

# Aspects of Deterministic and Probabilistic Modeling and Simulation in Earthquake Engineering

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# Outline

Introduction

Deterministic Modeling

Probabilistic Modeling

Summary

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Deterministic Modeling

Probabilistic Modeling

Summary

# The Problem

- ▶ Seismic response of Nuclear Power Plants
- ▶ 3D, inclined seismic motions consisting of body and surface waves
- ▶ Inelastic (elastic, damage, plastic behavior of materials: soil, rock, concrete, steel, rubber, etc.)
- ▶ Full coupling of pore fluids (in soil and rock) with soil/rock skeleton
- ▶ Buoyant effects (foundations below water table)
- ▶ Uncertainty in seismic sources, path, soil/rock response and structural response

## Solution

- ▶ **Physics based modeling and simulation** of seismic behavior of soil-structure systems (NPP structures, components and systems)
- ▶ Development and use of **high fidelity** time domain, nonlinear numerical models, in **deterministic** and **probabilistic** spaces
- ▶ Accurate following of the **flow of seismic energy** (input and dissipation) within soil-structure NPP system
- ▶ **Directing**, in space and time, with **high (known) confidence**, seismic energy flow in the soil-foundation-structure system

# NRC ESSI Simulator System

- ▶ **The NRC-ESSI-Program** is a 3D, nonlinear, time domain, parallel finite element program specifically developed for Hi-Fi modeling and simulation of Earthquake Soil/Rock Structure Interaction problems for NPPs on NRC-ESSI-Computer.
- ▶ **The NRC-ESSI-Computer** is a distributed memory parallel computer, a cluster of clusters with multiple performance processors and multiple performance networks.
- ▶ **The NRC-ESSI-Notes** represent a hypertext documentation system detailing modeling and simulation of NPP ESSI problems.

# NRC ESSI Simulator Program

- ▶ Based on a Collection of Useful Libraries (modular, portable)
- ▶ Library centric software design
- ▶ Various public domain licenses (GPL, LGPL, BSD, CC)
- ▶ Verification and Validation
- ▶ Detailed program documentation (part of NRC ESSI Notes)
- ▶ Target users: U.S.-NRC staff, UCD students, external users

## Collection of Useful Libraries (Modeling Part)

- ▶ Template3D-EP libraries for elastic and elastic-plastic computations (UCD, CC)
- ▶ FEMTools finite element libraries provide finite elements (solids, beams, shells, contacts/isolators, seismic input) (UCD, UCB, CU, CC)
- ▶ Loading, staged, self weight, service loads, seismic loads (the Domain Reduction Method, analytic input (incoming/outgoing) of 3D, inclined, un-correlated seismic motions) (UCD, CC)
- ▶ Domain Specific Language for input (UCD, CC)



## Collection of Useful Libraries (Simulation Part)

- ▶ Plastic Domain Decomposition (PDD) for parallel computing (UCD, CC)
- ▶ PETSc (ANL, GPL-like) and UMFPACK (UF, GPL) solvers
- ▶ Modified OpenSees Services (MOSS) for managing the finite element domain (UCD, CC; UCB, GPL?)
- ▶ nDarray (UCD, CC), LTensor (CIMEC, GPL), BLAS (UTK, GPL) for lower level computational tasks,
- ▶ Message Passing Interface (MPI, openMPI, new BSD license)

# NRC ESSI Simulator Computer

A distributed memory parallel (DMP) computer designed for high performance, parallel finite element simulations

- ▶ Multiple performance CPUs and Networks
- ▶ Most cost-performance effective
- ▶ Source compatibility with any DMP supercomputers
- ▶ Current system: 208 CPUs
- ▶ Near future: 784 CPUs



# NRC ESSI Simulator Version December 2010

- ▶ Operating System: Linux Fedora Core 14.
- ▶ Kernel: 2.6.35.10-74.fc14.x86\_64
- ▶ Compute Nodes (two):
  - ▶ CPU: 2 × Intel Xeon E5620 Westmere 2.4 GHz Quad Core (8 threads)
  - ▶ RAM: 6 × 4GB DDR3 1333 MHz ECC/Registered Memory (24GB Total Memory)
  - ▶ Disk: 8 × 500 GB Seagate Constellation ES 3.5" SATA/300 (Linux Software RAID10)
- ▶ Network: single GigaBit

# NRC ESSI Simulator Version April 2012

Operating System: Ubuntu

Kernel: 3.2

**Controller:** 1 node + **Compute:** 8 Nodes

- ▶ CPU: 2 x 12 cores Opteron 6234 = 24 cores
- ▶ RAM: 32GB (8 x 4GB)
- ▶ NICs:
  - ▶ GigaBit: Intel 82576 (Controller)
  - ▶ InfiniBand: ConnectX-2 QDR IB 40Gb/s (Controller+Compute)
- ▶ Disk: 8 × 2TB Toshiba MK2002TSKB (Controller)
- ▶ Disk: 1TB Toshiba MK1002TSKB (Compute)

Network (dual):

- ▶ GigaBit: HP ProCurve Switch 1810-48G 48 Port
- ▶ InfiniBand:: Mellanox MIS5030Q-1SFCA 36-port QDR

# NRC ESSI Simulator Notes

- ▶ A hypertext documentation system describing in detail modeling and simulations of NPP ESSI problems
  - ▶ Theoretical and Computational Formulations (FEM, EL-PL, Static and Dynamic solution, Parallel Computing)
  - ▶ Software and Hardware Platform Design (OO Design, Library centric design, API, DSL, Software Build Process, Hardware Platform)
  - ▶ Verification and Validation (code V, Components V, Static and Dynamic V, Wave Propagation V)
  - ▶ Application to Practical Nuclear Power Plant Earthquake Soil/Rock Structure Interaction Problems (ESSI with 3D, inclined, uncorrelated seismic waves, ESSI with foundation slip, Isolators)

# Outline

Introduction

**Deterministic Modeling**

Probabilistic Modeling

Summary

# High Fidelity Modeling

- ▶ Energy influx, body and surface waves, 3D, inclined
- ▶ Mechanical dissipation outside of SFS domain:
  - ▶ Radiation of reflected waves
  - ▶ Radiation of oscillating SFS system
- ▶ Mechanical dissipation inside SFS domain:
  - ▶ Plasticity of soil/rock subdomain
  - ▶ Viscous coupling of porous solid with pore fluid (air, water)
  - ▶ Plasticity and viscosity of foundation – soil/rock interface
  - ▶ Plasticity/damage of the structure
  - ▶ Viscous coupling of structure/foundation with fluids
- ▶ Numerical energy dissipation/production

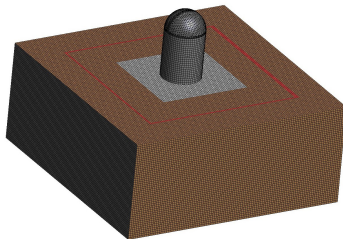
# High Performance, Parallel Computing

- ▶ The NRC ESSI Simulator can be used in both sequential and parallel modes
- ▶ For high fidelity models, parallel is really the only option
- ▶ High performance, parallel computing using Plastic Domain Decomposition Method
- ▶ Developed for multiple/variable capability CPUs and networks (DMP and SMPs)



# Representative NPP Example Problem

- ▶ Body and surface seismic waves
- ▶ Seismic wave frequencies up to 50Hz
- ▶ Elastic-plastic soil/rock and structural components,
- ▶ Inelastic contact/gap
- ▶ Seismic isolator effects
- ▶ Buoyant effects for deep foundation embedment
- ▶ High Fidelity Model: soil block:  $230m \times 230m \times 100m$ , foundation  $90m \times 90m$  Containment Structure:  $40m \times 50m$ , 2.1 Million DOFs, 700,000 elements,



# Finite Elements

- ▶ Linear and nonlinear truss element
- ▶ Linear and nonlinear beam (disp. based), variable BCs
- ▶ Linear shell Triangle and Quad with drilling DOFs
- ▶ Linear and nonlinear thick shell
- ▶ Single phase solid bricks (8, 20, 8-20, 27 node element)
- ▶ Two phase (fully coupled, porous solid, pore fluid) solid bricks (8 and 27 node:  $u - p - U$ ,  $u - p$ )
- ▶ Dry friction slip and gap element
- ▶ Saturated gap and (effective stress) slip element
- ▶ Seismic isolator (latex rubber, neoprene rubber, rubber with lead core, friction pendulum)

# Material Models for Solids and Structures

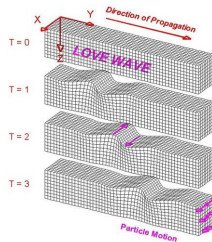
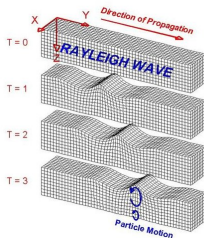
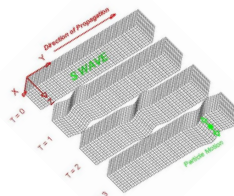
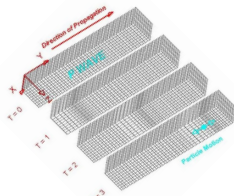
- ▶ Elastic: linear, nonlinear isotropic, cross anisotropic
- ▶ Elastic-Plastic: von Mises, Drucker–Prager, Cam–Clay, Rounded Mohr–Coulomb, Parabolic Leon, SANIsand (Dafalias–Manzari...), Pisanò–Jeremić.
- ▶ Isotropic and kinematic (translational and rotational) kinematic hardening

# Earthquake Ground Motions

## Realistic earthquake ground motions

- ▶ Body: P and S waves
- ▶ Surface: Rayleigh, Love waves, etc.
- ▶ Lack of correlation (incoherence)
- ▶ Inclined waves
- ▶ 3D waves

# Body (P, S) and Surface (Rayleigh, Love) Waves

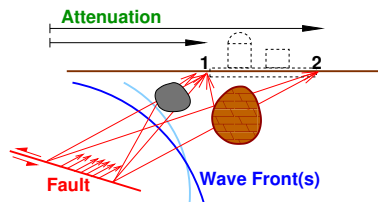


# Spatial Variability (Incoherence, Lack of Correlation)

Incoherence → frequency domain

Lack of Correlation → time domain

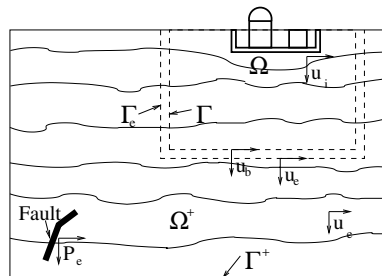
- ▶ Attenuation effects
- ▶ Wave passage effects
- ▶ Extended source effects
- ▶ Scattering effects
- ▶ Variable seismic energy dissipation



# Seismic Input

The Domain Reduction Method  
(Bielak et al.):

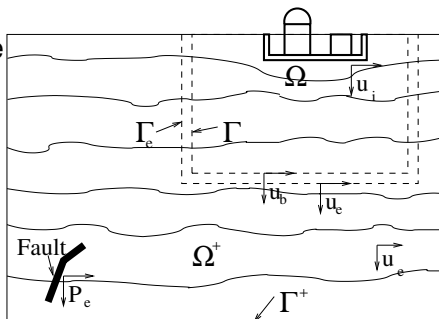
The effective force  $P^{eff}$   
is a dynamically consistent  
replacement for the dynamic  
source forces  $P_e$



$$P^{eff} = \begin{Bmatrix} P_i^{eff} \\ P_b^{eff} \\ P_e^{eff} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -M_{be}^{\Omega+} \ddot{u}_e^0 - K_{be}^{\Omega+} u_e^0 \\ M_{eb}^{\Omega+} \ddot{u}_b^0 + K_{eb}^{\Omega+} u_b^0 \end{Bmatrix}$$

# DRM

- ▶ Seismic forces  $P_e$  replaced by  $P^{eff}$
- ▶  $P^{eff}$  applied only to a single layer of elements next to  $\Gamma$ .
- ▶ The only outgoing waves are from dynamics of the NPP
- ▶ Material inside  $\Omega$  can be elastic-plastic
- ▶ All types of seismic waves (body, surface...) are properly modeled

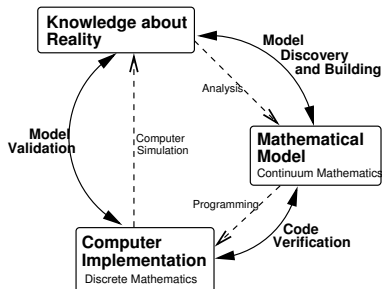




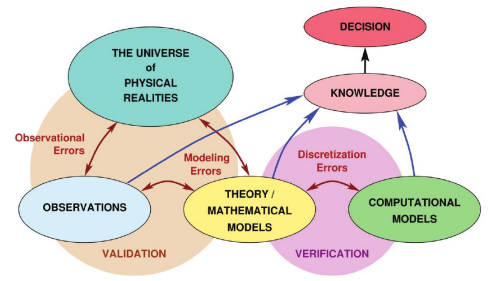
## Verification, Validation and Prediction

- ▶ Verification: the process of determining that a model implementation accurately represents the developer's conceptual description and specification. Mathematics issue. *Verification provides evidence that the model is solved correctly.*
- ▶ Validation: The process of determining the degree to which a model is accurate representation of the real world from the perspective of the intended uses of the model. Physics issue. *Validation provides evidence that the correct model is solved.*
- ▶ Prediction: use of computational model to foretell the state of a physical system under consideration under conditions for which the computational model has not been validated

# Role of Verification and Validation



Oberkampf et al.



Oden et al.

# Importance of V & V

- ▶ V & V procedures are the primary means of assessing accuracy in modeling and computational simulations
- ▶ V & V procedures are the tools with which we build confidence and credibility in modeling and computational simulations

# V & V for ESSI Modeling and Simulations

- ▶ Material modeling and simulation (elastic, elastic-plastic...)
- ▶ Finite elements (solids, structural, special...)
- ▶ Solution advancement algorithms (static, dynamic...)
- ▶ Seismic input and radiation
- ▶ Finite element model verification

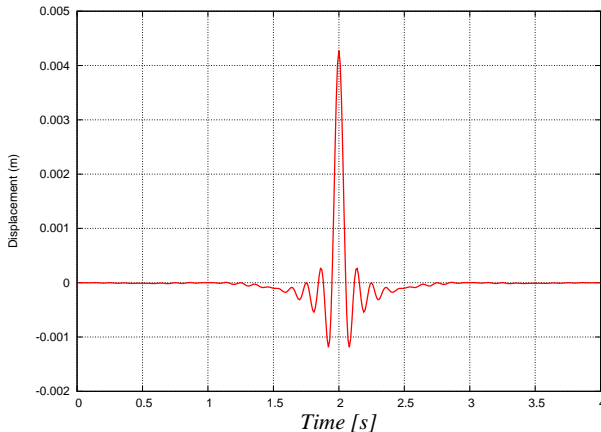
# Mesh Size Effects on Seismic Wave Propagation Modeling

- ▶ Finite element mesh "filters out" high frequencies
- ▶ Usual rule of thumb: 10-12 elements needed per wave length
- ▶ 1D wave propagation model
- ▶ 3D finite elements (same in 3D)
- ▶ Motions applied as displacements at the bottom

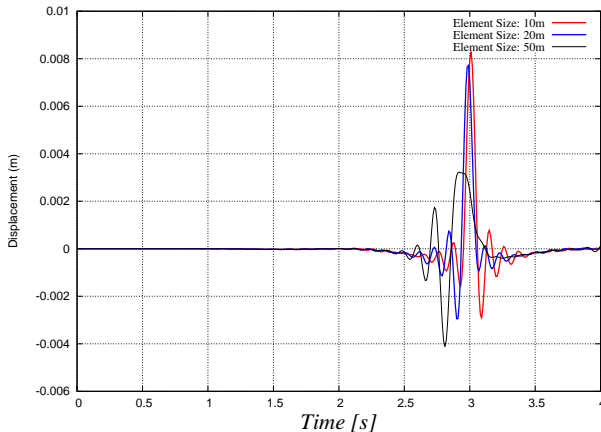


case	model height [m]	$V_s$ [m/s]	El.size [m]	$f_{max}$ (10el) [Hz]
3	1000	1000	10	10
4	1000	1000	20	5
6	1000	1000	50	2

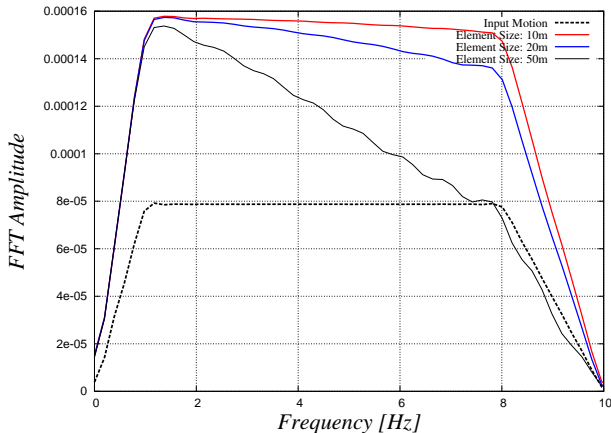
# Cases 3, 4, and 6, Ormsby Wavelet Input Motions



# Cases 3, 4, and 6, Surface Motions



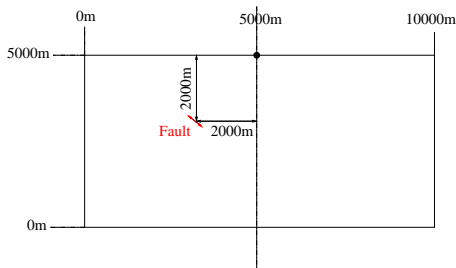
# Cases 3, 4, and 6, Input and Surface Motions, FFT





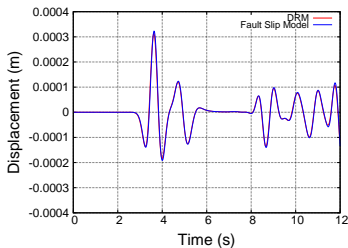
# Free Field, Inclined, 3D Body and Surface Waves

- ▶ Development of analytic and numerical 3D, inclined, uncorrelated seismic motions for verification
- ▶ Large scale models
- ▶ Point shear source
- ▶ Stress drop:
  - ▶ Wavelet (Ricker, Ormsby, etc)
  - ▶ Analytic
- ▶ Seismic input using DRM

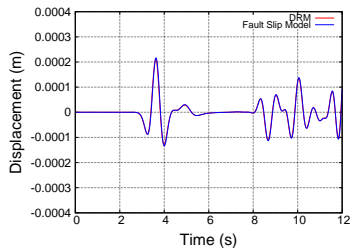


# Verification: Displacements, Top Middle Point

(X)

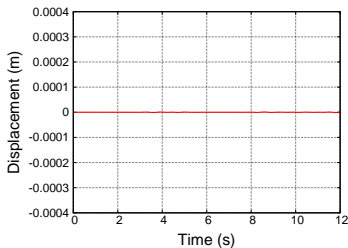


(Z)

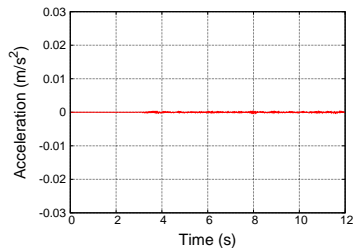


# Verification: Disp. and Acc., Out of DRM

## Displacement



## Acceleration



# Outline

Introduction

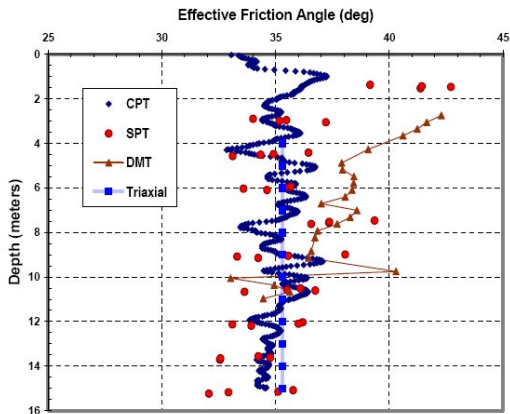
Deterministic Modeling

**Probabilistic Modeling**

Summary

# Material Behavior Inherently Uncertain

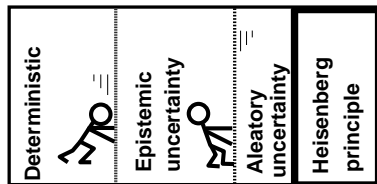
- ▶ Spatial variability
- ▶ Point-wise uncertainty, testing error, transformation error



(Mayne et al. (2000))

# Types of Uncertainties

- ▶ Aleatory uncertainty - inherent variation of physical system
  - ▶ Can not be reduced
  - ▶ Has highly developed mathematical tools
- ▶ Epistemic uncertainty - due to lack of knowledge
  - ▶ Can be reduced by collecting more data
  - ▶ Mathematical tools are not well developed
  - ▶ trade-off with aleatory uncertainty
- ▶ Ergodicity (exchanging ensemble averages for time average) assumed to hold



# Recent State-of-the-Art

- ▶ Governing equation
  - ▶ Dynamic problems  $\rightarrow M\ddot{u} + C\dot{u} + Ku = F$
  - ▶ Static problems  $\rightarrow Ku = F$
- ▶ Existing solution methods
  - ▶ **Random r.h.s** (external force random)
    - ▶ FPK equation approach
    - ▶ Use of fragility curves with deterministic FEM (DFEM)
  - ▶ **Random l.h.s** (material properties random)
    - ▶ Monte Carlo approach with DFEM  $\rightarrow$  CPU expensive
    - ▶ Perturbation method  $\rightarrow$  a linearized expansion! Error increases as a function of COV
    - ▶ Spectral method  $\rightarrow$  developed for elastic materials so far
- ▶ Original development of **Probabilistic Elasto-Plasticity**

# Uncertainty Propagation through Constitutive Eq.

- Incremental el-pl constitutive equation  $\Delta\sigma_{ij} = D_{ijkl}\Delta\epsilon_{kl}$

$$D_{ijkl} = \begin{cases} D_{ijkl}^{el} & \text{for elastic} \\ D_{ijkl}^{el} - \frac{D_{ijmn}^{el} m_{mn} n_{pq} D_{pqkl}^{el}}{n_{rs} D_{rstu}^{el} m_{tu} - \xi_* r_*} & \text{for elastic-plastic} \end{cases}$$

- What if all (any) material parameters are uncertain
- PEP and SEPFEM methods for spatially variable and point uncertain material



# Solution to Probabilistic Elastic-Plastic Problem

- ▶ Use of stochastic continuity (Liouville) equation (Kubo 1963)
- ▶ With cumulant expansion method (Kavvas and Karakas 1996)
- ▶ To obtain ensemble average form of Liouville Equation
- ▶ Which, with van Kampen's Lemma (van Kampen 1976): ensemble average of phase density is the probability density
- ▶ Yields Eulerian-Lagrangian form of the Forward Kolmogorov (Fokker-Planck-Kolmogorov) equation

# Probabilistic Stress Solution: Eulerian–Lagrangian form of FPK Equation

$$\begin{aligned}
 \frac{\partial P(\sigma_{ij}(x_t, t), t)}{\partial t} &= \frac{\partial}{\partial \sigma_{mn}} \left[ \left\{ \left\langle \eta_{mn}(\sigma_{mn}(x_t, t), E_{mnrs}(x_t), \epsilon_{rs}(x_t, t)) \right\rangle \right. \right. \\
 &+ \int_0^t d\tau \text{Cov}_0 \left[ \frac{\partial \eta_{mn}(\sigma_{mn}(x_t, t), E_{mnrs}(x_t), \epsilon_{rs}(x_t, t))}{\partial \sigma_{ab}}; \right. \\
 &\quad \left. \left. \eta_{ab}(\sigma_{ab}(x_{t-\tau}, t-\tau), E_{abcd}(x_{t-\tau}), \epsilon_{cd}(x_{t-\tau}, t-\tau)) \right] \right\} P(\sigma_{ij}(x_t, t), t) \Big] \\
 &+ \frac{\partial^2}{\partial \sigma_{mn} \partial \sigma_{ab}} \left[ \left\{ \int_0^t d\tau \text{Cov}_0 \left[ \eta_{mn}(\sigma_{mn}(x_t, t), E_{mnrs}(x_t), \epsilon_{rs}(x_t, t)); \right. \right. \right. \\
 &\quad \left. \left. \eta_{ab}(\sigma_{ab}(x_{t-\tau}, t-\tau), E_{abcd}(x_{t-\tau}), \epsilon_{cd}(x_{t-\tau}, t-\tau)) \right] \right\} P(\sigma_{ij}(x_t, t), t) \Big]
 \end{aligned}$$

# Eulerian–Lagrangian FPK Equation and (SEP)FEM

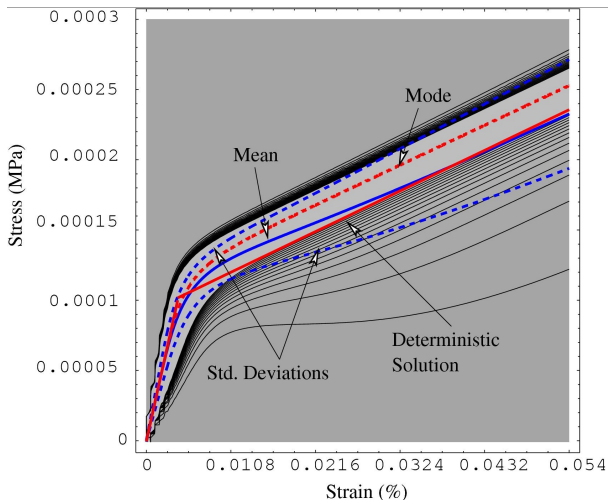
- Advection-diffusion equation

$$\frac{\partial P(\sigma_{ij}, t)}{\partial t} = -\frac{\partial}{\partial \sigma_{ab}} \left[ N_{ab}^{(1)} P(\sigma_{ij}, t) - \frac{\partial}{\partial \sigma_{cd}} \left\{ N_{abcd}^{(2)} P(\sigma_{ij}, t) \right\} \right]$$

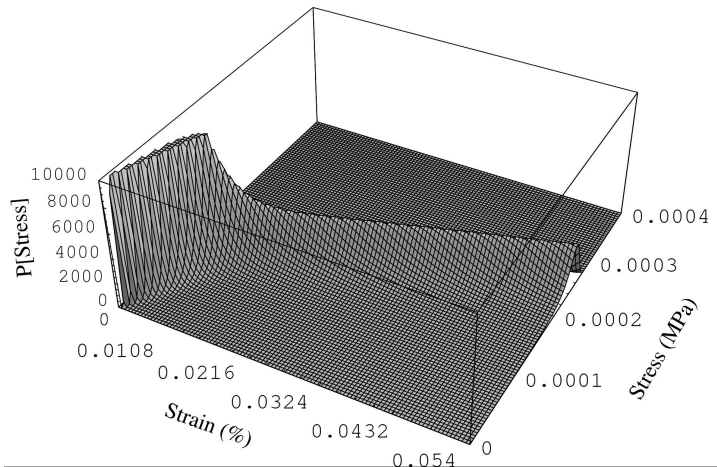
- **Complete** probabilistic description of response
- **Second-order exact** to covariance of time (exact mean and variance)
- Any uncertain FEM problem (**M** $\ddot{\mathbf{u}}$  + **C** $\dot{\mathbf{u}}$  + **K** $\mathbf{u}$  = **F**) with
  - uncertain material parameters (stiffness matrix **K**),
  - uncertain loading (load vector **F**)

can be analyzed using PEP and SEPFEM to obtain PDFs of DOFs, stress, strain...

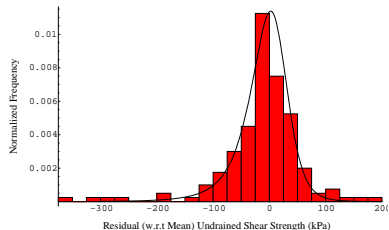
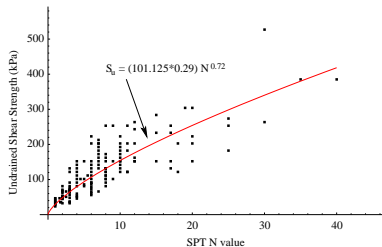
# Probabilistic Elastic-Plastic Response



# Probabilistic Elastic-Plastic Response

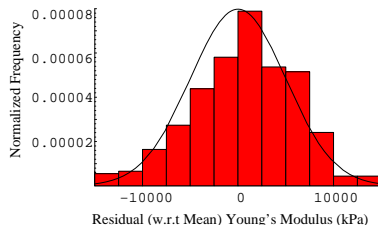
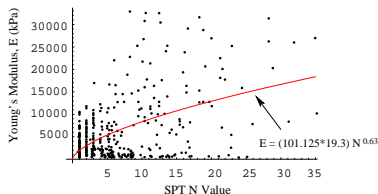


# SPT Based Determination of Shear Strength



Transformation of SPT  $N$ -value  $\rightarrow$  undrained shear strength,  $s_u$   
 (cf. Phoon and Kulhawy (1999B))  
 Histogram of the residual (w.r.t the deterministic transformation equation) undrained strength, along with fitted probability density function (Pearson IV)

# SPT Based Determination of Young's Modulus



Transformation of SPT  $N$ -value  $\rightarrow$  1-D Young's modulus,  $E$  (cf. Phoon and Kulhawy (1999B))

Histogram of the residual (w.r.t the deterministic transformation equation) Young's modulus, along with fitted probability density function

# Stochastic Finite Element Formulation

- Governing equations:

$$A\sigma = \phi(t); \quad Bu = \epsilon; \quad \sigma = E\epsilon$$

- **Spatial** and **stochastic** discretization
  - Deterministic spatial differential operators ( $A$  &  $B$ ) → Regular shape function method with Galerkin scheme
  - Input random field material properties ( $E$ ) → Karhunen–Loève (KL) expansion, optimal expansion, error minimizing property
  - Unknown solution random field ( $u$ ) → Polynomial Chaos (PC) expansion



# Spectral Stochastic Elastic–Plastic FEM

- ▶ Minimizing norm of error of finite representation using Galerkin technique (Ghanem and Spanos 2003):

$$\sum_{n=1}^N K_{mn}^{ep} d_{ni} + \sum_{n=1}^N \sum_{j=0}^P d_{nj} \sum_{k=1}^M C_{ijk} K_{mnk}'^{ep} = \langle F_m \psi_i[\{\xi_r\}] \rangle$$

$$K_{mn}^{ep} = \int_D B_n \mathbf{E}^{ep} B_m dV$$

$$C_{ijk} = \langle \xi_k(\theta) \psi_i[\{\xi_r\}] \psi_j[\{\xi_r\}] \rangle$$

$$K_{mnk}'^{ep} = \int_D B_n \sqrt{\lambda_k} h_k B_m dV$$

$$F_m = \int_D \phi N_m dV$$

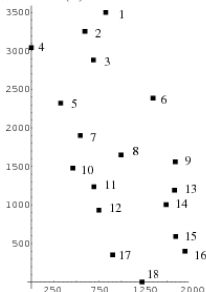
# Inside SSEPFEM

- ▶ Explicit stochastic elastic–plastic finite element computations
- ▶ FPK probabilistic constitutive integration at Gauss integration points
- ▶ Increase in (stochastic) dimensions (KL and PC) of the problem
- ▶ Excellent for parallelization, both at the element and global levels
- ▶ Development of the probabilistic elastic–plastic stiffness tensor

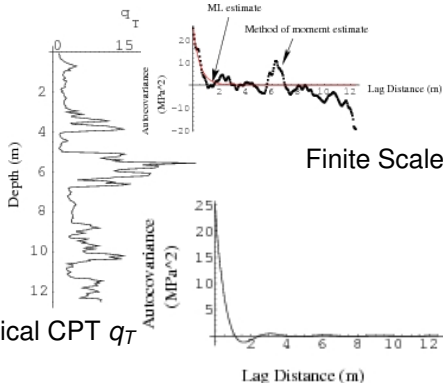
# Seismic Wave Propagation through Stochastic Soil

## ► Maximum likelihood estimates

S-N Coordinate (m)

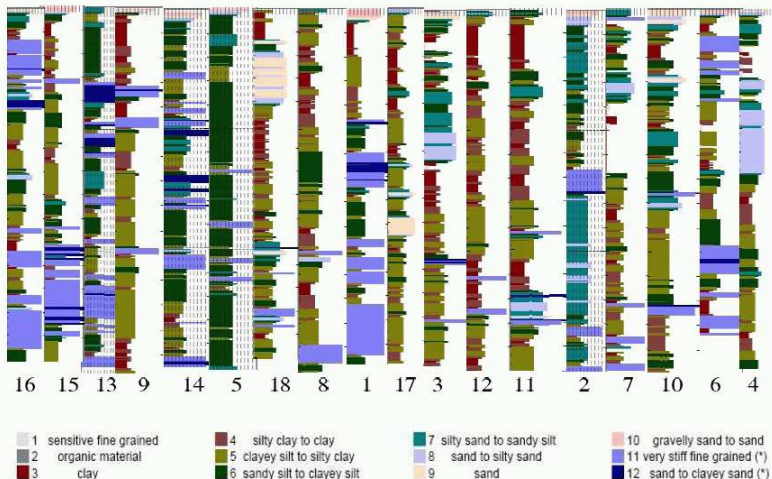


W-E Coordinate (m)



Fractal

# "Uniform" CPT Site Data



## Random Field Parameters from Site Data

- ▶ Soil as 12.5 m deep 1-D soil column (von Mises Material)
  - ▶ Properties (including testing uncertainty) obtained through random field modeling of CPT  $q_T$ 

$$\langle q_T \rangle = 4.99 \text{ MPa}; \quad \text{Var}[q_T] = 25.67 \text{ MPa}^2;$$

$$\text{Cor. Length } [q_T] = 0.61 \text{ m}; \quad \text{Testing Error} = 2.78 \text{ MPa}^2$$
  - ▶  $q_T$  was transformed to obtain  $G$ :  $G/(1 - \nu) = 2.9q_T$ 
    - ▶ Assumed transformation uncertainty = 5%
 
$$\langle G \rangle = 11.57 \text{ MPa}; \quad \text{Var}[G] = 142.32 \text{ MPa}^2$$

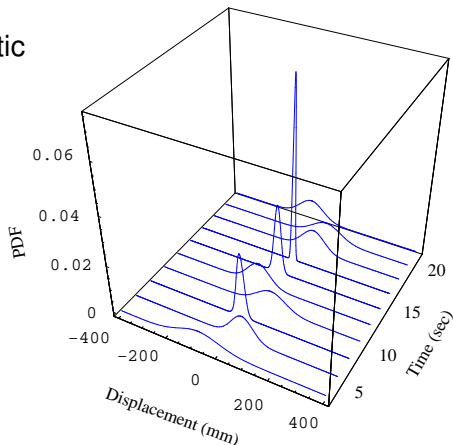
$$\text{Cor. Length } [G] = 0.61 \text{ m}$$
- ▶ Input motions: modified 1938 Imperial Valley

## Decision About Site (Material) Characterization

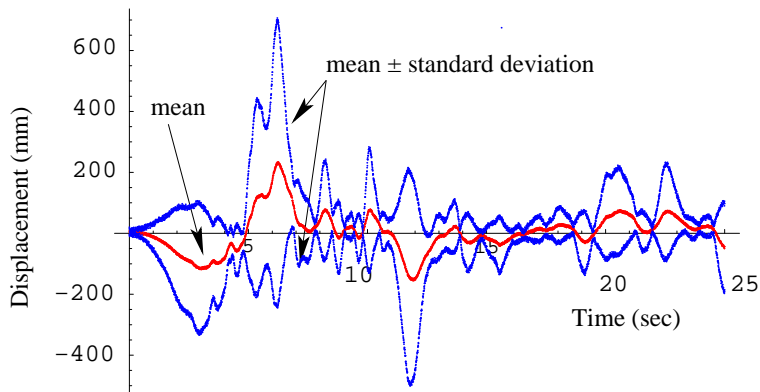
- ▶ Do nothing about site characterization (rely on experience): conservative **guess** of soil data,  $COV = 225\%$ , correlation length = 12m.
- ▶ Do better than standard site characterization:  $COV = 103\%$ , correlation length = 0.61m)
- ▶ Improve site (material) characterization if probabilities of exceedance are unacceptable!

## Full PDFs of all DOFs (and $\sigma_{ij}$ , $\epsilon_{ij}$ , etc.)

- ▶ Stochastic Elastic-Plastic Finite Element Method (SEPFEM)
- ▶ Dynamic case
- ▶ Full PDF at each time step  $\Delta t$

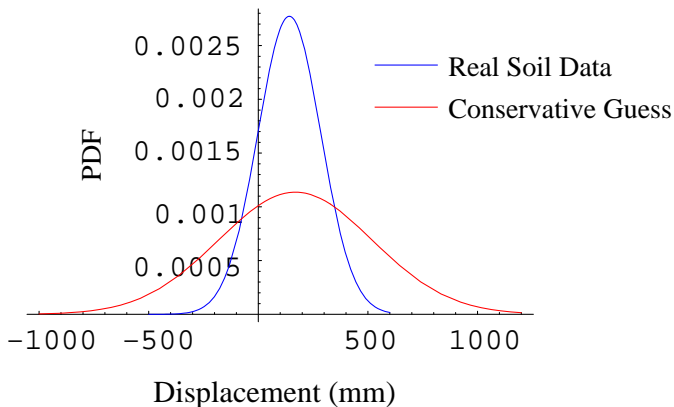


# Evolution of Mean $\pm$ SD for Guess Case

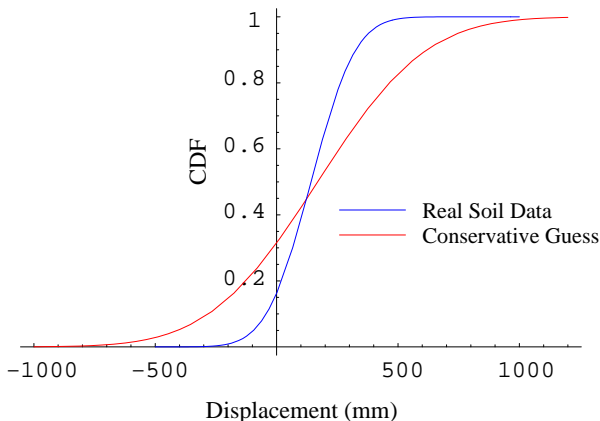




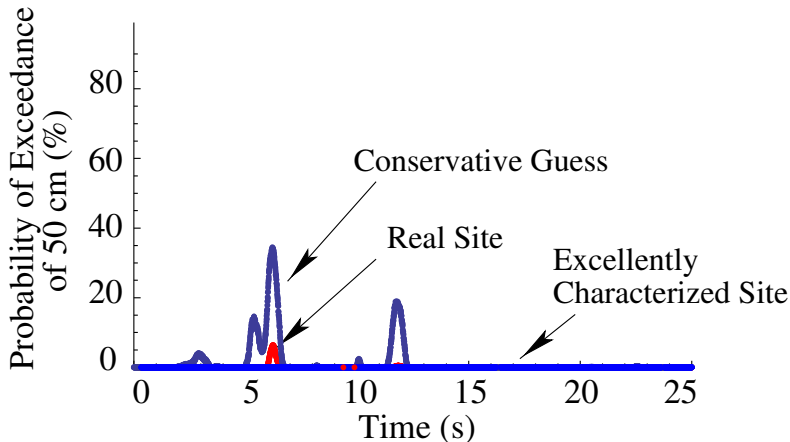
## PDF at each $\Delta t$ (say at 6 s)



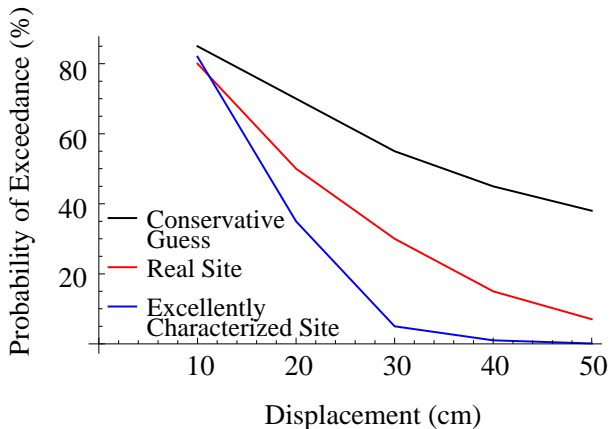
# PDF $\rightarrow$ CDF (Fragility) at 6 s



## Probability of Unacceptable Deformation (50cm)



# Risk Informed Decision Process



# Outline

Introduction

Deterministic Modeling

Probabilistic Modeling

Summary

## Summary

- ▶ **Interplay of Uncertain Earthquake, Uncertain Soil/Rock, and Uncertain Structure** in time domain **probably** plays a decisive role in seismic performance of structures (NPPs, etc.)
- ▶ Improve **risk informed decision making** through high fidelity **Deterministic** and **Stochastic Elastic-Plastic Finite Element** modeling and simulation
- ▶ **Education and training** of users will prove essential
- ▶ **Acknowledgement:** funding and collaboration with the US-NRC, and funding from NSF, DOE, CNSC