Verification Procedures for Simulation of Fully Coupled Behavior of Porous Media

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Outline

Introduction

Verification and Validation
  Verification

Saturated Soils
  Fully Coupled Formulation

Verification Suite
  Examples

Summary
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Summary
Introduction

- Numerical analysts, designers need the best available tools for performance assessment (numerical predictions)
- Verification and validation process ensures accuracy of numerical predictions
- How much can (should) we trust model implementations (verification)?
- How much can (should) we trust numerical simulations (validation)?
- How good are our numerical predictions?
- The T experiments
- Focus on verification
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Summary
Verification, Validation, Prediction

- Verification: provides evidence that the model is solved correctly.
- Validation: provides evidence that the correct model is solved.
- Prediction: use of computational model to foretell the state of a physical system under consideration under conditions for which the computational model has not been validated.
Role of Verification and Validation

Oberkampf et al.

Oden et al.
Verification

Verification: the process of determining that a model implementation accurately represents the developer’s conceptual description and specification. Mathematics issue. Verification provides evidence that the model is solved correctly.
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Fully Coupled Formulation

Dynamic Equilibrium for Saturated, Coupled Systems

- Effective stress principle $\sigma'_{ij} = \sigma_{ij} + \alpha \delta_{ij} p$; $(p = -1/3\sigma_{kk})$

- Equilibrium of the mixture
  $\sigma_{ij,j} - \rho \ddot{u}_i - \rho_f [\ddot{w}_i + \dot{w}_j \dot{w}_{i,j}] + \rho b_i = 0$; $(\rho = n\rho_f + (1 - n)\rho_s)$

- Equilibrium of the fluid
  $-\rho_{,i} - R_i - \rho_f \ddot{u}_i - \rho_f [\ddot{w}_i + \dot{w}_j \dot{w}_{i,j}] / n + \rho_f b_i = 0$; (Darcy: $n\dot{w}_j = Ki; i = h,j; R_i = k_i^{-1} \dot{w}_j; k_{ij} = K_{ij}/\rho_f g$ $[m^3/s/kg]$)

- Flow conservation $\dot{w}_{i,i} + \alpha \varepsilon_{ii} + \dot{p}/Q + n\dot{\rho}_f/\rho_f + \dot{s}_0 = 0$; $1/Q \equiv n/K_f + (1 - n)/K_s$
Fully Coupled $u - p - U$ Formulation

- Physical, velocity proportional damping from solid–fluid interaction (not using Rayleigh damping)
- Accelerations of pore fluid not neglected
  - Important for SFSI
  - Inertial forces of fluid allow liquefaction modeling
- Stable formulation for near incompressible pore fluid
Finite Element Discretization

\[
\begin{align*}
(M_s)_{KijL} & \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_{Lj} \\ \ddot{p}_N \\ \ddot{U}_{Lj} \end{bmatrix} \\
0 & 0 \\
0 & (M_f)_{KijL} \\
0 & 0
\end{align*} + \\
\begin{align*}
(C_1)_{KijL} & \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \dot{u}_{Lj} \\ \dot{p}_N \\ \dot{U}_{Lj} \end{bmatrix} \\
0 & 0 \\
0 & (C_2)_{KijL} \\
-\begin{bmatrix} (C_2)_{LjiK} & 0 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \dot{u}_{Lj} \\ \dot{p}_N \\ \dot{U}_{Lj} \end{bmatrix}
\end{align*} + \\
\begin{align*}
(K_{EP})_{KijL} & \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_{Lj} \end{bmatrix} \\
-\begin{bmatrix} (G_1)_{KijM} \\ -P_{MN} \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_{Lj} \end{bmatrix} \\
-\begin{bmatrix} (G_1)_{LjM} \\ -P_{MN} \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_{Lj} \end{bmatrix} \\
0 & -\begin{bmatrix} (G_2)_{KiL} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_{Lj} \end{bmatrix}
\end{align*} = \\
\begin{bmatrix} \ddot{f}_{Ki}^{\text{solid}} \\ \ddot{f}_{Ki}^{\text{fluid}} \end{bmatrix}
\]
Fully Coupled Formulation

Finite Element Discretization

\[
(M_s)_{KijL} = \int_{\Omega} N_K^u (1 - n) \rho_s \delta_{ij} N_L^u \, d\Omega \quad ; \quad (M_f)_{KijL} = \int_{\Omega} N_K^u n \rho_f \delta_{ij} N_L^u \, d\Omega \\
(C_1)_{KijL} = \int_{\Omega} N_K^u n^2 k_i^{-1} N_L^u \, d\Omega \quad ; \quad (C_2)_{KijL} = \int_{\Omega} N_K^u n^2 k_i^{-1} N_L^u \, d\Omega \\
(C_3)_{KijL} = \int_{\Omega} N_K^u n^2 k_i^{-1} N_L^u \, d\Omega \quad ; \quad (K^{EP})_{KijL} = \int_{\Omega} N_K^u n D_{ijmn} N_L^u, n \, d\Omega \\
(G_1)_{KiM} = \int_{\Omega} N_K^u, i (\alpha - n) N_M^p \, d\Omega \quad ; \quad (G_2)_{KiM} = \int_{\Omega} n N_K^u, i N_M^p \, d\Omega \\
P_{NM} = \int_{\Omega} N_M^p \frac{1}{Q} N_M^p \, d\Omega
\]
Finite Element Discretization

\[ f_{K_i}^{\text{solid}} = \int_{\Gamma_t} N_K^u n_j \sigma_{ij}'' \, d\Gamma - \int_{\Gamma_p} N_K^u (\alpha - n) n_i \rho d\Gamma \]

\[ + \int_{\Omega} N_K^u (1 - n) \rho_s b_i d\Omega \]

\[ f_{K_i}^{\text{fluid}} = -\int_{\Gamma_p} n N_K^U n_i \rho d\Gamma \]

\[ + \int_{\Omega} n N_K^U \rho_f b_i d\Omega \]
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Verification Suite

- Code Verification
  - Memory
  - Function call arguments
  - Code coverage
  - Argument bounds
  - Compiler warnings

- Computational Solution Verification
  - Drilling of a well [Coussy 04]
  - The Case of a Spherical Cavity [Coussy 04]
  - Consolidation of a Soil Layer [Coussy 95]
  - Line Injection of a fluid in a Reservoir [Coussy 95]
  - Wave propagation, step displacement [Gajo and Mongiovi 95]
  - Wave propagation, step velocity loading [de Boer et al. 93]
  - Wave propagation, step force loading [Hiremath et al. 88]
Examples

Vertical Consolidation

- Effective Stresses
- Hydrostatic Pressures
- Total Stresses

γ = 18.55 kN/m³

Normalized Excess Pore Pressures

Quicker dissipation towards the surface

Settlement of soil skeleton (m)

Numerical Analysis

Analytical Solution
Vertical Consolidation: Normalized Excess Pore Pressure

- **Analytical Solution**
- **Numerical Analysis**

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**Examples**

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Verification Procedures for Simulation of Fully Coupled Behavior of Porous Media
Shock Wave Propagation, Step Displacement

Boundary Condition:
\( \delta (0, t) = 1 \times 10^{-3} \text{ cm} \)

\( u_x = U_x = 1 \times 10^{-3} \text{ cm} \)

0.01 cm
4 cm

Semi-finite Soil column

No lateral flow.
No lateral displacements.

(a) (b) (c)
Shock Wave Propagation: Step Displacement
Shock Wave Propagation: Porous Solid

The graph illustrates the comparison between FEM (Finite Element Method) and analytical solutions for solid displacement over time. The x-axis represents time in $10^{-6}$ seconds, while the y-axis represents the solid displacement in centimeters. The graph includes multiple curves for different permeability values ($k$):

- **FEM** (k = $10^{-5}$ cm$^3$/s/g)
- **FEM** (k = $10^{-6}$ cm$^3$/s/g)
- **FEM** (k = $10^{-8}$ cm$^3$/s/g)
- **Analytical Solution** (k = $10^{-5}$ cm$^3$/s/g)
- **Analytical Solution** (k = $10^{-6}$ cm$^3$/s/g)
- **Analytical Solution** (k = $10^{-8}$ cm$^3$/s/g)

The graph shows how the solid displacement changes with time for different permeability values, demonstrating the effectiveness of both FEM and analytical solutions in simulating shock wave propagation in porous solids.
Shock Wave Propagation: Pore Fluid

FEM Vs Analytical Solution

- FEM (k = 10^{-5} \text{ cm}^3\text{s/g})
- FEM (k = 10^{-6} \text{ cm}^3\text{s/g})
- FEM (k = 10^{-8} \text{ cm}^3\text{s/g})
- Analytical Solution (k = 10^{-5} \text{ cm}^3\text{s/g})
- Analytical Solution (k = 10^{-6} \text{ cm}^3\text{s/g})
- Analytical Solution (k = 10^{-8} \text{ cm}^3\text{s/g})

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▶ Importance of verification and validation for numerical predictions

▶ Numerical predictions under uncertainty

▶ Would you trust numerical simulations (for design/regulation/evaluation) if your program of choice (simulation tool) did not follow (extensive) verification and validation procedures?