Verification Procedures for Simulation of Fully Coupled Behavior of Porous Media

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Outline

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Introduction

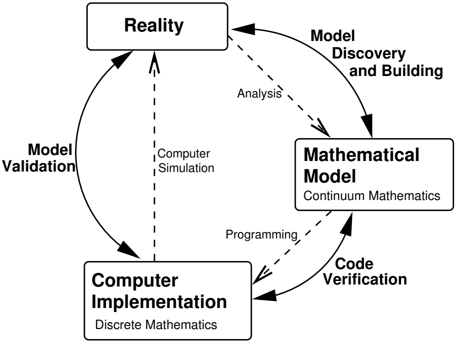
- Numerical analysts, designers need the best available tools for performance assessment (numerical predictions)
- Verification and validation process ensures accuracy of numerical predictions
- How much can (should) we trust model implementations (verification)?
- How much can (should) we trust numerical simulations (validation)?
- How good are our numerical predictions?
- The T experiments
- Focus on verification

Outline

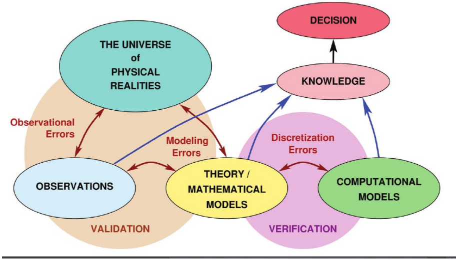
Verification, Validation, Prediction

- Verification: provides evidence that the model is solved correctly.
- Validation: provides evidence that the correct model is solved.
- Prediction: use of computational model to foretell the state of a physical system under consideration under conditions for which the computational model has not been validated

Role of Verification and Validation



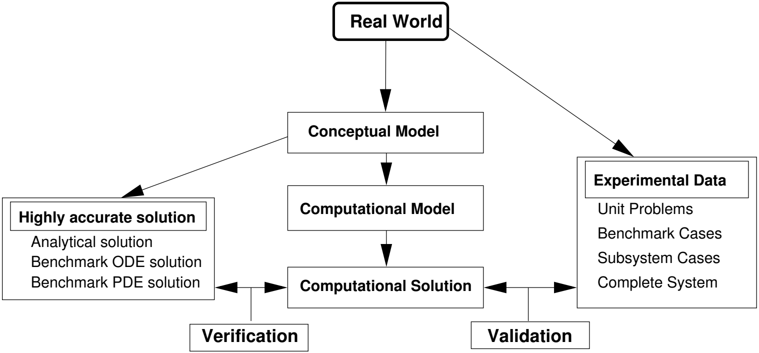
Oberkampf et al.



Oden et al.

Verification

Verification: the process of determining that a model implementation accurately represents the developer's conceptual description and specification. Mathematics issue.
Verification provides evidence that the model is solved correctly.



Outline

Dynamic Equilibrium for Saturated, Coupled Systems

- Effective stress principle $\sigma'_{ij} = \sigma_{ij} + \alpha\delta_{ij}p$; $(p = -1/3\sigma_{kk})$
- Equilibrium of the mixture
  $\sigma_{ij,j} - \rho\ddot{u}_i - \rho_f[\ddot{w}_i + \underline{\dot{w}_j\dot{w}_{i,j}}] + \rho b_i = 0$; $(\rho = n\rho_f + (1-n)\rho_s)$
- Equilibrium of the fluid
  $-p_{,i} - R_i - \rho_f\ddot{u}_i - \rho_f[\ddot{w}_i + \underline{\dot{w}_j\dot{w}_{i,j}}]/n + \rho_f b_i = 0$; (Darcy: $n\dot{w}_j = Ki$; $i = h_{,j}$; $R_i = k_{ij}^{-1}\dot{w}_j$; $k_{ij} = K_{ij}/\rho_f g$ $[m]^3[s]/[kg]$)
- Flow conservation $\dot{w}_{i,i} + \alpha\dot{\varepsilon}_{ii} + \dot{p}/Q + \underline{n\dot{\rho}_f/\rho_f + \dot{s}_0} = 0$;
  $1/Q \equiv n/K_f + (1-n)/K_s$

Fully Coupled $u - p - U$ Formulation

- Formulation: fully coupled by Zienkiewicz and Shiomi 1984), nonlinear dynamics by Argyris and Mlejnek (1991)
- Physical, velocity proportional damping from solid–fluid interaction (not using Rayleigh damping)
- Accelerations of pore fluid not neglected
  - important for SFSI
  - inertial forces of fluid allow liquefaction modeling
- Stable formulation for near incompressible pore fluid

Finite Element Discretization

$$
\begin{bmatrix} (M_s)_{KijL} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (M_f)_{KijL} \end{bmatrix}
\begin{bmatrix} \ddot{\bar{u}}_{Lj} \\ \ddot{\bar{p}}_N \\ \ddot{\bar{U}}_{Lj} \end{bmatrix} +
$$

$$
+ \begin{bmatrix} (C_1)_{KijL} & 0 & -(C_2)_{KijL} \\ 0 & 0 & 0 \\ -(C_2)_{LjiK} & 0 & (C_3)_{KijL} \end{bmatrix}
\begin{bmatrix} \dot{\bar{u}}_{Lj} \\ \dot{\bar{p}}_N \\ \dot{\bar{U}}_{Lj} \end{bmatrix} +
$$

$$
+ \begin{bmatrix} (K^{EP})_{KijL} & -(G_1)_{KiM} & 0 \\ -(G_1)_{LjM} & -P_{MN} & -(G_2)_{LjM} \\ 0 & -(G_2)_{KiL} & 0 \end{bmatrix}
\begin{bmatrix} \bar{u}_{Lj} \\ \bar{p}_M \\ \bar{U}_{Lj} \end{bmatrix} =
\begin{bmatrix} \bar{f}^{solid}_{Ki} \\ 0 \\ \bar{f}^{fluid}_{Ki} \end{bmatrix}
$$

Finite Element Discretization

$$(M_s)_{KijL} = \int_\Omega N_K^u (1-n)\rho_s \delta_{ij} N_L^u d\Omega \quad ; \quad (M_f)_{KijL} = \int_\Omega N_K^U n \rho_f \delta_{ij} N_L^U d\Omega$$

$$(C_1)_{KijL} = \int_\Omega N_K^u n^2 k_{ij}^{-1} N_L^u d\Omega \quad ; \quad (C_2)_{KijL} = \int_\Omega N_K^u n^2 k_{ij}^{-1} N_L^U d\Omega$$

$$(C_3)_{KijL} = \int_\Omega N_K^U n^2 k_{ij}^{-1} N_L^U d\Omega \quad ; \quad (K^{EP})_{KijL} = \int_\Omega N_{K,m}^u D_{imjn} N_{L,n}^u d\Omega$$

$$(G_1)_{KiM} = \int_\Omega N_{K,i}^u (\alpha - n) N_M^p d\Omega \quad ; \quad (G_2)_{KiM} = \int_\Omega n N_{K,i}^U N_M^p d\Omega$$

$$P_{NM} = \int_\Omega N_N^p \frac{1}{Q} N_M^p d\Omega$$

Finite Element Discretization

$$\bar{f}_{Ki}^{solid} = \int_{\Gamma_t} N_K^u n_j \sigma_{ij}'' d\Gamma - \int_{\Gamma_p} N_K^u (\alpha - n) n_i p d\Gamma + \int_{\Omega} N_K^u (1-n) \rho_s b_i d\Omega$$

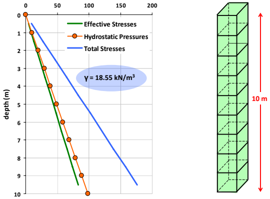
$$\bar{f}_{Ki}^{fluid} = -\int_{\Gamma_p} n N_K^U n_i p d\Gamma + \int_{\Omega} n N_K^U \rho_f b_i d\Omega$$

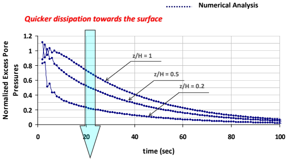
Outline

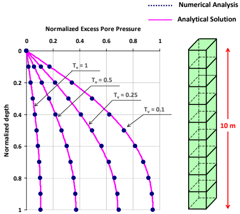
Verification Suite

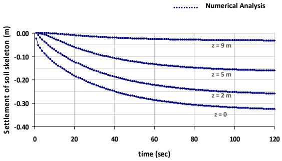
- Code Verification
  - Memory
  - Function call arguments
  - Code coverage
  - Argument bounds
  - Compiler warnings
- Computational Solution Verification
  - Drilling of a well [Coussy 04]
  - The Case of a Spherical Cavity [Coussy 04]
  - Consolidation of a Soil Layer [Coussy 95]
  - Line Injection of a fluid in a Reservoir [Coussy 95]
  - Wave propagation, step displacement [Gajo and Mongiovi 95]
  - Wave propagation, step velocity loading [de Boer et al. 93]
  - Wave propagation, step force loading [Hiremath et al. 88]

Vertical Consolidation

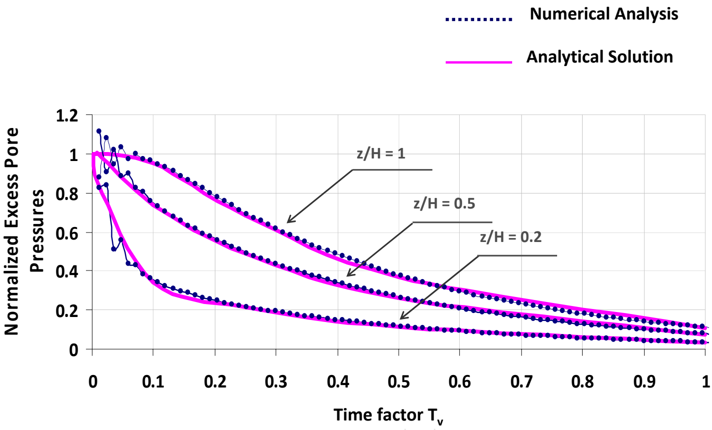




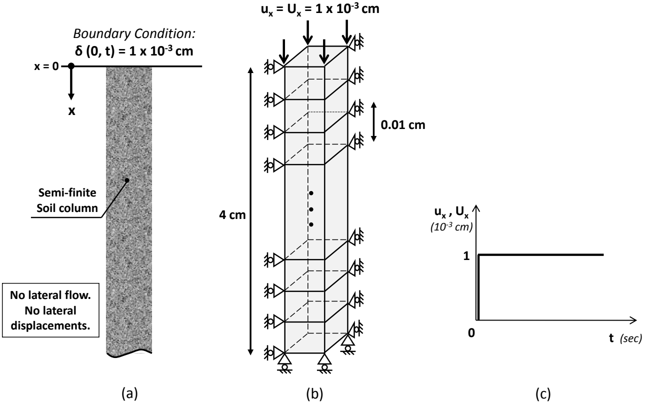




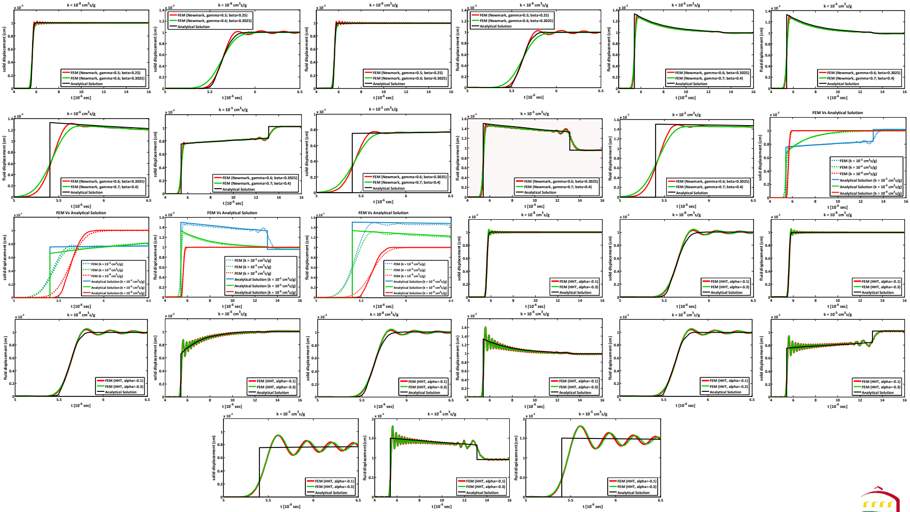
Vertical Consolidation: Normalized Excess Pore Pressure



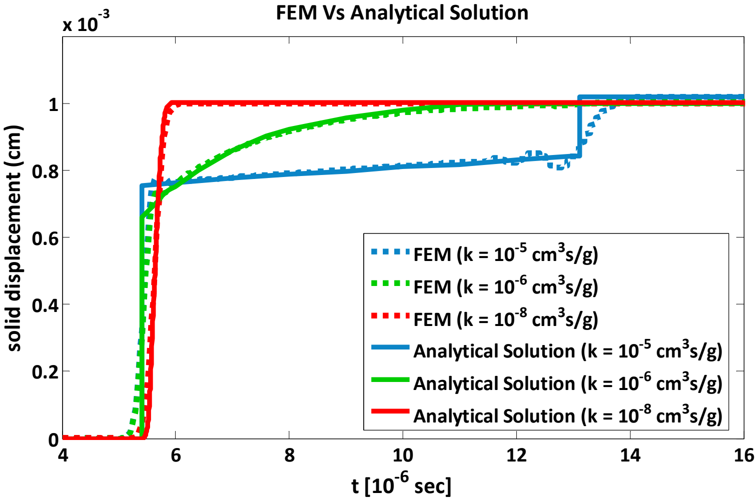
Shock Wave Propagation, Step Displacement



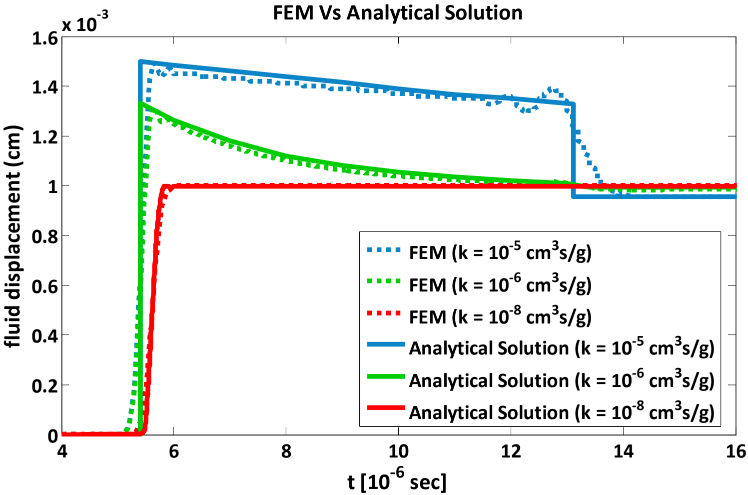
Shock Wave Propagation: Step Displacement



Shock Wave Propagation: Porous Solid



Shock Wave Propagation: Pore Fluid



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Summary

- Importance of verification and validation for numerical predictions
- Numerical predictions under uncertainty
- Would you trust numerical simulations (for design/regulation/evaluation) if your program of choice (simulation tool) did not follow (extensive) verification and validation procedures?