y \le \sigma]N^{el-pl}_{(1)}$
- Cumulative Density Function (CDF) of the yield function
- Similar to European Pricing Option in financial simulations (Black–Scholes options pricing model ’73, Nobel prize for Economics ’97)

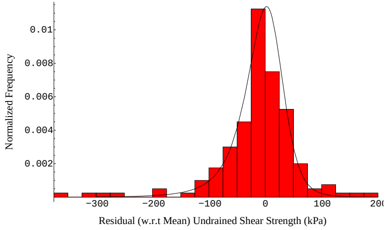


Bi–Linear von Mises Response



SPT Based Determination of Shear Strength

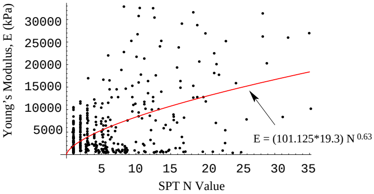


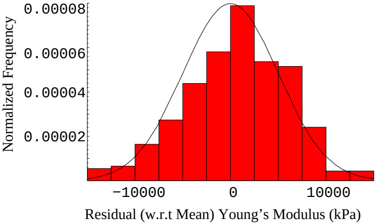


Transformation of SPT N-value → undrained shear strength, s_u (cf. Phoon and Kulhawy (1999B)

Histogram of the residual (w.r.t the deterministic transformation equation) undrained strength, along with fitted probability density function (Pearson IV)

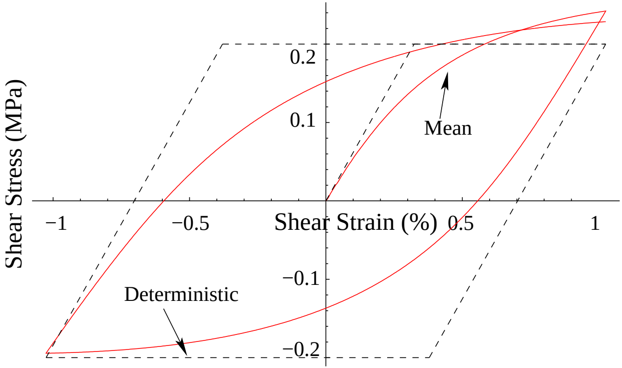
SPT Based Determination of Young's Modulus



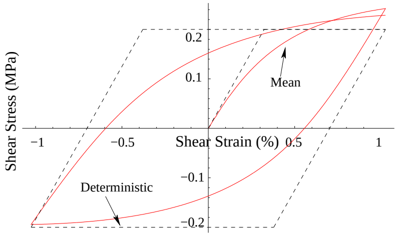


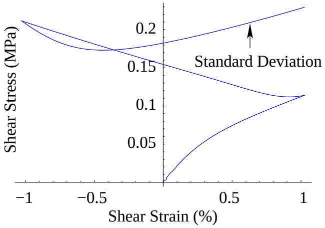
- Transformation of SPT *N*-value → 1-D Young's modulus, *E* (cf. Phoon and Kulhawy (1999B))
- Histogram of the residual (w.r.t the deterministic transformation equation) Young's modulus, along with fitted probability density function

Probabilistic Material Response (von–Mises)

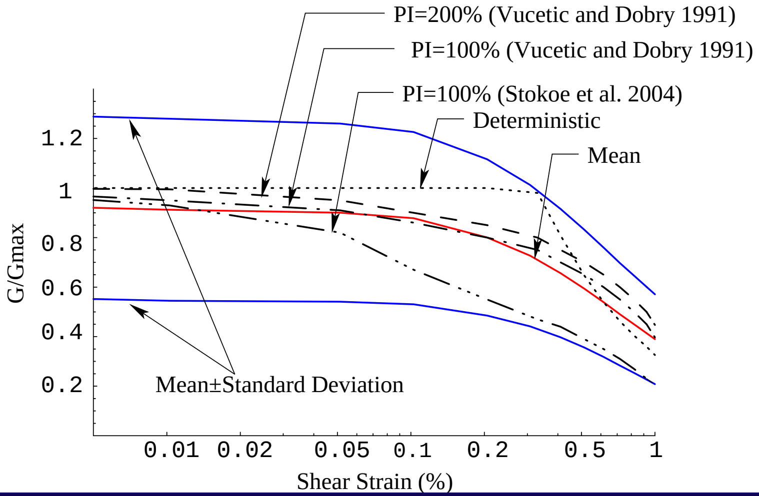


Probabilistic Material Response, Standard Deviation

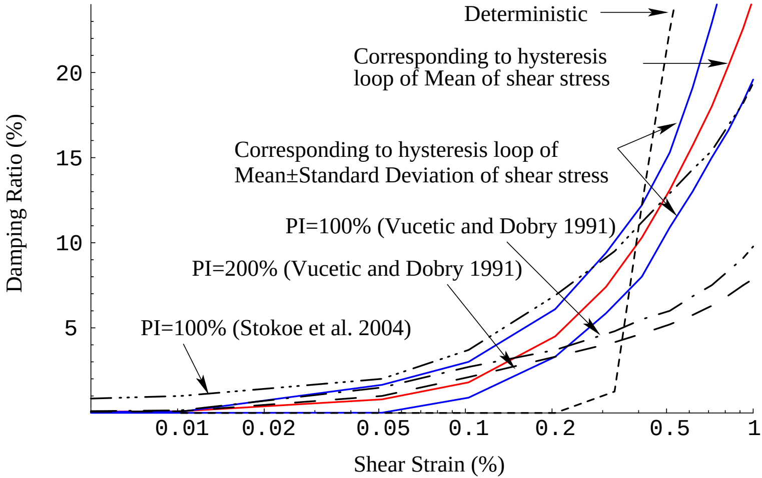




G/G_{max} Response



Damping Response



Outline

Stochastic Finite Element Formulation

- Governing equations:

$$A\sigma = \phi(t); \quad Bu = \epsilon; \quad \sigma = D\epsilon$$

- **Spatial** and **stochastic** discretization
  - Deterministic spatial differential operators (A & B) $\rightarrow$ Regular shape function method with Galerkin scheme
  - Input random field material properties (D) $\rightarrow$ Karhunen–Loève (KL) expansion, optimal expansion, error minimizing property
  - Unknown solution random field (u) $\rightarrow$ Polynomial Chaos (PC) expansion

Spectral Stochastic Elastic–Plastic FEM

- Minimizing norm of error of finite representation using Galerkin technique (Ghanem and Spanos 2003):

$$\sum_{n=1}^{N} K_{mn}^{ep} d_{ni} + \sum_{n=1}^{N} \sum_{j=0}^{P} d_{nj} \sum_{k=1}^{M} C_{ijk} K_{mnk}^{\prime ep} = \langle F_m \psi_i[\{\xi_r\}] \rangle$$

$$K_{mn}^{ep} = \int_D B_n D^{ep} B_m dV \qquad K_{mnk}^{\prime ep} = \int_D B_n \sqrt{\lambda}_k h_k B_m dV$$

$$C_{ijk} = \langle \xi_k(\theta) \psi_i[\{\xi_r\}] \psi_j[\{\xi_r\}] \rangle \qquad F_m = \int_D \phi N_m dV$$

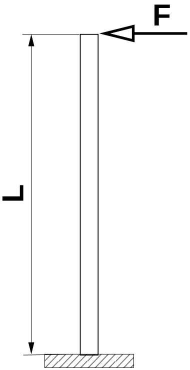
Inside SSEPFEM

- Explicit stochastic elastic–plastic finite element computations
- FPK probabilistic constitutive integration at Gauss integration points
- Increase in (stochastic) dimensions (KL and PC) of the problem
- Excellent for parallelization, both at the element and global levels
- Development of the probabilistic elastic–plastic stiffness tensor

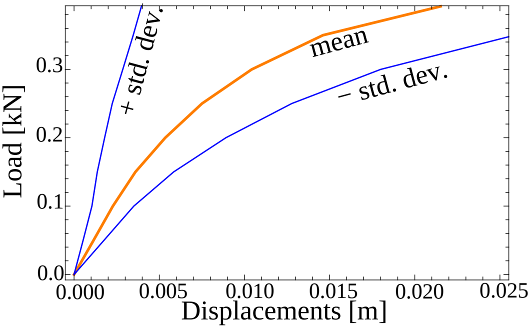
Outline

1–D Static Pushover Test Example

- Linear elastic model:
  $< G >= 2.5$ kPa,
  $Var[G] = 0.15\ \mathrm{kPa}^2$,
  correlation length for $G = 0.3$ m.
- Elastic–plastic material model, von Mises, linear hardening,
  $< G >= 2.5$ kPa,
  $Var[G] = 0.15\ \mathrm{kPa}^2$,
  correlation length for $G = 0.3$ m,
  $C_u = 5$ kPa,
  $C'_u = 2$ kPa.

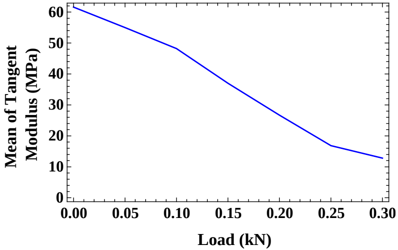


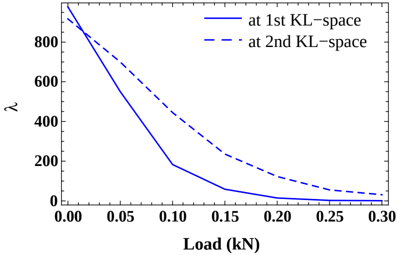
SSEPFEM Response



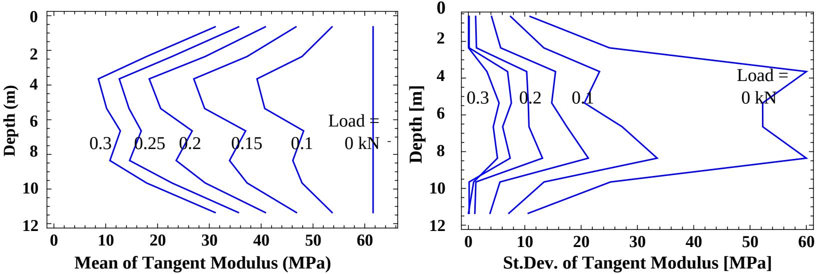
Mean and standard deviations of displacement at the top node, von Mises elastic-perfectly plastic material model, KL-dimension=2, order of PC=2.

Evolution of Probabilistic Stiffness at -6.645m





Probability for Softening!



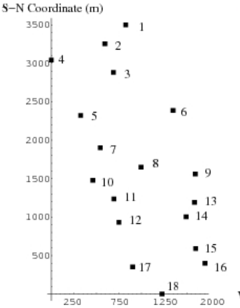
Outline

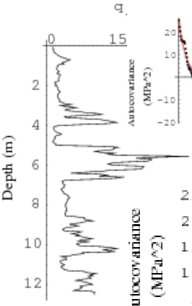
Seismic Wave Propagation through Stochastic Soil

- Soil as 12.5 m deep 1–D soil column (von Mises Material)
  - Properties (including testing uncertainty) obtained through random field modeling of CPT q_T
    $\langle q_T \rangle = 4.99\ MPa$; $Var[q_T] = 25.67\ MPa^2$;
    Cor. Length $[q_T] = 0.61\ m$; Testing Error $= 2.78\ MPa^2$
- q_T was transformed to obtain G: $G/(1-\nu) = 2.9 q_T$
  - Assumed transformation uncertainty = 5%
    $\langle G \rangle = 11.57 MPa$; $Var[G] = 142.32 MPa^2$
    Cor. Length $[G] = 0.61 m$
- Input motions: modified 1938 Imperial Valley

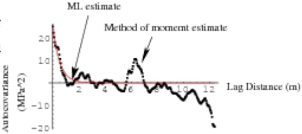
Random Field Parameters from Site Data

- Maximum likelihood estimates

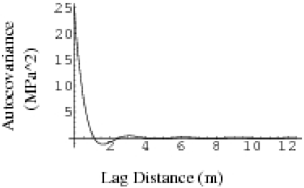




Typical CPT q_T

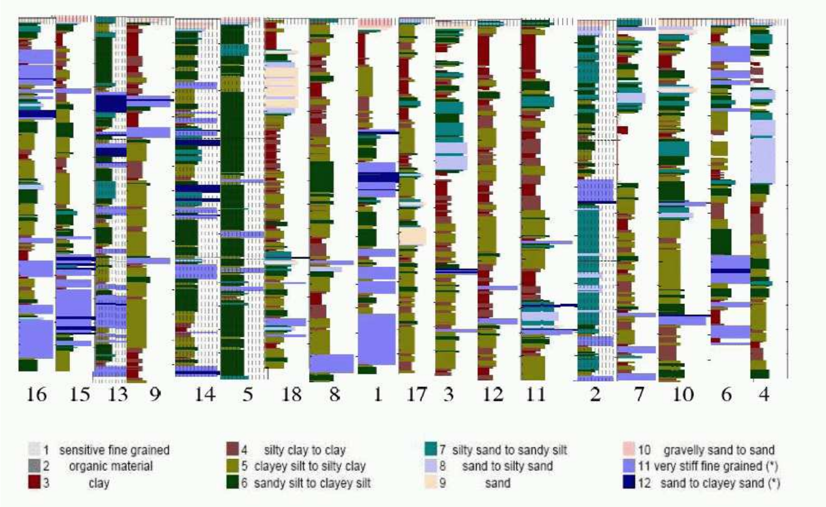


Finite Scale

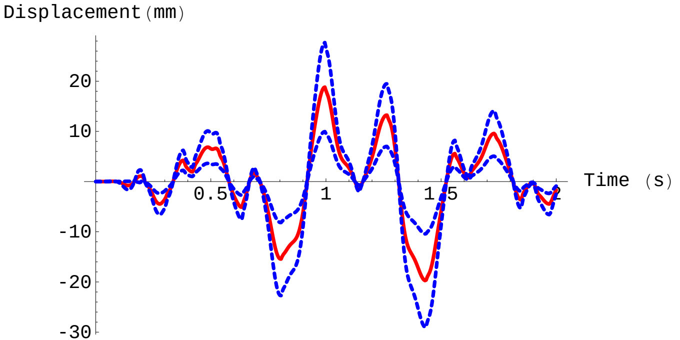


Fractal

"Uniform" CPT Site Data



Seismic Wave Propagation through Stochastic Soil



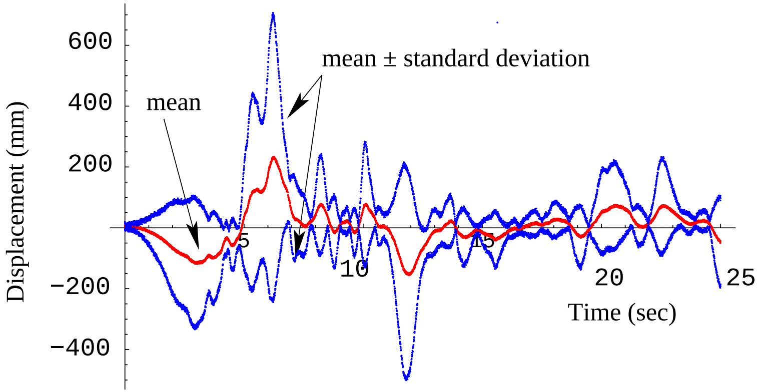
Mean± Standard Deviation

Outline

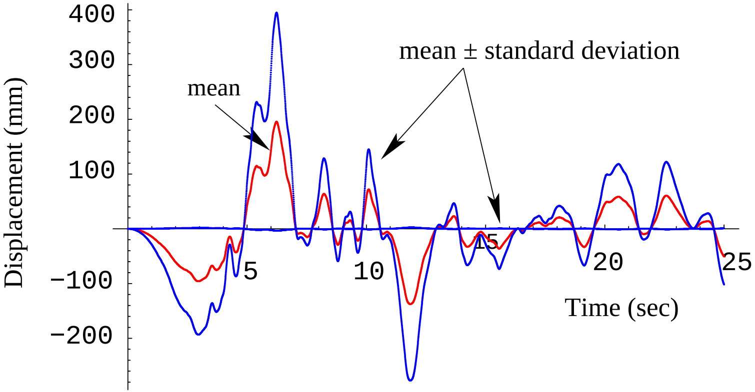
Decision About Site (Material) Characterization

- Do nothing about site characterization (rely on experience): conservative **guess** of soil data, $COV = 225\%$, correlation length $= 12$m.
- Do better than standard site characterization: $COV = 103\%$, correlation length $= 0.61$m)
- Improve site (material) characterization if probabilities of exceedance are unacceptable!

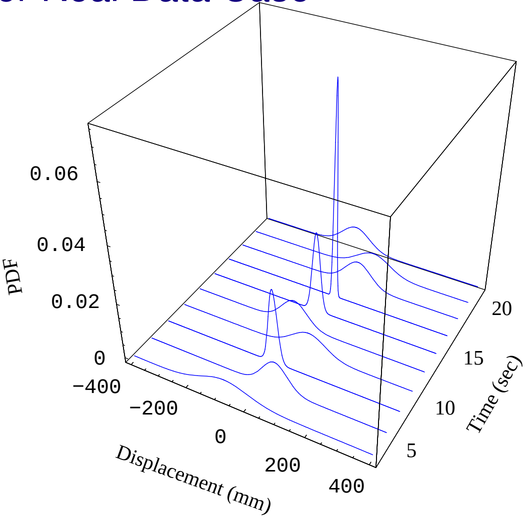
Evolution of Mean ± SD for Guess Case



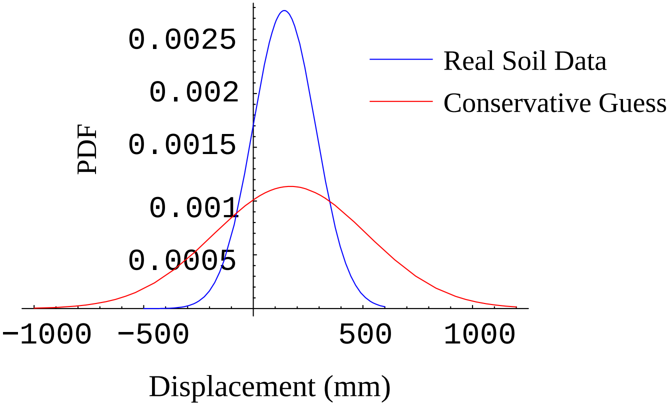
Evolution of Mean ± SD for Real Data Case



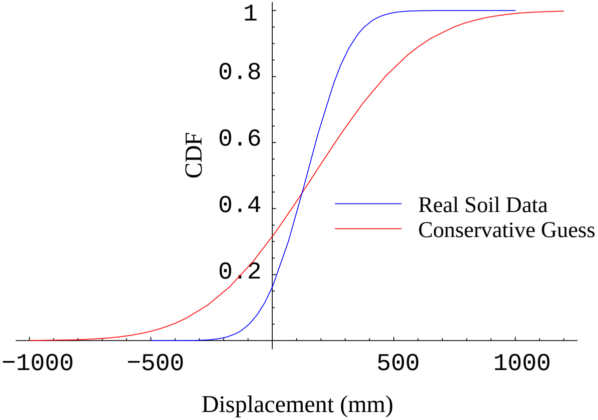
Full PDFs for Real Data Case



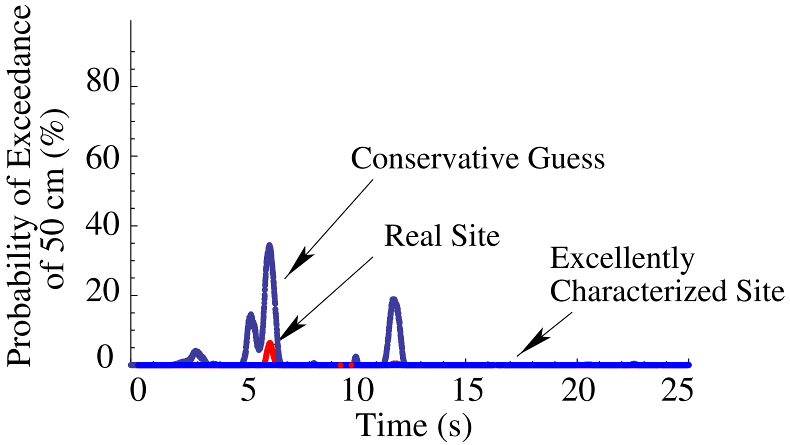
Example: PDF at 6 s



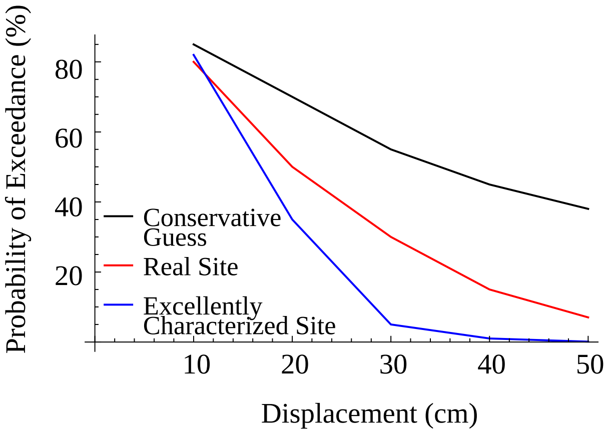
Example: CDF at 6 s



Probability of Unacceptable Deformation (50cm)



Risk Informed Decision Process



Summary

- Material (solids and structures) behavior is uncertain (probably!)
- Simulation of behavior for Geotechnical/Structural system needs to be done probabilistically
- Methods for such simulations do exist (shown today)
- Problem might be with the Human Nature! (how much do you want or do not want to know about potential problem?!)

Risk Information

- Risk informed decisions, very valuable and sought after in

  Nuclear Engineering
  Aerospace Engineering
  Mechanical Engineering
  Biomechanics
  Civil Engineering (Geotech/Struct)

- Owners, Banks and Insurance agencies (will) require it
- Improve infrastructure economy and safety through rational probabilistic mechanics