The arc spectrum of nitrogen is not developed in the presence of an excess of argon under the same conditions that it appears in the presence of helium.

Special precautions have to be taken to ensure the purity of the gases and the proportion of nitrogen present has to be carefully regulated.

Measurements have been made of the wave-lengths of the lines of the nitrogen arc spectrum (NI).

When nitrogen is excited by electron impacts there appears to be a direct transition as the energy of the impacts is increased from the negative band spectrum to the spark spectrum, which would imply that the rupture of nitrogen molecules is generally into ions rather than neutral atoms.

The action of the rare gases in modifying spectra is discussed, and the principal phenomena appear to be open to a simple explanation.

We wish to express our thanks to the Department of Scientific and Industrial Research for a grant which has been made to one of us (J.G.P.) during the course of this investigation.

The Heat Developed during Plastic Extension of Metals.

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1. *Introduction.*

When a soft metal, such as annealed copper or aluminium, is deformed while cold, either by stretching, hammering, rolling, or other method of "cold working," it hardens; that is, the forces necessary to deform it increase as the amount of plastic deformation increases. The physical state of "cold worked" metal is undoubtedly different from that of the metal in its original soft or annealed state, and various explanations have been put forward to account for the difference. Some of these explanations involve the hypothesis that the process of hardening is associated with the formation of amorphous material at the crystal planes where slipping occurs during the deformation. The formation of amorphous material from a crystalline mass would involve a phase change, which would in general be accompanied by a change in the internal energy of the material. It has been suggested that the phase change could

be detected by measuring the heat evolved during a deformation, and comparing with the heat equivalent of the work done on the metal by the forces producing the deformation. Any difference between the two would imply a change in the internal energy of the metal.

It is curious that very few measurements of this type appear to have been made. The only reference which we have been able to find occurs in Dr. Rosenhain's article on "Metals," in the 'Dictionary of Physics,'* where he quotes some previously unpublished observations made by Dr. Sinnat.

According to these observations, only one-tenth of the work done reappears in the form of heat, the remaining 90 per cent. being presumably used up in changing the phase of the material. This result, if true, would be of very great interest, but so far no further details have been published. Dr. Rosenhain has informed us that he is carrying out further experiments on the subject.

On the other hand, as will be seen later, the experimental difficulties in making measurements of this kind are considerable, and if several independent workers performed such experiments along parallel lines their time would not be wasted, even if they got identical results.

2. *Methods of Measurement.*

The simplest way to deform a metal is to take a straight bar or "test-piece" and stretch it by a longitudinal pull. The work done in any portion of the bar during an operation of this kind can be determined by measuring the pull P and the length l of the portion concerned at successive stages of the test. The work done, namely, $\int P dl$, is represented by the area of the stress-strain curve. The stress-strain relation is usually determined in engineering laboratories by means of a testing machine in which the stress is measured by balancing the tension in the specimen against a weight on a lever, and the strain by direct measurement on the specimen.

This method was not suitable for the experiments described here, because it was necessary to do the whole of the stretching in a few seconds of time, in order that the rise in temperature of the metal might be measured before any appreciable cooling had taken place. For this reason a self-recording testing machine was made which automatically produced a stress-strain curve as the specimen was being stretched. This machine is described later in some detail, because the value of our results depends entirely on the accuracy of the measurements, and it is necessary to show that the accuracy of our stress-strain curves is at least equal to that of our temperature measurements.

* Vol. 5, p. 398.

To measure the heat evolved during the stretching, the use of a calorimeter naturally suggests itself, but the experimental difficulties in the way of getting accurate results in this manner appear insuperable; and even if they could be overcome the result obtained would give only the total heat evolved throughout the whole specimen, while what is wanted is the heat evolved in the middle of the bar, where the stretching is uniform.

The method adopted was to measure the rise in temperature which occurs during a rapid plastic extension of the bar. For this purpose a thermocouple was used and the temperature was recorded photographically on a moving plate.

Preliminary experiments showed that, when fixed in the testing machine and exposed to the air, the specimens first used cooled through about 4 per cent. of their excess of temperature over that of the room in 1 second. It was clear, therefore, either that large cooling corrections would have to be introduced or that the temperature measurements would have to be made rapidly. The second alternative is preferable, if possible, and the first part of the work was devoted to a study of the heat losses immediately after the extension of the specimen, and to a search for the method of measuring temperature which suffered least from lag.

The cooling of the specimen was due almost entirely to conduction of heat through the ends, only a very small fraction of the heat being radiated or carried away by convection. This might have been guessed beforehand, but it was proved by observing the rate of cooling of the specimen when hung up horizontally by a silk thread, and comparing it with the rate of cooling of the specimen when fixed on the testing machine. In the former case the excess of temperature of the specimen over that of the atmosphere was reduced in the ratio 1 to e* in 18 minutes, while in the latter the time taken was 28 seconds. It appears, therefore, that only $\frac{28}{18 \times 60}$, *i.e.*, $\frac{1}{40}$ of the heat loss, was due to convection and radiation, and that the remaining $\frac{39}{40}$ must have been due to conduction of heat from the central uniform part of the specimen to the ends and grips of the testing machine.

The coldness of the ends takes some time to reach the middle, and it was during the interval between the time of generation of the heat and the time of arrival of the cold waves from each end of the bar that the temperature measurements were made.

* e the base of Napierian logarithms.

To find how long this interval might be expected to be it is sufficient to idealise the condition slightly. Consider a uniform bar of length l, coefficient of conductivity k, specific heat σ, density ρ, and suppose that at time $t = 0$ the temperature of the whole bar is suddenly raised by an amount T_0, the ends being maintained subsequently at their original temperature, which may be taken as $T = 0$. The subsequent temperature at distance x from either end is represented by the series

$$T = \frac{4}{\pi} T_0 \left\{ e^{-at} \sin \frac{\pi x}{l} + \tfrac{1}{3} e^{-3^2 at} \sin \frac{3\pi x}{l} + \tfrac{1}{5} e^{-5^2 at} \sin \frac{5\pi x}{l} + \ldots \right\} \quad (1)$$

where

$$a = \frac{\pi^2 k}{l^2 \rho \sigma}.$$

When $t = 0$, $T = T_0$ for all values of x.

When t is large the series reduces to the first term

$$T = \frac{4T_0}{\pi} e^{-at} \sin \frac{\pi x}{l}. \quad (2)$$

When t is small this series is inconvenient for numerical calculation and the expression

$$T = T_0 \left\{ 2 - \frac{2}{\sqrt{\pi}} \int_0^{\zeta_1} e^{-\mu^2} d\mu - \frac{2}{\sqrt{\pi}} \int_0^{\zeta_2} e^{-\mu^2} d\mu \right\} \quad (3)$$

may be used, where

$$\zeta_1 = \frac{x}{2} \sqrt{\frac{\rho\sigma}{\kappa t}} \quad \text{and} \quad \zeta_2 = \frac{l - x}{2} \sqrt{\frac{\rho\sigma}{\kappa t}}.$$

The expression (3) is approximately the same as (1) for small values of t. The maximum errors occur first at the ends, $x = 0$ and $x = l$. When $t = 0{\cdot}077 \rho\sigma l^2/k$ the error is 1 per cent. at the ends but is still quite inappreciable in the middle, where (3) gives $T = 0{\cdot}594\ T_0$ while (2) gives $T = 0{\cdot}595\ T_0$. It appears that the error in using (3) from $t = 0$ to $t = 0{\cdot}077 \rho\sigma l^2/k$ and (2) from this value to $t = \infty$ is never more than a fraction of 1 per cent.

The theoretical cooling curve for points in the middle of a bar, cooled by conduction of heat to both ends, is shown in fig. 1. It will be seen that from

$$t = 0 \text{ to } t = 0{\cdot}014 \rho\sigma l^2/k \quad (4)$$

the temperature is practically constant. At $t = 0{\cdot}014 \rho\sigma l^2/k$ it has fallen only 1/166th of its initial value. After this the temperature begins to fall rapidly, and at $t = 0{\cdot}04 \rho\sigma l^2/k$ the curve becomes practically indistinguishable from the ordinary exponential curve (shown as a dotted line in fig. 1), which would result from Newton's law of cooling.

In the second part of the curve, when the temperature of the middle of the bar is falling exponentially, the time t_e taken to fall to $1/e$ of its value is $\rho\sigma l^2/\pi k^2$

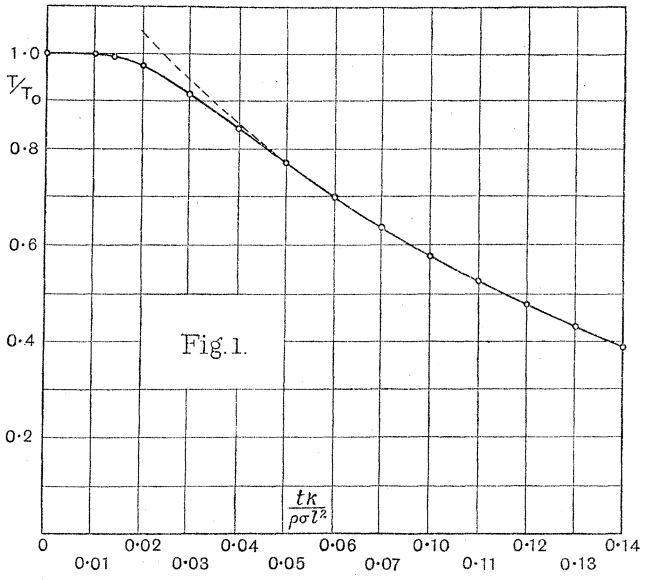


FIG. 1.—Theoretical cooling curve for the middle point of a bar initially at a uniform temperature and subsequently cooled by maintaining the ends at a steady temperature.

or $1/a$. It appears, therefore, that if this time t_e is observed experimentally the temperature will not fall through more than 1/166th of its initial value during the time $0{\cdot}014\,\pi^2 t_e$ after the instant when the heat is generated. If the temperature measurements can be made within this interval of time the error due to neglecting cooling corrections will not amount to 0·6 per cent.

In the first experiment with aluminium bars having a parallel portion in the middle 11 cm. long before the stretching began, the time t_e was observed to be 28 seconds, so that the time available was $0{\cdot}014 \times 28 = 4$ seconds, whereas using the expression (4) and inserting the known physical constants for aluminium, namely, $\rho = 2{\cdot}7$, $\sigma = 0{\cdot}212$, $k = 0{\cdot}5$, and putting $l = 11$, the time comes out to be 1·9 seconds. The difference is due no doubt to the fact that the ends of the parallel middle portion are not in practice maintained at the temperature of the atmosphere, and it seems clear that the time during which the temperature of the middle of the bar might be expected to remain practically constant is considerably underestimated by the expression (4). This is confirmed by measurements of temperature records taken in the course of the experiments.

In some of the experiments here described it was possible to use specimens containing as much as 30 cm. of parallel part in the middle, and with these there was ample time for measuring the temperature. It was not possible, however, to obtain single crystal bars of aluminium with a parallel middle portion greater than 16 cm. For this reason it was necessary to have a temperature recording system which could be relied on to give a correct reading within two or three seconds of the time of application of the strain.

For this purpose an iron-constantan thermojunction was used. This was connected directly to a galvanometer, the moving part of which reflected light from a vertical slit on to a fine horizontal slit placed close in front of a photographic plate which was made to move vertically at a constant speed by means of an electric motor fitted with a speed governor.

The galvanometer used was of the moving magnet type, but the suspended system was specially designed and made for us by Dr. P. Kapitza, with a view to reducing the period to the minimum value consistent with the required sensitivity. It consisted of an astatic pair of cobalt-steel magnets, each $2\frac{1}{2}$ mm. long $\times$ 0·02 mm. thick. These were fixed to a fine straight glass thread which was hung vertically by a quartz fibre. The reflecting mirror was made of thin coverslip glass, 2 mm. square. A magnet was used to control the sensitivity of the system, and in the present experiments the period of oscillation was 2·3 seconds, and the damping per half-period was 0·44. The records obtained with this apparatus could be reduced to eliminate the effects of periodicity and damping in the galvanometer; in most cases, however, this was found to be unnecessary, the temperature of the middle of the bar remaining constant for a time which was long enough to allow the oscillation to die away.

3. *Method of Using a Thermojunction.*

The usual method of using a thermojunction for measuring the temperature of solid bodies is either to insert it in a small hole in the body, or, if possible, to solder it to the surface. Neither of these methods was available in the present instance. To bore a small transverse hole in the specimen would alter the type of distortion which the metal would experience in the immediate neighbourhood of the hole. This would give rise to an error whose magnitude it would be impossible to estimate. Solder could not be used because many of the experiments were made with aluminium. It was, therefore, necessary to devise some method of attaching or pressing the thermojunction to the surface of the metal.

When a thermojunction touches a specimen on one side only, unless it can be soldered on, some other material, preferably a non-conductor, must be in contact with its other side in order to press it against the specimen. The temperature of the thermojunction will, therefore, be intermediate between that of the specimen and that of the non-conductor, and if the resistance to the passage of heat at the contact is small, presumably the temperature recorded will be close to that of the specimen, but there will be an error whose magnitude must be determined. In the experiments here described this source of error was reduced to a minimum by pressing the thermojunction against the inside of a $\frac{3}{16}$-inch hole bored symmetrically from end to end of the specimen. The total amount of non-conducting material used for pressing the thermojunction against the inside of the hole was very small, and in any case it was certain that everything inside the hole would warm up to the temperature of the metal, whereas it was by no means certain that a thermojunction pressed on the outside of the specimen would do so.

The chief difficulty in the way of getting an accurate record of the temperature of the metal immediately after stretching was due to the lag in temperature between the thermojunction and the specimen. To test the efficiency of different ways of fitting the thermojunction a very thin-walled metal tube with a bore of $\frac{3}{16}$-inch was made. The thermojunction was fitted into this, and a sudden change in temperature was produced by removing the tube from a jar of water and suddenly plunging it in a jar of water whose temperature was about 2° higher, and stirring vigorously. The temperature record produced in this way had to be analysed to allow for the lag in the galvanometer system. The method adopted for this purpose will next be described.

If there were no lag in the heating of the thermojunction, the record would show a damped harmonic oscillation, the characteristics of which would be determined solely by the galvanometer. In the present case the galvanometer had a period of 2·3 seconds and a damping of 0·44 per $\frac{1}{2}$ period. It is clear, therefore, that if the temperature of the thermojunction were suddenly raised from $T = 0$ to $T = T_0$, and then kept at temperature T_0, the record would overshoot the true temperature by an amount $0{\cdot}44\ T_0$, so that the maximum temperature shown on the record would be $1{\cdot}44\ T_0$. The next minimum would be $\{1 - (0{\cdot}44)^2\}\ T_0 = 0{\cdot}81\ T_0$, and so on.

If the lag in the temperature of the thermojunction were large, the recorded temperature would be close to the true temperature and there would be no maximum on the record till the true steady temperature had been attained.

For intermediate cases, where the lag in the temperature and the period of the galvanometer were of the same order of magnitude, there might be a first maximum on the record, which would necessarily be less than $1{\cdot}44\ T_0$, but might be greater than T_0 by an amount which would depend on the temperature lag in the thermojunction.

If T_m represents the first maximum temperature shown on the record, it is possible by measuring T_m/T_0 to measure the amount of the temperature lag in the thermojunction.

If y represents the reading of the galvanometer record when multiplied by the calibration factor, so as to reduce it to a measure of temperature, the equation which represents the galvanometer record, in the case when there is no lag in the temperature of the thermojunction, is

$$\frac{d^2y}{dt^2} + \kappa \frac{dy}{dt} + \mu y = 0, \tag{5}$$

and the solution of this is

$$y = T_0 (1 - e^{-\frac{1}{2}\kappa t} \cos nt) \tag{6}$$

where $n^2 = \mu - \frac{1}{4}\kappa^2$.

$2\pi/n$ is the period of the galvanometer, while $e^{-\frac{1}{2}\kappa}$ is the damping factor.

When there is a lag, (5) becomes

$$\frac{d^2y}{dt^2} + \kappa \frac{dy}{dt} + \mu (y - T) = 0, \tag{7}$$

where T is the temperature of the thermojunction.

If $1/\lambda$ represents the lag in the temperature of the thermojunction when the temperature outside the case containing it is suddenly changed, the equation for T may be assumed to be

$$T = T_0 (1 - e^{-\lambda t}). \tag{8}$$

This equation in fact defines the "lag."

Substituting for T in (7), solving, and inserting the condition, that when $t = 0$, $y = 0$ and $dy/dt = 0$, an equation is formed for y. Using the values for n and $\frac{1}{2}k$ obtained from the experiments on the period and damping of the galvanometer, the solution of (7) which fits the conditions is found to be

$$\frac{y}{T_0} = 1 + \frac{-8{\cdot}17e^{-\lambda t} + (1{\cdot}42\ \lambda - \lambda^2)\, e^{-0{\cdot}71t} \cos 2{\cdot}77t - (0{\cdot}256\ \lambda^2 + 2{\cdot}59\lambda)\, e^{-0{\cdot}71t} \sin 2{\cdot}77t}{\lambda^2 - 1{\cdot}42\ \lambda + 8{\cdot}17}.$$

Taking a series of different values of λ, it is found that if λ is less than $0{\cdot}5$ there is no maximum value for y, but that when λ is greater than $0{\cdot}71$, y has a maximum T_m, the values of which are given in Table I.

Table I.

λ	∞	2	1	0·71
T_m/T_0	1·44	1·157	0·932	0·759

The value T_m/T_0 was measured for a number of different types of thermojunction and the results given in Table I were used to estimate the lag. It was found that the lag varied very greatly, according to the method used for pressing the thermojunction against the heated surface. In the first place the lag was always far too great unless there was actual metallic contact between the junction and the heated metal. For this reason it would be impossible to obtain accurate results if a multiple junction were used, because of the necessity in that case for having insulating material between the metal and the junction.

Various methods were tried for pressing the thermojunction directly against the inside of the hole through the specimen, but they were all failures. In every case a lag of several seconds occurred, though the lag was not so great as when there was no metallic contact.

The method which proved most successful consisted in soldering the junction to a plate of silver foil 0·1 mm. thick, 7 mm. long and 9 mm. wide. This plate was bent round a small cylindrical piece of cork which could be forced into the hole. In this way the silver was pressed hard up against the inside of the specimen.

This method had the advantage that when the hole contracted as the specimen lengthened the only effect was to make the cork press the silver plate still harder against the metal, and even when the hole became elliptical, as it did when single crystal specimens were used, the silver foil was still pressed against the metal all over its surface.

Using this method the lag was reduced to 0·4 second (*i.e.*, $1/\lambda = 0{\cdot}4$). It will be noticed that this includes any lag which may have existed between the temperature of the copper tube and that of the water into which it was plunged, so that the lag due to the passage of heat to the thermojunction must have been less than this. In order to ensure that the temperature of the thermojunction was within 1 per cent. of that of the specimen with which it was in contact it was therefore necessary to read the record at a time greater than $-0{\cdot}4 \log_e 0{\cdot}01$, or 1·8 seconds, after the stretching was finished.

5. *Method of Carrying Out an Experiment.*

The thermojunction was first fitted inside the middle of the specimen. The iron and constantan wires passed out through one end to the cold junction

which was contained in an oil-filled tube dipping into a thermos flask full of water. The other end of the axial hole through the specimen was made watertight with wax, and the thermojunction was calibrated by moving the specimen from one large jar of water at the temperature of the room to another containing water about 2° C. higher. The temperature of the water was found by means of Beckmann thermometers graduated in hundredths of a degree Centigrade. The water in both jars was stirred vigorously by means of small fans, run by an electric motor, but even then the specimen took some 18 seconds to attain the temperature of the surrounding water.

An actual stretching experiment lasted a few seconds only, so the speed of the falling plate on which the records were taken was reduced when the calibrations were being done (*see* fig. 2). In order to provide a base line from which measurements of the plate could be made, an image of the same vertical slit which was used to illuminate the galvanometer mirror was cast by a fixed miror on the horizontal slit of the recording apparatus, and the light was cut off by a shutter from both images, at regular intervals of 6 seconds, so that small simultaneous gaps appear in the temperature record and the base line.

In most of the experiments two records were taken on each plate. In fig. 2 the outer and thicker line, marked A, is the calibration record.

The next step was to put the specimen in the machine and stretch it by winding the handle as rapidly as possible, till the temperature of the specimen had risen through about 2° C. The machine was then left for about 10 minutes, with the specimen still under tension, in order that it might have time to cool. It was then again stretched, till its temperature rose again by about 2° C. With the steel, copper, and aluminium specimens three such extensions were made. With the single crystals of aluminium there were five.

The load was then released suddenly and a record taken of the reversible adiabatic rise in temperature which accompanies the sudden elastic contraction. This usually amounted to about 0·3° C., or about 3 per cent. of the total rise which would have occurred if the whole extension had been done at once.

The main object of the experiments was to determine how much of the energy put into the metal by external forces was used in increasing the internal energy of the material. For this reason it was thought that the ideal way to carry out an experiment would be to stretch the material and then immediately to release the stress, so that the material would be unstressed as a whole both before and after the experiment. This was inconvenient

and it was decided to divide up the adiabatic heating which occurred when the stress was finally released, and to distribute it among the successive stages

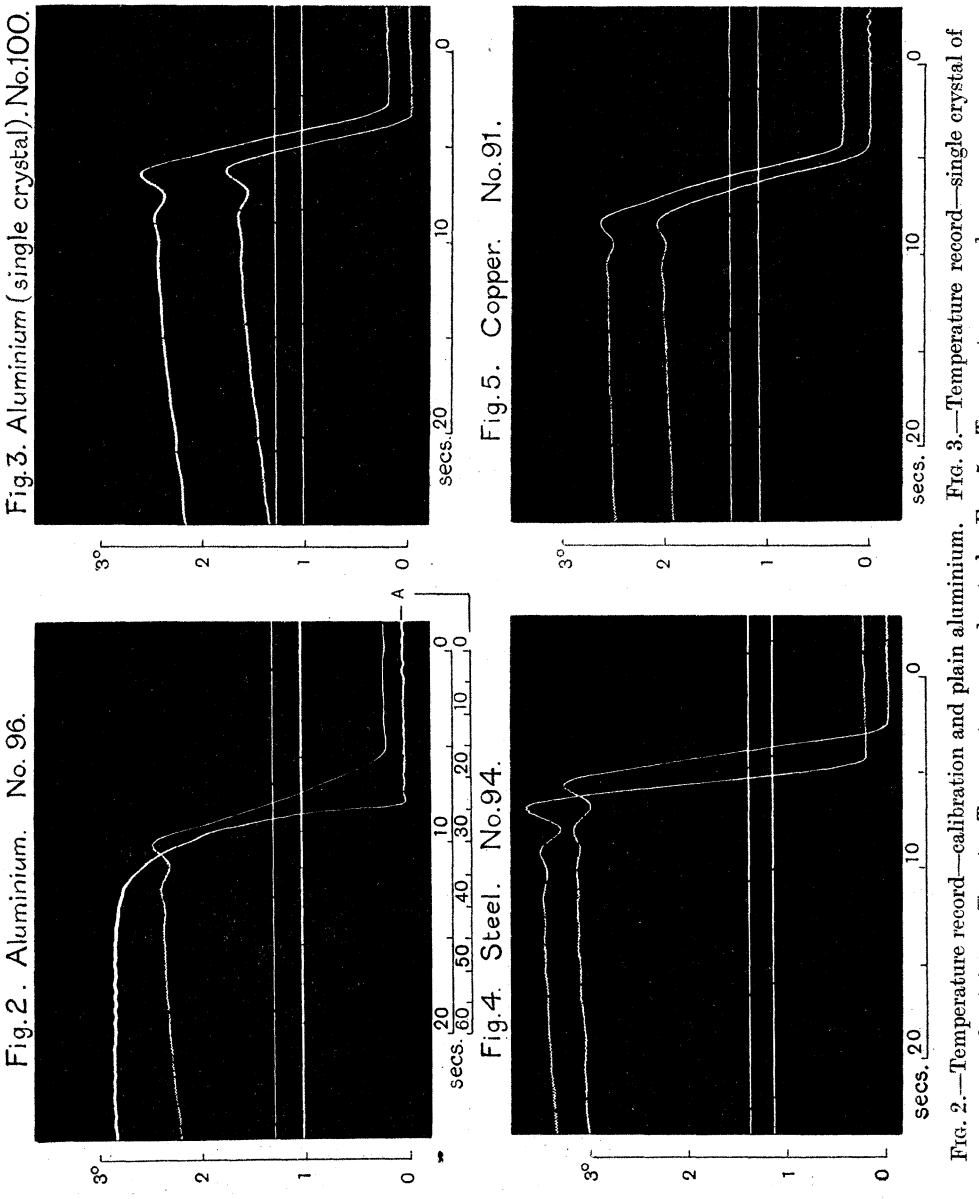


Fig. 2.—Temperature record—calibration and plain aluminium. Fig. 3.—Temperature record—single crystal of aluminium. Fig. 4.—Temperature record—steel. Fig. 5.—Temperature record—copper.

of the stretching in proportion to the change in stress which occurred in each stage. Thus if the stresses at the ends of the first three stages were p_1, p_2 and p_3, and if the change in temperature due to adiabatic heating on releasing

the stress p_3 was T_a, an amount $p_1 T_a/p_3$ was added to the observed rise of temperature during the first stretching, $(p_2 - p_1) T_a/p_3$ for the second stretching, and $(p_3 - p_2) T_a/p_3$ for the third. Except for the first extension in each experiment these corrections are small, but they are founded on experimental evidence that the elastic strain, and hence the adiabatic heating, is proportional to the change of stress and does not depend on the hardness of the material.

6. *Temperature Records.*

Some of the temperature records are shown in figs. 2–5. In each the stretching of the specimen was complete before the spot of light had travelled its maximum distance, but since the stretching was not instantaneous (it occupied about 2 seconds) the amount by which the galvanometer mirror overshot the mark was small. It is a comparatively simple matter to analyse the record so as to make allowance for the galvanometer lag and inertia, but an inspection of the records shows that the oscillation has practically died away before the specimen begins to cool, so that in most cases no correction is required. In each of the records the temperature curve has the flat top which was predicted on p. 426 (fig. 1), but it is most obvious in fig. 3 which is the record of the stretching of a single-crystal specimen of aluminium. It will be seen that the temperature did not begin to decrease till at least 5 seconds after the stretching was complete, and by that time the galvanometer oscillation had practically disappeared. In figs. 4 and 5, which are the records for annealed steel and copper, respectively, the specimens were 30 cm. long, and the rate of cooling was much slower than for the aluminium specimen, which had only 16 cm. of parallel part in the middle. The flat top in these records extends over about 8 seconds.

7. *Discussion of Results.*

In order to compare the observed rise in temperature with the heat evolved the heat equivalent of the work done in each experiment has been calculated. If l_0 is the original length of the specimen between its marks, and ε its extension at any stage, its length is then $l_0 (1 + \varepsilon)$. If P is the pull in the testing machine and a_0 the original cross-section of the specimen, the work done on unit volume of the material during a stretching between extensions ε_1 and ε_2 is $\frac{1}{l_0 a_0} \int_{\varepsilon_1}^{\varepsilon_2} P l_0 d\varepsilon$. This integral is evaluated from the stress-strain records (see p. 448).

The cross-section of the specimen was measured in square inches and the

work done in foot-pounds. Using for Joule's equivalent J the value $4{\cdot}18 \times 10^7$ ergs per calorie, the factor for converting the above integral to the equivalent rise in temperature is $\frac{1{\cdot}65 \times 10^{-3}}{\rho\sigma}$.

The densities of the materials used were determined by weighing in water. The specific heats were not measured, but their values were taken from the latest edition of Landoldt and Börnstein's tables. The values adopted for ρ and σ are shown in Table II below.

Table II.

Material.	ρ.	σ.
Aluminium	2·70	0·212
Steel	7·80	0·106
Copper	8·93	0·092

The results of measurements on eleven specimens, namely three each of annealed copper, steel and plain aluminium, and two single crystal specimens of aluminium are given in Table III.

In this table the number of the specimen is given at the top so that the figures may be compared with the reproductions of some of the temperature records (figs. 2–5, p. 432) and stress-strain records (fig. 17, p. 448). In the first column is given the extension ε of the material (per cent. of its unstrained length) at the end of each experiment. The second gives the observed rise in temperature T_0. The third gives the correction T_a for adiabatic heating which must be added in order to give the rise in temperature which would be observed if the material were unstressed at the beginning and end of each experiment. The fourth column gives the corrected temperature $T_1 = T_0 + T_a$. The fifth column gives the temperature rise T_2 corresponding to the heat equivalent of the work done. The sixth column gives the final result, namely the ratio T_1/T_2, *i.e.*, the proportion of the work done which is converted into heat. The bottom line of each Table gives the total results for each specimen.

It will be seen that in every case the observed evolution of heat falls short of the heat equivalent of the work done. The difference which represents increase in the internal energy of the material amounts to $13\frac{1}{2}$ per cent. of the work done in the case of steel, 8 to $9\frac{1}{2}$ per cent. in the case of copper, 7 to 8 per cent. in the case of aluminium, and $4\frac{1}{2}$ to 5 per cent. in the case of the single-crystal specimens of aluminium.

Table III.

	No. 81.						No. 94.						No. 95.					
	ϵ.	T_0.	T_a.	T_1.	T_2.	T_1/T_2.	ϵ.	T_0.	T_a.	T_1.	T_2.	T_1/T_2.	ϵ.	T_0.	T_a.	T_1.	T_2.	T_1/T_2.
Steel	5·92	3·41	0·28	3·69	4·20	0·88	3·99	2·14	0·25	2·39	2·79	0·86	6·11–10·50*	3·65	0·06	3·71	4·29	0·865
	9·70	2·77	0·06	2·83	3·36	0·84	8·13	3·12	0·06	3·18	3·72	0·855	13·37	2·54	0·03	2·57	3·00	0·855
	13·10	2·82	0·03	2·85	3·28	0·87	11·83	3·23	0·04	3·27	3·72	0·88	16·22	2·66	0·02	2·68	3·03	0·88
Total	13·10	9·00	0·37	9·37	10·84	0·865	11·83	8·49	0·35	8·84	10·23	0·865	16·22*	8·85	0·11	8·96	10·32	0·865

	No. 91.						No. 92.						No. 93.					
Copper	7·66	1·91	0·22	2·13	2·31	0·92	7·78	1·79	0·22	2·01	2·25	0·895	8·76	2·18	0·23	2·41	2·72	0·885
	12·67	2·06	0·05	2·11	2·29	0·92	13·63	2·38	0·07	2·45	2·69	0·91	15·35	2·88	0·06	2·94	3·23	0·91
	17·45	2·32	0·04	2·36	2·57	0·92	19·90	3·09	0·04	3·13	3·44	0·91	20·20	2·50	0·03	2·53	2·76	0·915
Total	17·45	6·29	0·31	6·60	7·17	0·92	19·90	7·26	0·33	7·59	8·38	0·905	20·20	7·56	0·32	7·88	8·71	0·905

	No. 96.						No. 97.						No. 98.					
Aluminium (plain)	11·12	2·16	0·25	2·41	2·59	0·93	10·41	1·99	0·21	2·20	2·42	0·91	9·16	1·64	0·24	1·88	2·03	0·925
	16·88	1·67	0·02	1·69	1·82	0·93	16·70	1·80	0·01	1·81	1·96	0·925	15·91	1·94	0·03	1·97	2·09	0·945
	23·06	1·85	0·01	1·86	2·01	0·93	21·95	1·58	0·01	1·59	1·71	0·93	21·88	1·82	0·01	1·83	1·95	0·94
Total	23·06	5·68	0·28	5·96	6·42	0·93	21·95	5·37	0·23	5·60	6·09	0·92	21·88	5·40	0·28	5·68	6·07	0·935

	No. 99.						No. 100.					
Aluminium (single crystal)	14·58	1·58	0·13	1·71	1·80	0·95	16·19	2·12	0·16	2·28	2·37	0·96
	24·10	1·59	0·02	1·61	1·67	0·965	25·75	1·66	0·01	1·67	1·77	0·945
	35·25	2·07	0·02	2·09	2·20	0·95	37·70	2·24	0·01	2·25	2·38	0·945
	43·20	1·57	0·01	1·58	1·68	0·94	46·52	1·73	0·00	1·73	1·82	0·95
	52·72	1·96	0·01	1·97	2·09	0·94	55·72	1·82	0·00	1·82	1·93	0·945
Total	52·72	8·77	0·19	8·96	9·44	0·95	55·72	9·57	0·18	9·75	10·27	0·945

ϵ = per cent. extension at end of each experiment.
T_0 = observed temperature rise.
T_a = "adiabatic" correction.
$T_1 = T_0 + T_a$.
T_2 = temperature rise equivalent to observed work in extension ϵ.

* Owing to an accident, no temperature record was obtained for the first extension, 6·11 per cent., of this specimen.

One remarkable feature of the table is the constancy of the ratio of the increase in internal energy to the work done on the specimens during different stages of the test. It is the same during the first extension when the metal is very soft and hardening rapidly as it is in the last extension when it is quite hard and only hardening slowly. It seems therefore that the increase in internal energy does not bear any very direct relationship to the hardening.

8. *Description of Machine for Extending Specimen, and Recording Extensometer.*

The mechanism by which the specimens were stretched was designed specially for these experiments, particularly for use with single crystals of aluminium. These have, at the breaking load of about 1 ton on an initial cross-section of $\frac{1}{4}$ square inch, an elongation of the order of 70 per cent. on a length of 6 inches. Standard tensile testing machines will not deal satisfactorily with large and relatively weak specimens. The wedge grips deform the ends unduly, and require a large initial load. The force is generally applied by a weight, through a lever, and is difficult to control with such large extensions at almost constant loads. The vertical position of the specimen is inconvenient. Finally a rapid time rate of loading is almost out of the question.

In the machine described in detail below the specimen is horizontal. Its ends are screwed into steel end-pieces with spherical seatings, which fit into corresponding recesses in two moving carriages. Of these, one rests against a pair of steel springs, whose compression measures the applied load. These springs are very short and stiff, so that the energy stored is small and there is no tendency towards instability with plastic specimens. The second carriage is moved by a square thread screw, the nut being rotated by hand through a reduction gear. Loads of 2,500 lbs. can be conveniently applied in about 2 seconds. The recording mechanism consists of a drum on which a celluloid film is stretched, the trace being made on it by a fine steel point. This method (due originally to Mr. Collins of the Cambridge Instrument Company) has proved very satisfactory. The record is immediately available, requires no treatment, is permanent, and can be measured with great accuracy. The drum has both axial and rotational motion, the former being proportional to the compression of the springs referred to above, and the latter to the absolute movement of a point on the specimen. The scribing point has a rotational motion only, proportional to the absolute motion of another point on the specimen, situated similarly to, and some 10 cm. from, the first point.

Providing certain geometrical conditions are fulfilled the co-ordinates of the

resulting diagram are proportional to the compression of the spring and elongation of the specimen. Actually the abscissæ are 1/3·59 × the elongation of the specimen between the selected points, the length of the diagram in this direction varying from 6 to 15 mm. (see fig. 17). The ordinates (2,000 lbs. is represented by about 9 mm.) are approximately twice the compression of the springs, but as the springs do not obey Hooke's Law exactly, areas are not exactly proportional to work done. The consequent distortion of the diagram from a true stress-strain figure is unimportant so far as concerns deductions from the shape of the curves, as the departure from Hooke's Law is mainly at low loads. For estimation of work done the diagrams are measured on a double stage microscope and areas of the true stress-strain curves calculated.

The calibration of the mechanism was done directly. By a system of levers described below loads were applied to the spring in the same way and through the same members as when the specimen is in place. The extension co-ordinate was calibrated by substituting for the specimen an arrangement of telescopic brass tubes incorporating a vernier reading to 1/1000 inch.

The whole of the testing machine and recorder is self-contained and occupies a space approximately 4 feet by 1 foot 8 inches. The weight is sufficient to enable the maximum load to be applied in a few seconds without the necessity for fixing to the ground. On the other hand, the whole apparatus can be lifted by four men with ease.

9. *Tensile Testing Machine.* (Fig. 6.)

The main frame A is rectangular, approximately 4 feet × 1 foot 8 inches, and is made of 4-inch × 3-inch × $\frac{1}{2}$-inch channel steel, bolted together with angle pieces. The longer sides A_2 are extended, carrying the recording instrument at one end and the gearing for extending the specimen at the other. To the shorter sides A_1, A_4, are bolted, parallel to the longitudinal axis of the machine, two bars of $1\frac{1}{2}$-inch round steel A_3, which act as guides for the cross head D. This cross head is moved by means of the screw B (1 inch diameter × $\frac{1}{4}$ inch pitch). The phosphor-bronze nut through which this screw passes is mounted in bearings attached to the cross member A_1, and is rotated by the 3 to 1 chain gearing C. The end thrust is taken by a ball bearing.

On the cross member A_4 rest two large helical springs F, held in position by short circular pieces of steel fitting freely inside them. Across their outer ends is a piece of channel steel G provided with two similar discs. To this cross bar, and to the cross head D, are attached two pieces of channel steel E_1, E_2, forming carriages for the ends of the specimen. E_1 is sufficiently

supported by its attachment to D. To E_2 are fixed two wheels e which roll on the guide bars A_3.

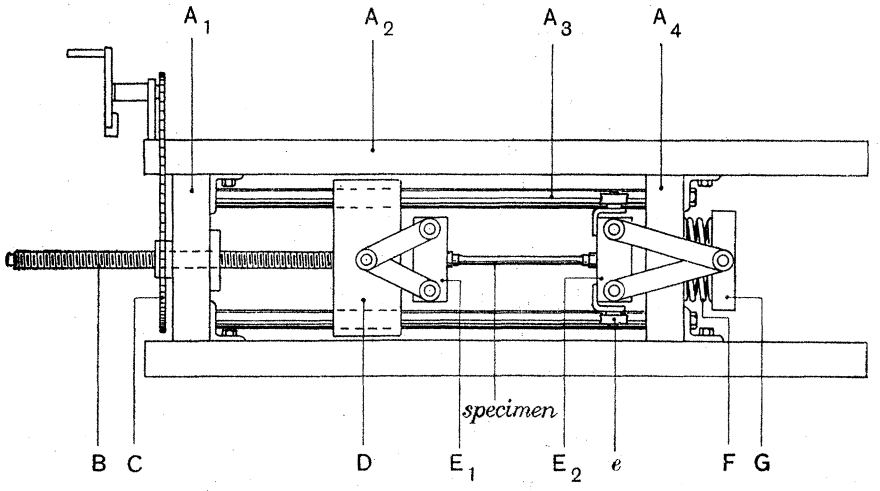


FIG. 6.—Arrangement of testing machine.

The ends of the specimen are screwed into end-pieces H (fig. 7), which form, together with the parts E_1, E_2, a species of bayonet-joint. In the face of E_1 is a rectangular hole through which the end-piece H passes freely. On then

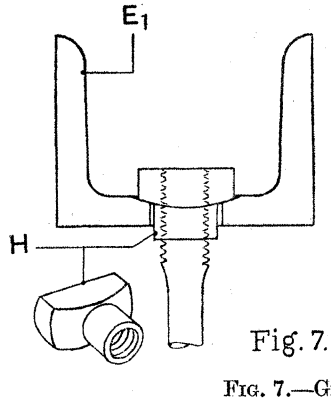


Fig. 7.

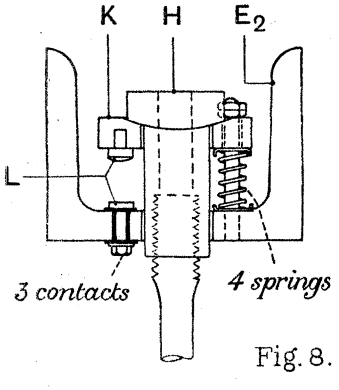


Fig. 8.

FIG. 7.—Grip for end of specimen.

FIG. 8.—Grip for end of specimen with spring link.

rotating the latter through approximately 90° the spherical seating on it bears on a corresponding recess in the back of E_1. The arrangement at the other end E_2 is similar, with an additional complication described below.

The spiral springs F have each three coils of $\frac{1}{2}$-inch diameter wire, wound on

a 2-inch mandrel, with a pitch of 0·4 inch. The pitch of the end coils decreases to zero and the ends of the spring are ground flat. This forms a suitable bearing surface and gives the spring, regarded as a strut, some stability. But it causes the law of the spring to be non-linear on account of the closing of the end coil in the early stages of compression. Since there is no actual fixture of the ends either of the specimen or of the springs, a small initial tension was necessary both in the actual experiment and in calibration. To ensure that the same amount was used always, a spring link was introduced which closed hard up at a known load, its closing being indicated electrically. In addition, in order to avoid any uncertainty which might have arisen owing to the springs seating themselves differently at different times, stops were inserted between E_2 and A_4 which prevented the springs from ever becoming slack. The initial tension (72 lbs., of the order of one-fifth of the force required to produce permanent set in the weakest specimens) was sufficient to pull E_2 away from these stops.

The spring link is shown in fig. 8. The spherical seating K, at the end nearer the springs F, is separate from the carriage E_2, being supported on four light springs, suitably guided. Three pairs of stops L make contact when the load of 72 lbs. is applied to H, causing three lamps to light. Provided the load is applied axially, so that the lamps light nearly simultaneously, this device is very sensitive, a decrease of 2 lbs. being enough to extinguish the lamps. It was calibrated directly, the carriage E_2 being removed and suspended.

10. *Mechanism for Calibrating Springs.* (See fig. 9.)

The springs were calibrated in place in the machine by the application of weights through the system of levers shown diagrammatically in fig. 9. A vertical lever N, suspended by an adjustable wire, is pivoted at its lower end on a hook-piece O, which engages with the lower flange of the carriage E_1. To this lever are attached the links M and P. The former screws into the standard end-piece H and engages, *via* the spring link (fig. 8), with the carriage E_2, and so to the springs F. To P is attached a bell-crank lever Q, pivoted at R and carrying weights at S. All pivots (indicated by ⊗) are hardened knife-edges.

The arms of the levers are so proportioned that the force at M is approximately 10 times that at P, which is approximately double the weight S. The overall ratio is thus 20 to 1, enabling forces of 2,500 lbs. to be applied to the springs by a weight of 125 lbs. at S.

The geometry of the mechanism was restored to standard by the following

adjustments. Weights being hung at S, the cross-head D was moved, carrying with it the hook O, until the lever N was parallel to a plumb line. The turn-

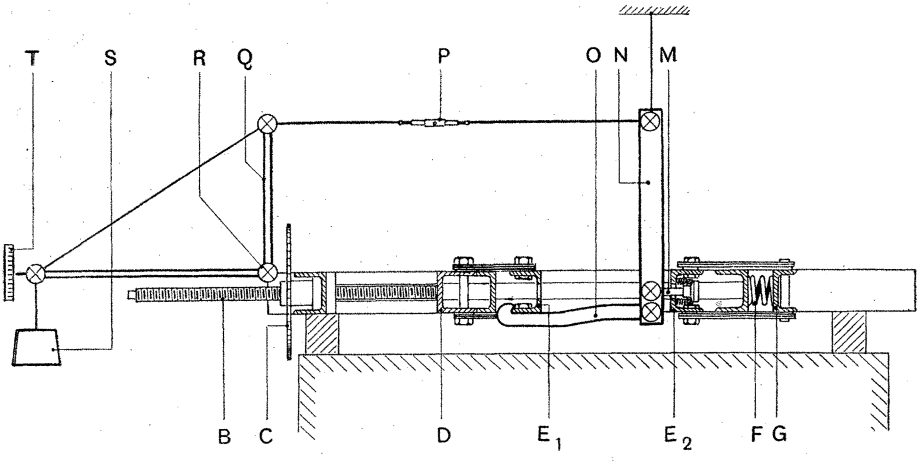


FIG. 9.—Arrangement of mechanism for calibrating springs.

buckle in P was then adjusted till the bell-crank lever was in the standard position, as indicated by the index and scale T.

The accuracy of the calibration is dealt with below (§14).

11. *Extensometer.* (See figs. 10–12.)

The essential problem of ordinary extensometers is the multiplying mechanism by which the minute extensions occurring within the elastic range are made visible. In this instrument, with extensions of the order of 50 per cent. on 10 cm., it was found desirable to use a reducing gear (actually 3·59 to 1, in all) in order to produce a convenient diagram. On account of the inconvenience of very long levers, a mechanism was developed which kept the ratio of reduction constant to a high degree of accuracy with levers of reasonable length. This is shown in fig. 10. Two levers, A_1, A_2, of special construction to avoid errors due to deflection, engage at their lower ends with the selected points on the specimen (the method of attachment is described below). Their upper ends pivot in cross-heads sliding in guides B_1, B_2. From a point on each of these levers a connection is taken to the recorder. Provided the following conditions are fulfilled, the ratio is invariable:—

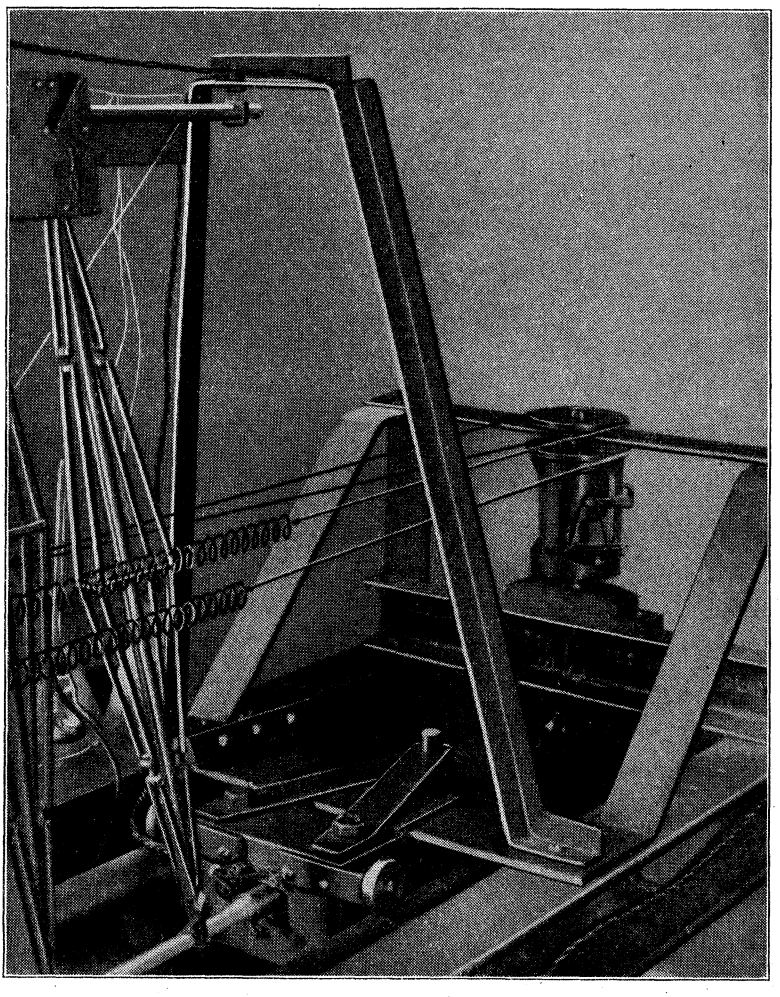
(1) The points of attachment of the connections must divide the levers in the same ratio.

(2) The guides for the upper ends of the levers must be perpendicular to the axis of the specimen and must be fixed in relation to one another.

(3) The connections to the recorder must be parallel to the axis of the specimen.

The levers may be, and in fact are, of different lengths.

Motion of the specimen, as a whole, in relation to the two guides B_1, B_2 produces no movement of the tracing-point on the drum, and hence it is not



essential, from this point of view, that the guides should be fixed, except relatively. As, however, they experience the reaction due to the force required to move the recorder, they are, in fact, fixed to the framework shown in the accompanying photograph.

The levers A_1, A_2 are tubular, stiffened by kingpost bracing tensioned

up by adjusting screws. The general arrangement can be seen in the above photograph.

One of the problems of the experiment was the attachment of the lower ends of the levers A_1, A_2 to the selected points in the specimen. Since normal plane sections of a single crystal do not remain normal when the specimen is stretched, it is impossible to use the conventional form of attachment—namely, two screws engaging in punch-marks at opposite ends of a diameter. On the other hand, the deformation of a single crystal is so closely uniform along its whole length that the extension of any generating line of the original cylinder may be taken as the extension of the whole specimen. Two punch-marks are made about 10 cm. apart, on a generating line, the punch used being a gramophone needle. Into each of these fits a similar needle fixed into the lower end

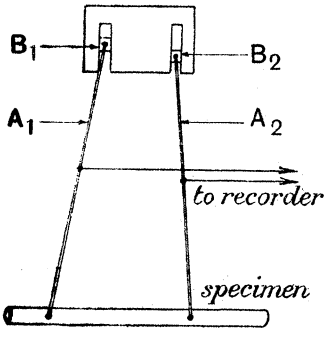


Fig. 10.

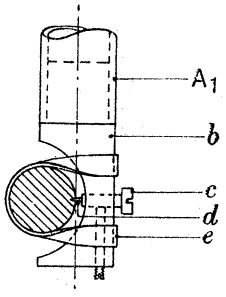


Fig. 11.

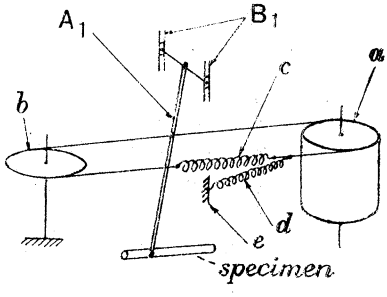


Fig. 12.

FIG. 10.—Arrangement of extensometer mechanism.
FIG. 11.—Method of attachment of extensometer to specimen.
FIG. 12.—Arrangement of connections to recorder.

of the lever A_1 or A_2, the details being shown in fig. 11. In the end of the tube forming the lever is soldered a piece of brass b, shaped as shown, carrying the brass plug c, in which the needle-point d is soldered. The point is held firmly in the punch-mark by a rubber band e. It will be seen that the position of the point, practically on the axis of the tube, forming the lever A_1 reduces the torsion on the lever to a negligible amount, and any tendency of the lever to twist as a whole is reduced by arranging the trunnions which slide in the guides B_1, B_2 at some distance from the axis of the tube. (See fig. 12, B_1.)

The intermediate points on the lever A_1 A_2 are connected to the recorder by steel bands 0·1 inch wide by 0·004 inch thick. In order to avoid errors due to the slight flexural rigidity of this band, a certain minimum tension is needed. This is obtained by making each band a complete loop (*see* fig. 12),

passing from the attachment to the lever A_1, round the pulley a, by which the recorder is driven, back to a free pulley b at the other end of the machine, and finally returning to the lever A_1, which therefore experiences no force due to any tension in the band. A long spiral spring c is inserted in each loop between the two pulleys mentioned above. Finally, in order to ensure that there is no lost motion in the system a light adjustable spiral spring d connects a point on each band to the framework e.

It will appear from the calibrations referred to in §14 below that these precautions had the desired result, the x co-ordinate of the records being a constant fraction of the extension between the selected points on the specimen to a high order of accuracy.

12. *Recorder.* (See figs. 13–15.)

A diagrammatic sectional view of the recorder is shown in fig. 13, and its external appearance in the above photograph. It is mounted on a cross bar of channel steel attached to the extended sides of the main frame of the testing machine. On this is a bearing A, in which turns the external drum B, which carries the scribing point b_1. On a parallel cross bar of flat steel, firmly supported by arms which are clearly visible in the photograph, is a second bearing C.

On the upper end of the drum B is formed a groove for the steel band e_2 from which it derives its motion. Three large oval panels are removed from the drum in order to enable the celluloid film to be mounted on the internal drum F. The scribing point b_1 is fixed in a triangular frame which pivots on pointed screws b_2, attached to the drum B. It is pressed on the film by a flat spring (*see* photograph), which is so arranged that the point can be swung back while the film is put in place, and restored, with the same pressure, when required.

The inner drum F carries the film on which the record is made. This is ordinary cinematograph film, the gelatine being first removed. The axial motion of the drum, approximately twice the compression of the springs which measure the force on the specimen, is obtained as follows. To the cross bar by which these springs are compressed (*see* fig. 14) is attached a compensating lever a, from whose ends two steel bands pass to two equal pulleys L_1, mounted on a spindle fixed below the recorder. In between, and integral with, these pulleys is a larger one L_2, from whose circumference a steel band passes up to the recorder. Each band is attached to the circumference of its pulley, the compensating lever a eliminating the effect of any slight inequality in the

diameters of the smaller ones. The bands are kept taut by a spring K (fig. 13) so arranged that it does not contribute to the friction of the recorder.

The band from the larger pulley L_2 is attached to the lower end of a tube H which passes up through both drums and has on its upper end a collar, against which the inner drum is pressed by a spring G, a ball thrust bearing reducing

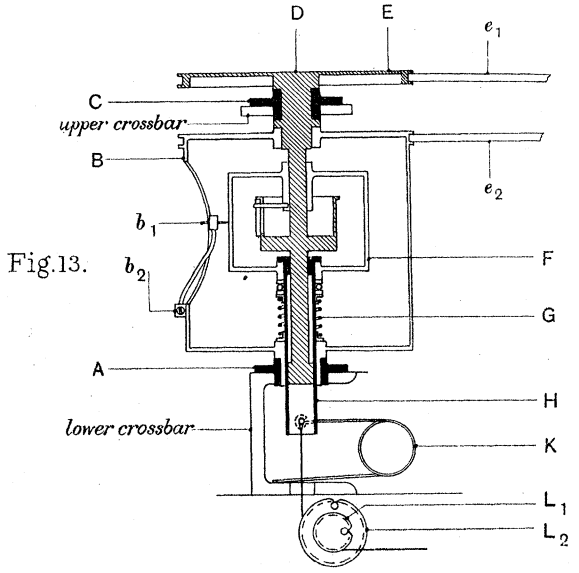


Fig.13.

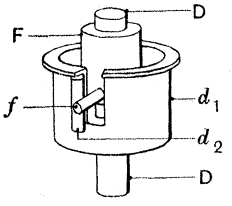


Fig.15.

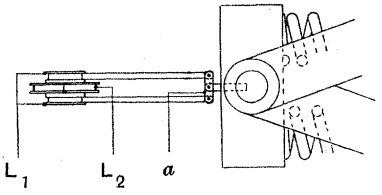


Fig.14.

Fig. 13.—Arrangement of recorder.

Fig. 14.—Connection of extensometer to springs.

Fig. 15.—Rotational drive to drum of recorder.

the friction. The spring G also serves to press the outer drum B down on to the bearing A.

The most difficult problem of the instrument was the rotational motion of the inner drum F. This is derived from the upper pulley E and steel band e_1. The spindle D of this pulley passes down through the upper bearing C, both drums, B and F, and the tube H, the bearing surfaces being arranged

as shown in fig. 13 in order that the tension in the bands may not cause the mechanism to bind.

On the part of this spindle D which is inside the drum F is mounted a cylinder d_1 (fig. 15) with a diameter approximately half that of the drum. This cylinder is open at the upper end and is slotted parallel to its axis, as shown in the sketch. To its outer surface is soldered a piece of silver steel rod d_2 which was carefully tested for straightness. Its position is indicated in fig. 15. It was adjusted to be as nearly as possible parallel to the spindle D. Through the slot in d_1 there projects a similar piece of rod f, which is fastened to a projection of the end of the drum F. The rods f and d_2 are kept in contact by springs (not shown) to avoid backlash, and form a "key" and "keyway" connecting D and F rotationally.

This construction reduces the errors in the axial motion of the drum F to a minimum. As the result of the precautions described it is not possible to detect, on a Hilger measuring microscope, any departure from straightness in fiducial lines on the records parallel to either x or y axis (*see* fig. 17, below). The angle between the axes is almost exactly 90°, showing that the "key way" is practically parallel to the axis of the drum. As however the angle between the stages of the microscope could be adjusted so that the movements were parallel to the fiducial lines, the exact magnitude of this angle is not important.

13. *Instrumental Errors.*

The errors to be anticipated in such a mechanism as has been described may be classified as follows:—

(1) *Due to strain.*—The supports of all parts are very firm. The stresses in the main frame are very small and any deflection which takes place when the force is applied to the specimen is certainly elastic and, since the springs are calibrated *in situ*, is absorbed in the calibration. The steel bands driving the recorder are of ample size, and are under sufficient initial tension to ensure that their flexural rigidity is not a source of error.

(2) *Due to friction.*—There is very little frictional resistance to the rotational movements of the recorder, but there is an appreciable frictional resistance to the axial motion of the inner drum. Since however the forces to be measured are of the order of 1,000 lbs. and this friction is of the order of ounces, no appreciable error arises. All the surfaces on the recorder are copiously lubricated with a fairly viscous oil, and no tendency to stick has ever been observed. It may be noted that in all experiments the motion of every part of the mechanism

is in one direction only (the fall of stress at the elastic limit with steel is the only exception and cannot give rise to any appreciable error).

(3) *Due to slack.*—The precautions taken to avoid slack at every point have been described. The points which engage in the punch marks on the specimen fit very firmly, and though the latter must extend into an oval shape when the specimen is stretched, it appears from separate measurements made on the stretched specimen that the points remain in the centre of the punch marks.

(4) *Due to change of conditions between calibration and experiments.*—As has been mentioned in describing the methods of calibrating, elaborate precautions were taken to ensure that, so far as the condition of the machine and recorder were concerned, the calibrations and experiments were identical.

14. *Order of Accuracy of Results.*

The magnitude of the probable error of the final result (the area of the stress-strain figure) is influenced chiefly by the calibration of the springs. The levers used in this calibration (*see* §10) were found to have ratios of 1·959 and 10·21 as the mean of several calibrations by weights, giving a combined ratio of 20·00. The variations in the figures for the individual levers were of the order of 1 in 1,000 and the combined result may be relied on to 1 in 500. The "out of balance" of the bellcrank lever was determined directly to be equivalent to 21·5 lbs. at the springs.

The calibration curve of the springs is shown in fig. 16. It includes the

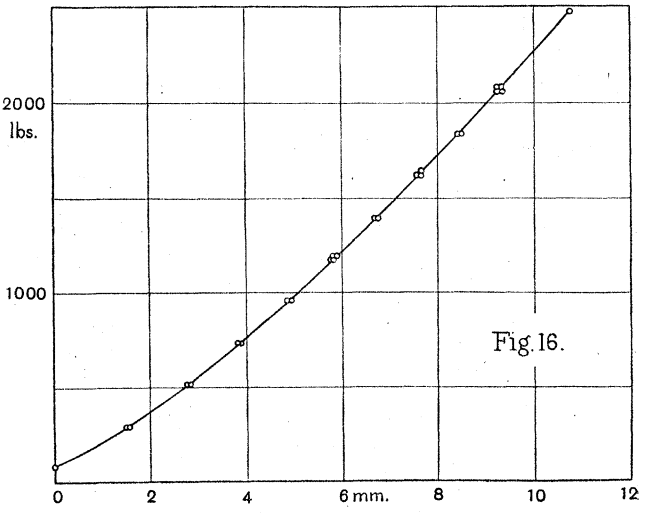


Fig. 16.—Calibration curve of springs.

results of calibrations taken both before and after the experiments, showing that the characteristics of the springs did not alter appreciably with time. The points nowhere depart from the curve adopted by more than 1 per cent., and it is considered that this represents the maximum error in a force measurement.

The calibration of the extension co-ordinate by the method described in §8 (p. 437) gave a ratio of 3·59 within 1 in 500, the ratio calculated from the measurements of the mechanism involved being 3·58. The former figure was used as it corresponded to a direct comparison of specimen and record, whereas the latter involved several separate measurements, one being of a type in which great accuracy could not be attained. The constancy of this ratio for extensions of both large and small amounts was within the figure given : " large " meaning of the order of 5 cm. on the specimen, and " small " of the order of 5 mm. In the experiments the actual extensions corresponding to each temperature measurement had the following average values for the materials tested :—

	mm.
Copper	6
Steel	4
Aluminium (plain)	7
Aluminium (single crystal)	10

The *total* extensions of each specimen averaged 18, 12, 22 and 50 mm. respectively.

The actual records were measured on a Hilger comparator, on which readings could be repeated within one-hundredth of a millimetre. The final areas were computed from measurements of ordinates sufficiently closely spaced.

It is considered that the above figures enable it to be claimed that the final areas, representing the work done, are determined within 1 per cent.

15. *Stress-Strain Records.*

Some examples of these are shown, enlarged 3·1 times, in fig. 17.* The upper three are from steel, copper, and plain aluminium, respectively, and the lower two from single crystal specimens of aluminium. The numbers correspond to those in the table of results (Table III) and in the temperature records, figs. 2–5.

* The definition of the lines has suffered in reproduction. As seen by transmitted light the trace of the scribing point appears as two dark lines, separated by a white line. The overall width is about 0·04 mm. In measuring the records, the cross wires of the microscope were focussed on the outer edge of one of the dark lines.

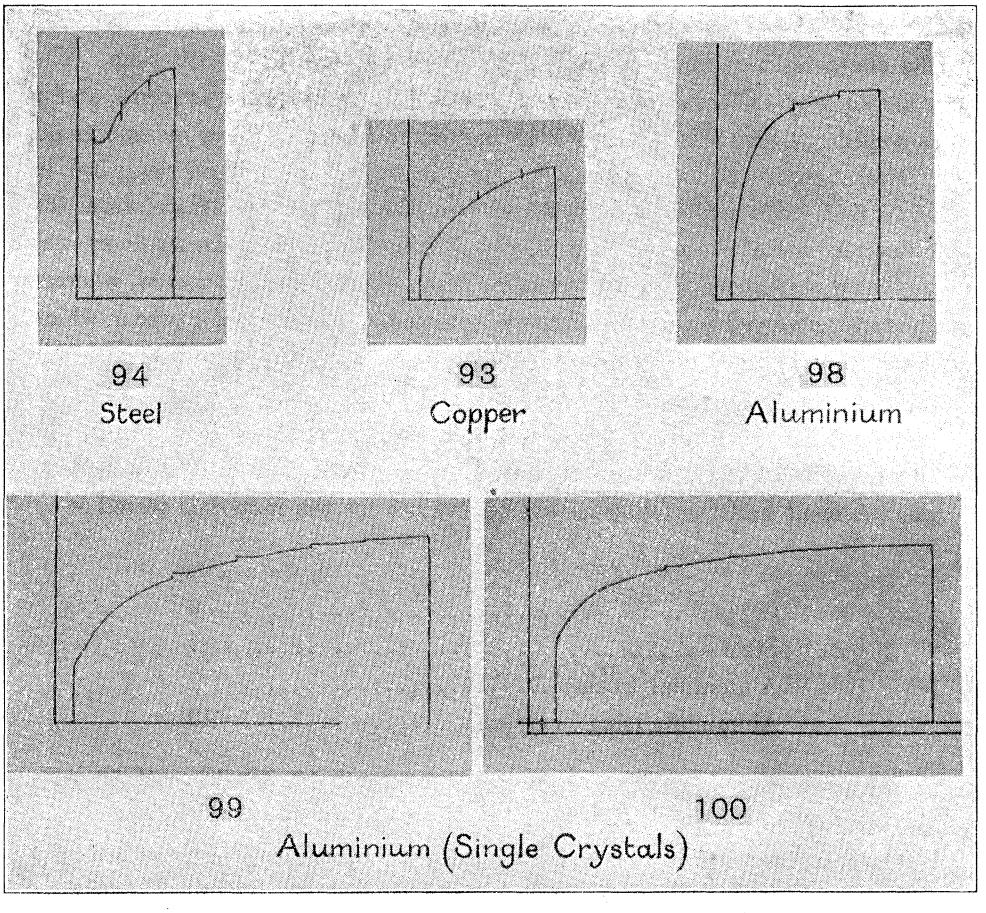


FIG. 17.—Stress-strain records.

The approximate scales of the diagrams as reproduced are :—

Extension (x axis) 1 cm. = 11·6 per cent. on the original length of 10 cm. between marks.

Forces (y axis) 1 cm. = 715 lbs. approximately. The force scale is not exactly linear, for reasons which have been described above.

The original dimensions of the specimens are given in the following Table IV.

Table IV.

Material.	Length of parallel part.	Outside diameter.	Inside diameter.	Cross sectional area.
	cm.	in.	in.	sq. in.
Steel	30	0·31	0·21	0·0422
Copper	29	0·25	0·12	0·0384
Aluminium	21	0·50	0·19	0·1688
Aluminium (single crystal)	16	0·56	0·19	0·2151

The approximate stress scales are therefore as follows :—

Steel	1 cm. =	17,000	lbs./square inch,
Copper	1 cm. =	18,600	,, ,,
Aluminium	1 cm. =	4,200	,, ,,
Aluminium (single crystal)	1 cm. =	3,300	,, ,,

referred, in all cases, to the original cross-sectional area.

The short vertical marks on the records represent the end of each experiment. With steel and aluminium they appear on the record automatically and were thought at first to be due to a defect in the instrument, but as they did not appear in experiments on copper (the marks on the record for copper were made deliberately in order to give greater precision to the measurements) they are presumably characteristic of certain materials only. Their significance is discussed below.

The interval between two extensions of any one specimen was about 10 minutes, and during this time the specimen was under tension, constant except in so far as any " creep " of the specimen allowed the springs to extend. The records show that this creep occurred with steel, and (to a much smaller extent) with plain aluminium, but not with copper or single-crystals of aluminium. It should be noted that the circumstances during this " creep " are very different from those which occur when a specimen is left under load in an ordinary testing machine. The load is then independent of any extension which may occur, whereas here extension of the specimen relieves the load rapidly. In fact, the springs extend the same amount as the specimen. The recorder magnifies extensions of the spring twice (approximately) and reduces extensions of the specimen, between the selected points, 3·59 times. Hence, if the whole length of the specimen is n times the length between the points, the line on the record representing the " creep " referred to above should slope down from left to right at an angle $\tan^{-1} 2 \times 3{\cdot}59\, n$. For the steel

specimen ($n = 3$) this angle is $\tan^{-1} 21 \cdot 5$. Examination of the record will show that these lines do in fact slope to about this extent.

On increasing the load, for the next extension, it will be seen that with steel and aluminium (both plain and single-crystal) no extension occurs until the load has risen appreciably above that previously reached. There is then a rapid extension at almost exactly constant load, followed by a period in which both load and extension increase. The fact that the curve corresponding to the last part joins smoothly on to that representing the end of the previous extension suggested that the "kink" in the curve was instrumental, but as the phenomenon is not observed with copper (*see* records) it is presumably characteristic of certain materials only, corresponding to something analogous to sticking.

With copper the absence of the "kink" is so definite that it was sometimes found difficult to decide exactly where each separate extension ended, though the approximate point could be seen without difficulty, owing to the slightly deeper depression in the celluloid made by the scribing point resting in one position for some minutes. The marks seen in the copper records were therefore made deliberately, as mentioned above.

It will be seen that the "kinks" in the curves for single-crystals of aluminium decrease regularly as the load increases, and practically disappear when the breaking load is reached—*see* in particular No. 100, which was on the point of breaking at the end of the experiment.

The record for mild steel No. 94 shows the characteristic fall of stress at the yield-point. An actual *fall* of stress was not observed in all specimens, the variation being presumably due to slight differences in the rate of extension. The "ripples" on the record are due to slight irregularity in turning the handle of the machine. They do not appear in the records for the other materials, presumably because these have a much smaller tendency to "creep."

All the specimens were thoroughly annealed before the experiments, but there is nevertheless an appreciable "elastic range" for all except the plain aluminium. This material, however, hardens remarkably rapidly, having after 1 per cent. extension an "elastic range" of about 4000 lbs./sq. in., and after 5 per cent. extension a range of about 9000 lbs./sq. in., these stresses being about one-third and three-fourths respectively of its ultimate breaking stress.

When plotted on true stress-strain co-ordinates the records for all the copper and all the plain aluminium specimens tested agree very closely with

one another. The single-crystal specimens of aluminium differ considerably from one another, No. 99 having a lower yield point than No. 100, though it is ultimately stronger.

The work was carried out in the Cavendish Laboratory through the kindness of Sir Ernest Rutherford, to whom we wish to express our thanks.

We wish also to thank Prof. Carpenter and Miss Elam for presenting us with single-crystal specimens of aluminium, and Mr. W. W. Hackett, of Messrs. Accles & Pollock, Ltd., who presented us with specially annealed steel tubing from which the test-pieces were made. An analysis of this material (for which we are indebted to Mr. W. E. Woodward, M.A.) showed that it contained 0·17 per cent. carbon and 0·76 per cent. manganese.

An Experimental Study of the Vibrations in the Blades and Shaft of an Airscrew.

By A. Fage, A.R.C.Sc., of the Aerodynamics Department, National Physical Laboratory.

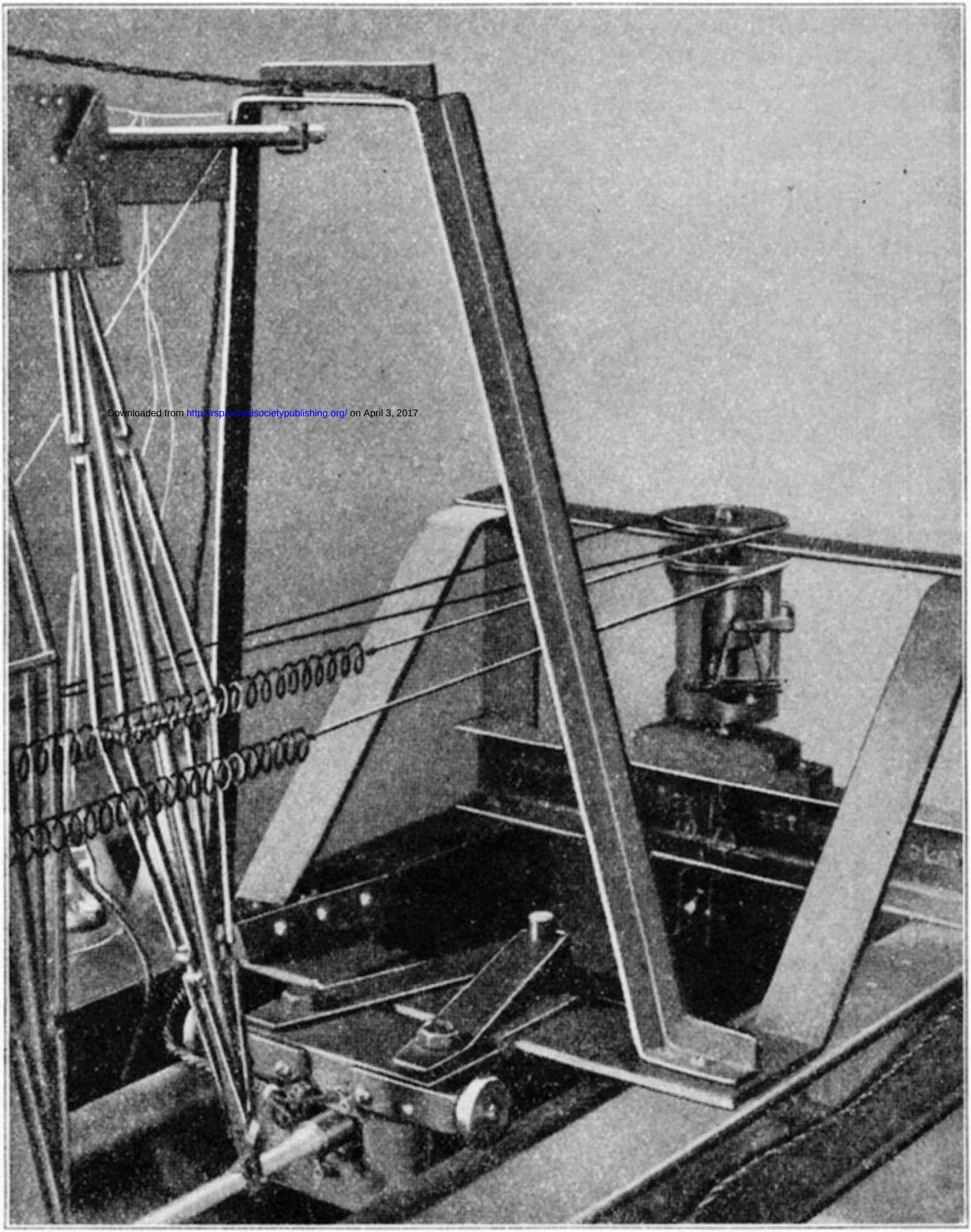
(Communicated by H. Lamb, F.R.S.—Received November 11, 1924.)

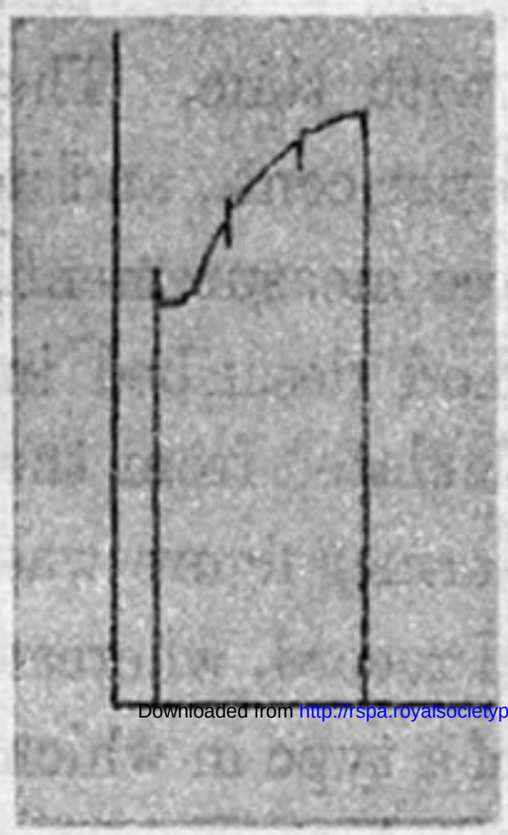
CONTENTS.

§1. *Introduction.*

This investigation* deals with the natural frequencies of flexural vibration of the blades and shaft of a rotating airscrew, and includes a comparison of

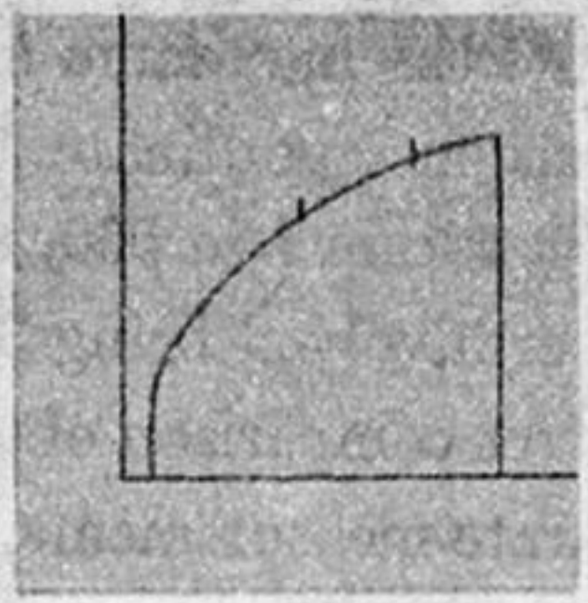
* The work described in this paper was carried out in the Aerodynamics Department of the National Physical Laboratory, and permission to communicate the results was kindly granted by the Aeronautical Research Committee.





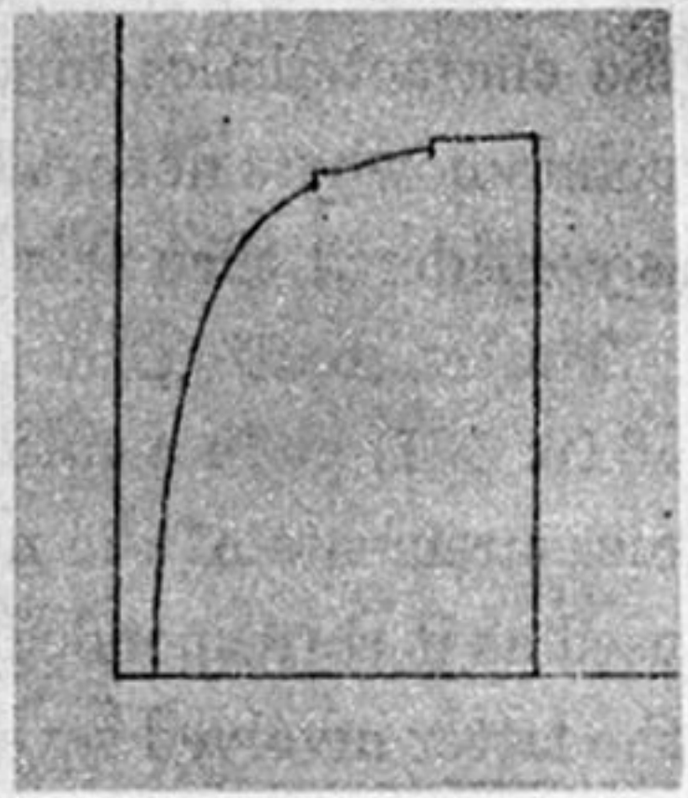
94

Steel



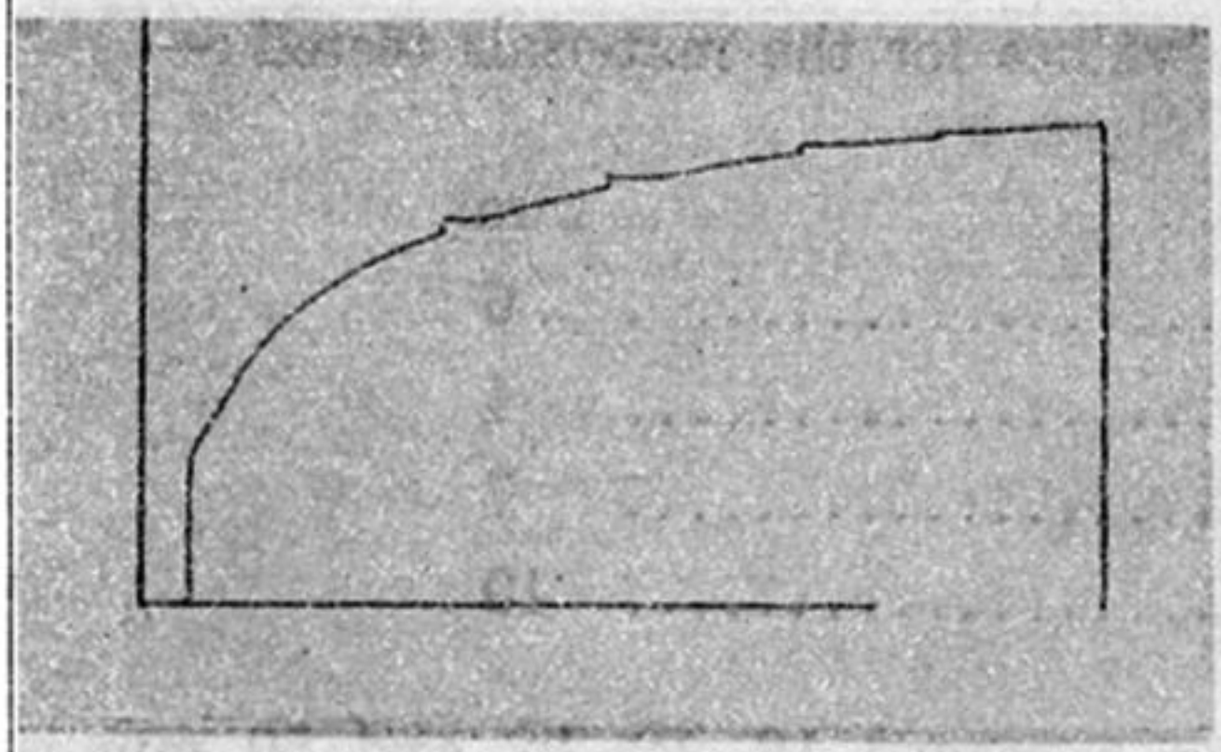
93

Copper



98

Aluminium



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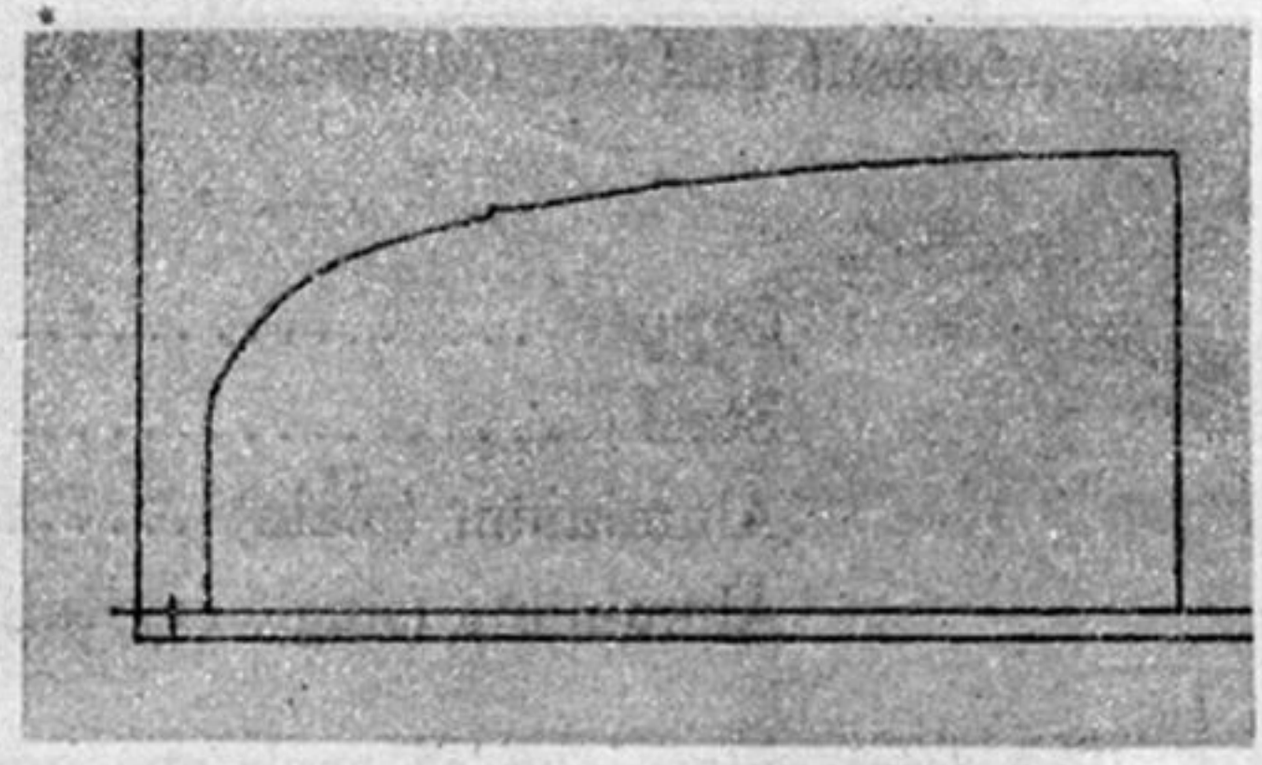

100

Aluminium (Single Crystals)

FIG. 17.—Stress-strain records.