The Latent Energy Remaining in a Metal after Cold Working

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The Latent Energy Remaining in a Metal after Cold Working.

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Summary.

Measurements of the latent energy remaining in metal rods after severe twisting are described. Very much more cold work can be done on a metal in torsion than in direct tension. It is found that as the total amount of cold work which has been done on a specimen increases the proportion which is absorbed decreases. Though saturation was not fully reached even with twisted rods, curves representing the experimental results for copper indicate that it would have been reached at a plastic strain very little greater than the strain at fracture. The amount of cold work necessary to saturate copper with latent energy at 15° C. is thus found to be slightly greater than 14 calories per gram.

By using compression instead of torsion, it was found possible to do much more cold work on copper than this, and compression tests revealed the fact that the compressive stress increases with increasing strain till the total applied cold work was equivalent to 15 calories per gram. No further rise in compressive stress occurred with further compression even though the specimen was compressed till its height was only 1/53rd of its original height.

The fact that the absorption of latent energy and the increase in strength with increasing strain both cease when the same amount of cold work has been applied suggests that the strength of pure metals may depend only on the amount of cold work which is latent in them.

When a metal is subjected to plastic distortion (cold working) most of the work done reappears in the form of heat, but a certain proportion remains latent and is no doubt associated with the changes to which cold working give rise in the physical properties of the metal. When the metal is heated all this latent heat must be released before the melting point is reached, and when it is dissolved the latent heat must appear as a heat of solution. It is therefore possible to measure the latent heat of cold working either when energy is put into the metal or when it is released. In the former case, the work done and the heat evolved during plastic deformation are measured. The difference is the latent energy of cold working. This method has been adopted by Farren and Taylor* and by Hort†, who found that within the range of their experiments $5\frac{1}{2}$ to $13\frac{1}{2}$% of the work done remains latent in the metal.

Some attempts have been made to measure the latent energy of cold working when it is released, but the results are not always comparable with those of Farren and Taylor and when they are the two kinds of measurements do not seem to be in agreement. In the experiments of Farren and Taylor the cold work was done by stretching a rod of metal by a direct load. With such a system of loading only a very limited amount of work can be done on the specimen before it breaks. With copper, for instance, the maximum latent energy which they measured was equivalent to a rise in temperature of only 0·83° C. In order to release the latent energy of copper it is probably necessary to raise its temperature to 500° C. or more. The latent energy would appear as the difference between the amounts of heat necessary to raise the temperature of two equal specimens to 500° when one of them had been subjected to cold work and the other had not. To measure it would be equivalent to measuring the difference between the specific heat of two metals when that difference is only 1 part in 600.

It is difficult to attain such accuracy in heat measurements, but by using metals which have been subjected to much more severe cold working than is attainable under a direct load, greater latent heat can be obtained and the accuracy of the measurements of the energy released on heating correspondingly increased.

It is well known that much more cold work can be done on a metal rod by twisting it than by stretching it. The amount of work which can be done in direct extension depends on the nature of the load-extension curve. The condition for fracture by instability owing to the formation of a local " neck " is

$$\frac{l_0}{\mathrm{T}}\frac{d\mathrm{T}}{dl} < 1 \qquad \text{or} \qquad \frac{d \log \mathrm{T}}{d \log (l/l_0)} < 1,$$

where T is the tensile stress, l is the length, and l_0 the initial length of the test specimen. The criterion for fracture can therefore be found by plotting the load-extension curve on a logarithmic scale. Fracture will then occur when the tangent to the curve makes an angle 45 degrees with the axes. When a round bar is twisted it will not fail owing to the formation of a local neck until

* 'Proc. Roy. Soc.,' A, vol. 107, p. 422 (1925).
† 'Mitt. Forsch. Arb. deuts. Ingenieur,' vol. 41 (1907).

the shear stress has ceased to increase with increasing strain. (In fact, twisted specimens usually fail for reasons other than this kind of instability.)

If the tensile stress rises very rapidly for a very small extension and then continues to rise much more slowly as the strain increases to large values, a tensile specimen will break at a very small extension, but a twisted rod may suffer very great distortion before breaking. The results of tensile and torsion tests on two identical nickel rods are shown in fig. 1. The nickel was used in the hardened condition as supplied by Messrs. Henry Wiggin & Co. In the tensile test the rod suffered no measurable plastic distortion till a stress of 76,800 lbs. per sq. in. was reached. It fractured when the stress was 84,800 lbs. per sq. in., and the extension was then $0 \cdot 012 l_0$.

In the torsion test the first measurable plastic strain occurred when q, the mean shear stress was 46,000 lbs. per sq. in.; q then gradually increased with increasing twist to 68,000 lbs. per sq. in. when fracture occurred. The amount

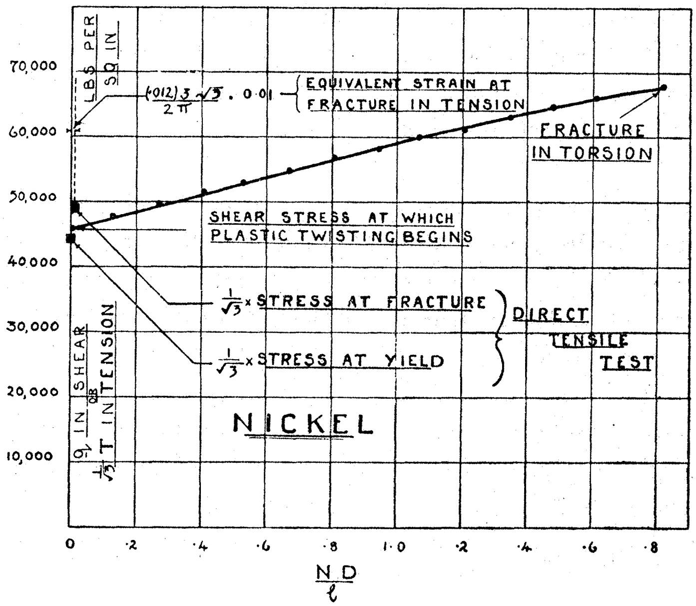


FIG. 1.

of twist is expressed in fig. 1 by the non-dimensional quantity ND/l, where N is the number of turns, l the length of the specimen, and D its diameter. It will be shown later that this is π times the shear strain in the surface layers of the rod. The maximum value of ND/l attained before fracture was $0 \cdot 82$.

In order to represent these two tests on the same diagram the theory of v. Mises may be used. According to this theory if T is the stress at which plastic flow begins in a rod subjected to direct tension and q the shear stress for plastic flow under the action of a pure shear, $q = \mathrm{T}/\sqrt{3}$. Similarly in order to find the small increment in ND/l which is equivalent, so far as work done on the specimen is concerned, to a small direct extension $(l - l_0)/l_0$ it is necessary to multiply* by $3\sqrt{3}/2\pi$. The results of the tensile test are represented in fig. 1 by points whose ordinates are $76{,}800/\sqrt{3} = 44{,}300$ lbs. per sq. in. for the plastic yield point and $84{,}800/\sqrt{3} = 49{,}000$ lbs. per sq. in. at fracture. The abscissa representing the strain at fracture is $(0\cdot012)3\sqrt{3}/2\pi = 0\cdot01$.

It will be seen that the total work done on the specimen during the test, which is proportional to the area under the stress-strain curve in each case, is about 80 times as great for the twisted rod as it was for the rod broken in direct tension.

In the work to be described later cold work was done by twisting instead of direct extension. The maximum latent energy left in copper after severe twisting was equivalent to a rise in temperature of about 15° C., *i.e.*, 18 times as much as that used in Farren and Taylor's experiments. The release of this amount of energy can be measured and in a later communication we hope to be able to describe the apparatus with which such measurements have been made.

In the experiments of Farren and Taylor it was found that the energy left in a metal after distortion is a definite fraction of the work done on it. This fraction varied with the nature of the metal, but appeared to be constant for various amounts of distortion in spite of the fact that the resistance to extension rapidly increased as the extension increased. It seems unlikely that it would be possible to increase the amount of latent energy indefinitely by doing cold work; accordingly one of our objectives in measuring the latent energy due to cold work in twisted bars was to find out whether it goes on increasing proportionately to the work done when the cold working is very severe. Work recently published by Rosenhain and Stott† shows that when copper or aluminium wire is drawn through a die a rather smaller proportion of energy is absorbed than was found by Farren and Taylor. In their apparatus the measured work done included the work done against friction between the wire and the die so than the proportion absorbed was necessarily smaller than that measured with apparatus in which all the work is expended in straining the material, but after making due allowance for the friction they still found that the pro-

* For $\rho\delta\mathrm{W} = \mathrm{T}(l - l_0)/l_0 = \dfrac{2\pi}{3} q\delta(\mathrm{ND}/l)$, see equation (2), p. 315.

† 'Proc. Roy. Soc.,' A, vol. 140, p. 9 (1933).

portion of energy absorbed was rather smaller than that found by Farren and Taylor. The present experiments confirm this result.

Measurement of Latent Energy Produced by Cold Work.

To measure the latent energy it is necessary to measure simultaneously the work done and the heat evolved, and in order to avoid loss of heat it is necessary to perform the whole experiment rapidly.

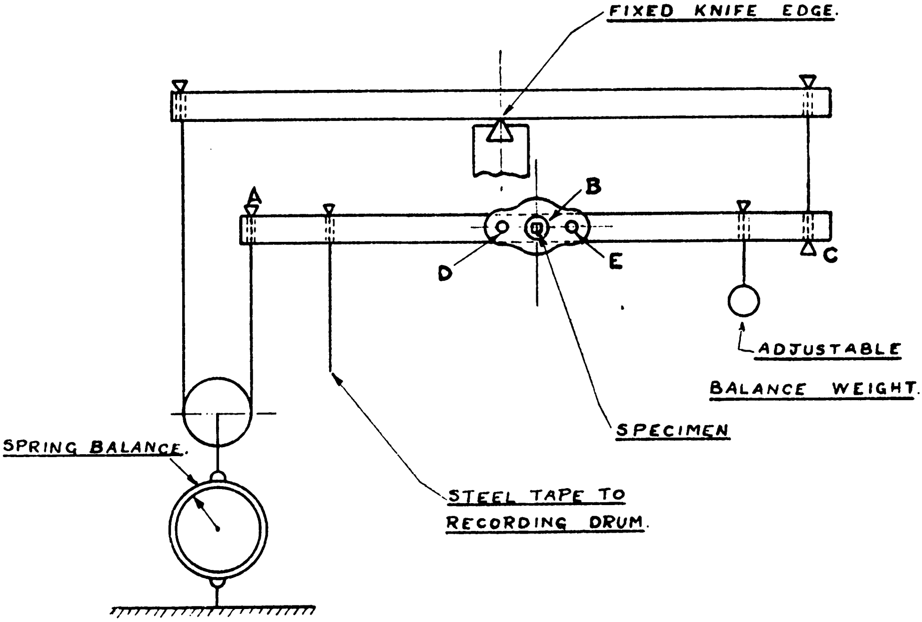


FIG. 2.—Arrangement of torque recorder.

Measurement of Work Done.

To measure the work done during a rapid twisting of a bar, a self-recording machine was designed which produced diagrams in which the ordinates represent torque and the abscissæ angle of twist between two sections of the twisted bar.

The specimens were round in section, $\frac{3}{8}$-inch diameter, and had square ends $\frac{3}{4}$-inch × $\frac{3}{4}$-inch. Their length was 15 inches, but the round part only occupied 11 inches of this. One square end fitted into the headstock of a lathe and could be twisted at any desired speed by a geared electric motor. The torque was applied at the other end by the lever arrangement and spring balance shown in fig. 2. This was designed to apply a pure couple in such a way that it would be recorded directly as a movement of the spring balance. The diagram is self-explanatory, except for the arrangement of the part where the torque is transmitted to the specimen. The arm, ABC, carries a ball race at B which is housed on the loose headstock of the lathe. It also carries two pins, D, E, which engage with a carrier fitting over the square end of the specimen. This arrangement permits the specimen to extend longitudinally* and ensures that no bending moments are applied. The torque was recorded by means of a steel tape (fig. 2), one end of which was attached to the torque arm, while the other passed round a recording drum which carried the paper on which the record was made.

The twist was recorded by means of two discs which were attached to the specimen at points 10 inches apart. These carried steel tapes, one of which drove a spindle and the other a nut. This nut carried a slide rest to which the recording pencil was attached so that the position of the pencil on the recording drum depended only on the relative rotations of the nut and spindle.

Measurement of Heat Evolved.

Two methods were used. The first was to measure the rise in the temperature of the surface of a specimen after rapidly twisting it through about one turn. The second was to remove it so rapidly from the lathe after finishing the twisting that the heat generated in the central part of the specimen had not penetrated through the square ends, and drop it into a calorimeter. The two methods proved to give results in good agreement with one another.

The first method is the same as that used by Farren and Taylor, but the fact that the specimen is now twisted instead of being pulled introduces a new difficulty. More cold work is done in the outer layers than near the middle of the specimen so that if the experiment is done very rapidly the outer layers are heated more than the inner ones. A certain time must elapse before the temperature is equalized over the cross-section, but the reading must be made before the wave of cooling penetrates from the ends of the specimen to the central part where the temperature measurements are made. The temperature was measured by means of an iron-constantan thermocouple connected directly to a galvanometer.

The movement of a spot of light reflected from the galvanometer mirror was recorded photographically on a drum rotating at a uniform speed.

In the experiments of Farren and Taylor the thermocouple was inside the specimen and its leads passed down the middle of the hollow specimen so that

* When a bar is twisted plastically it usually grows in length.

the junction itself and its leads certainly took up the temperature of the specimen. In the case of solid specimens this method of ensuring good thermal contact is not possible. The method adopted in the present experiment was to solder the thermo-junction to the middle of a small square of sheet silver. This was tightly bound by silk threads to the outer surface of the specimen so that the junction itself was on the under side of the silver. The thermocouple wires were insulated from the silver and the specimen by enamel except at the point where they were soldered. This method ensured that the junction was not cooled by conduction along the thermocouple wires.

In order to remove any further uncertainties that may exist in the use of a thermocouple applied to the outside of a specimen and to allow for cooling of the specimen during the time taken to twist it and for possible cooling of the thermocouple by the silk threads which bound it to the specimen, the heat input corresponding with the observed temperature-time curve was determined independently after each experiment. For this purpose the temperature-time record and the torque-angle curve were taken for, say, one turn of the lathe head. The specimen and its thermocouple were then removed from the torque apparatus and after cooling a heavy current up to 100 amperes was passed through the specimen and maintained for the same length of time as that for which the torque had been applied. By adjusting this heating current it was found possible to reproduce exactly the temperature-time curve obtained in the torque test provided the twisting had not been carried out so quickly that the equalization of temperature between the outside and inside of the specimen could not take place. The current and the potential drop between the square ends of the specimen were then measured and the energy input calculated.

The cooling is due chiefly to conduction from the ends of the specimen. It was shown by one of the present writers that during a time equal to $T = 0 \cdot 014 \rho \sigma l^2 / \kappa$ after the generation of heat the cooling effect of the ends causes a drop in temperature in the middle of the specimen which is less than $0 \cdot 6\%$ of the total rise. In this expression l is the length of the specimen which in the present case is 30 cm., for iron $\rho = 7 \cdot 8$, κ the conductivity is $0 \cdot 14$ and σ the specific heat is $0 \cdot 106$ so that $T = 80$ seconds.

In the temperature-time record, fig. 3, A represents the time at which the twisting began, B represents the time it was complete. The time occupied by the operation, namely, 45 seconds, is considerably less than the 80 seconds necessary for the cooling effect of the ends to penetrate to the middle, yet it is sufficiently long to ensure that the temperature is uniform across the section.

It will be seen in fig. 3 that no measurable drop in temperature occurred till the time represented by C. The time interval between the beginning of the experiment and the first observable drop in temperature was found to be about 95 seconds. This agrees well with the theoretical value T = 80 seconds.

For copper specimens, owing chiefly to their greater conductivity, T was only 11·5 seconds; the twisting being carried out in 6 or 7 seconds, leaving 4 or 5 seconds available for the temperature measurement, before the waves of cooling from the ends produced an appreciable effect in the middle.

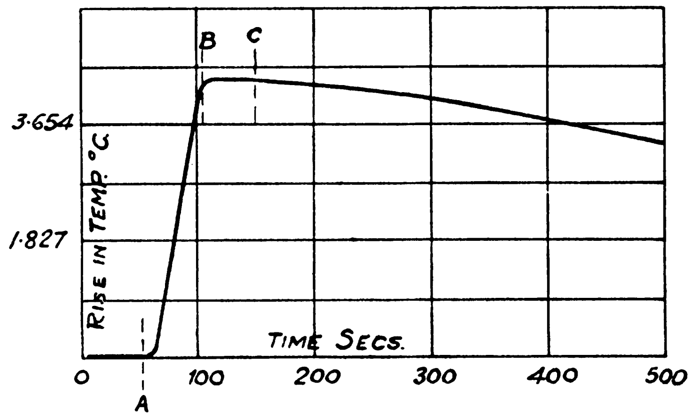


FIG. 3.—Temperature-time record during plastic twisting.

Expression of Results in Non-dimensional Form.

The shear strain at any point in the twisted rod is $2\pi Nr/l$, where N is the number of turns in length l. A non-dimensional expression for representing twist is therefore ND/l. If the shear stress depends only on the shear strain, it may be represented by an expression of the form $q = 12G/\pi D^3$, where D is the diameter of the specimen, G is the applied torque and q is the uniform shear stress which would give rise to the torque G. q is approximately the average shear stress over the cross-section.

Results.—The results of tests on an annealed mild steel bar and on a decarburized mild steel bar are given in Tables I and II. Each bar was twisted through successive small amounts, usually about 1 turn, and the value of ND/l given in column 1 corresponds with the total strain from the initial annealed condition of the bar. The stress given in column 2 is expressed in lbs. per sq. in. and is the mean value of q between the beginning and end of one experiment, thus referring to Table I the mean value of q during the fifth stage of twisting was 43,700 lbs. per sq. ft. and during this stage the strain increased from $ND/l = 0 \cdot 1126$ to $ND/l = 0 \cdot 1452$. The work done during a twist of δN turns is $2\pi G \delta N$ so that the work done on unit mass is

$$\delta W = 8G\delta N/D^2 l\rho. \tag{1}$$

Expressed in non-dimensional form

$$\delta W = \frac{2\pi q}{3\rho}\,\delta\left(\frac{ND}{l}\right), \tag{2}$$

where $\delta(ND/l)$ is the change in ND/l during the experiment under consideration. If q is expressed in lbs. per sq. in. and δW is expressed in calories per gram of metal

$$\delta W = \frac{2\pi q}{3\rho J}\,\frac{(453 \cdot 6)\,(981)}{(2 \cdot 54)^2}\,\delta\left(\frac{ND}{l}\right). \tag{3}$$

For steel $\rho = 7 \cdot 85$ and since $J = 4 \cdot 18 \times 10^7$

$$\delta W = 0 \cdot 000436 q \delta\,(ND/l). \tag{4}$$

Thus in the fifth stage of twisting of the mild steel specimens of Table I, $q = 43{,}700$ lbs. per sq. in., $\delta(ND/l) = 0 \cdot 1452 - 0 \cdot 1126 = 0 \cdot 0326$ so that $\delta W = 0 \cdot 621$ calories per gram. The values of δW calculated in this way are given in column 3.

The observed rise in temperature δT is given in column 4 and the quantity of heat δH necessary to raise the metal through $\delta T°$ C., namely, $\sigma\delta T$, where σ is the specific heat, is given in column 5. For steel $\sigma = 0 \cdot 106$ so that the figures in column 5, Table I, are derived from those of column 4 by multiplying by $0 \cdot 106$.

The difference between the work done on the specimen and heat given out is $\delta W - \delta H$ so that the proportion of the work done on the metal which remains latent in it is $(\delta W - \delta H)/\delta W$. This is expressed as a percentage in column 6. It will be seen that during successive stages the proportion of heat remaining latent is very nearly constant and equal to 11% of the work done on the steel. There is, however, some evidence of a slight falling off during the last stage of twisting, when the proportion of latent energy falls to 9%. In this case the total work done W is found by adding all the figures in column 3. Thus $W = 6 \cdot 75$ gram calories. This is equivalent to a rise in temperature of 64° C. H, the total heat emitted, is found by adding the figures in column 5. H is $5 \cdot 99$ calories, so that the total latent energy in the specimen at the end of the experiment is $W - H = 0 \cdot 76$ calories per gram, which is equivalent to a rise in temperature of $7 \cdot 1°$ C.

The corresponding results for decarburized mild steel (nearly pure iron) are given in Table II. It will be seen, referring to the last column of the table, that there is a very definite falling off in the proportion of energy which remains latent. From 12% in the early stages it falls to about 7·5% in the last stage of the test. The energy latent in the metal at the end of the experiment was 0·67 calories per gram which is equivalent to a rise in temperature of 6·3° C.

Table I.—Mild Steel Annealed *in vacuo*. Test using Thermo-junction.

1. ND/L.	2. q. lb. per sq. in.	3. δW. cal./gm.	4. δT. ° C.	5. δH. cal./gm.	6. $\frac{\delta W - \delta H}{\delta W} \times 100$.
0·01655	14,730	0·106	0·975	0·1033	2·55
0·0455	26,200	0·331	2·76	0·292	11·76
0·0824	35,400	0·569	4·7	0·498	12·6
0·1126	39,250	0·515	4·3	0·456	11·4
0·1452	43,700	0·621	5·4	0·545	12·2
0·1768	44,250	0·606	4·98	0·528	12·85
0·2068	45,750	0·600	5·05	0·535	10·85
0·2447	47,400	0·781	6·60	0·700	10·37
0·2757	48,800	0·658	5·45	0·577	12·3
0·3052	49,100	0·632	5·25	0·556	12·0
0·3355	49,700	0·656	5·50	0·583	11·1
0·3660	50,800	0·676	5·52	0·615	9·02

W = 6·75 calories per gram. H = 5·99 calories per gram.
W − H = 0·76 calories per gram.

Table II.—Decarburized Mild Steel. Test using Thermo-junction.

1. ND/L.	2. q. lbs. per sq. in.	3. δW. cal./gm.	4. δT. ° C.	5. δH. cal./gm.	6. $\frac{\delta W - \delta H}{\delta W} \times 100$.
0·0429	16,650	0·311	2·9	0·3075	—
0·0808	26,050	0·430	3·61	0·383	13·5
0·1175	31,400	0·502	4·16	0·441	11·7
0·1537	33,850	0·534	4·42	0·469	12·1
0·1896	35,250	0·552	4·65	0·493	10·7
0·2255	36,800	0·576	5·04	0·534	7·3
0·2599	39,050	0·586	4·92	0·522	10·9
0·2940	39,700	0·590	4·90	0·520	11·9
0·3296	40,400	0·627	5·24	0·556	11·7
0·3655	41,500	0·650	5·46	0·579	12·45
0·4014	42,200	0·660	5·60	0·594	10·0
0·4369	42,200	0·654	5·70	0·605	7·5
0·4724	43,050	0·667	5·80	0·615	7·8
0·5084	43,400	0·681	5·98	0·634	6·9

W = 7·92 calories per gram. H = 7·25 calories per gram.
W − H = 0·67 calories per gram.

Heat Measurement with a Calorimeter.

The method just described has two definite defects. Firstly, it assumes that the specific heat of the specimen remains constant during the test and, secondly, there is always a certain element of uncertainty in measurements made with a thermocouple applied to the outside of a specimen. For these reasons it was decided to repeat the experiments using a calorimeter to measure δH directly. For this purpose it was necessary to reduce the loss of heat from the ends of the specimen as much as possible. This was accomplished (*a*) by making the large square section ends long so that the heat which was generated in the small diameter central part of the specimen would remain in it till the wave of temperature had penetrated down the square ends as far as the grips; (*b*) by inserting a heat-insulating material between the grips and the specimen; (*c*) by carrying out each stage of the twisting as rapidly as possible; (*d*) by re-designing the grips so that the specimen could be removed and dropped in the calorimeter within 1 second of the end of the twisting test.

By varying the time during which the specimen remained on the machine after twisting and before dropping it into the calorimeter, we were able to estimate the loss of heat and we found that when the experiment was carried out as rapidly as possible, the loss was considerably less than 1% of the heat generated. This is so small that we did not attempt to use such corrections. The rise in temperature of the water in the calorimeter was measured by means of a Beckmann thermometer.

Results.—Measurements using a calorimeter were carried out with bars made of pure copper, mild steel, and decarburized mild steel (*i.e.*, nearly pure iron). The results are given in Tables III, IV, and V. In these tables the columns 1, 2, and 3 give ND/l, q, and δW. Column 4 gives δH which is now measured directly (in Tables I and II it was found by multiplying the observed temperature rise δT by a value for the specific heat of the metal taken from physical tables). Column 5 gives the percentage of the energy used during each stage of twisting which remains latent in the specimen. Column 6 gives the total work done on the specimen expressed in calories per gram of metal. Column 7 gives the total latent energy due to twisting, expressed in calories per gram, which remains in the metal at the end of each stage of the test.

In the previous work of Farren and Taylor, it was pointed out that the proportion $(\delta W - \delta H)/\delta W$ of the work done during cold working which remains latent in the metal is nearly constant over the range covered by these experiments, though the change in the strength of the material in the same range is very great.

Table III.—Annealed Pure Copper. Test using Calorimeter.

1. ND/L.	2. q. lb. per sq. in.	3. δW. cal./gm.	4. δH. cal./gm.	5. $\frac{\delta W - \delta H}{\delta W} \times 100$.	6. W. cal./gm.	7. W − H. cal./gm.
0·0594	8,680	0·1974	0·1785	9·58	0·1974	0·0189
0·1230	14,320	0·3488	0·3235	7·25	0·5462	0·0439
0·1870	17,000	0·4168	0·3885	7·64	0·9630	0·0722
0·2670	19,550	0·5995	0·554	7·60	1·5625	0·1177
0·3598	21,690	0·7710	0·718	8·17	2·3335	0·1707
0·4195	23,250	0·5310	0·4805	9·52	2·8645	0·2212
0·4875	24,290	0·6328	0·5815	8·10	3·4973	0·2725
0·577	25,330	0·8690	0·791	8·58	4·3663	0·3505
0·642	26,380	0·6570	0·590	10·20	5·0233	0·4175
0·7035	26,900	0·6340	0·576	9·15	5·6573	0·4755
0·756	27,320	0·5495	0·490	8·58	6·2068	0·5350
0·817	27,750	0·648	0·587	9·42	6·8548	0·5960
0·876	28,200	0·637	0·584	8·32	7·4918	0·6490
0·972	28,800	1·058	0·945	10·67	8·5498	0·7620
1·067	29,160	1·062	0·965	9·08	9·6118	0·8590
1·153	30,200	0·995	0·911	8·45	10·6068	0·9430
1·253	30,350	1·162	1·066	8·25	11·7688	1·0390
1·278	30,550	0·2924	0·268	8·34	12·0612	1·0634
1·340	30,700	0·730	0·696	4·66	12·7912	1·0974
1·395	31,220	0·6578	0·634	4·19	13·4490	1·1210
1·426	31,400	0·3729	0·355	4·80	13·8219	1·1391
1·448	31,580	0·266	0·256	3·72	14·0879	1·1490

Table IV.—Annealed Mild Steel. Test using Calorimeter.

1. ND/L.	2. q. lb. per sq. in.	3. δW. cal./gm.	4. δH. cal./gm.	5. $\frac{\delta W - \delta H}{\delta W} \times 100$.	6. W. cal./gm.	7. W − H. cal./gm.
0·067	30,000	0·870	0·769	11·62	0·870	0·161
0·1353	39,200	1·158	1·070	8·23	2·028	0·189
0·201	43,800	1·240	1·117	9·92	3·268	0·312
0·267	46,650	1·336	1·209	9·51	4·604	0·439
0·3335	49,000	1·420	1·293	8·95	6·024	0·566
0·4322	51,700	2·20	2·020	8·19	8·224	0·746
0·499	52,580	1·529	1·398	8·57	9·753	0·877
0·566	53,800	1·558	1·427	8·42	11·311	1·008
0·635	54,680	1·605	1·507	6·10	12·916	1·106
0·702	55,500	1·620	1·519	6·72	14·536	1·267

It has been suggested that the latent energy due to cold work which can be retained in a metal may not increase indefinitely as the amount of cold work increases, but that a stage may ultimately be reached in which cold work can still be done on the metal, but no further latent heat can be absorbed. To show how far the present results support this view the values of $100(\delta W - \delta H)/\delta W$ given in Tables I to V are plotted against ND/l in figs. 4, 5, and 6. It will be seen from those diagrams that for annealed mild steel and decarburized mild steel the proportion of applied cold work which remains latent decreases as ND/l increases. For decarburized mild steel, fig. 5, the absorption has decreased to $2 \cdot 9\%$ of the applied work when $ND/l = 0 \cdot 59$ and the curve seems to suggest that at the maximum observed value of ND/l, namely, $0 \cdot 59$, the metal has reached a state in which it is nearly saturated with latent energy. From Table II it will be seen that the total latent energy is then $0 \cdot 66$ calories per gram.

Table V.—Decarburized Mild Steel. Test using Calorimeter.

1. ND/L.	2. q. lb. per sq. in.	3. δW. cal./gm.	4. δH. cal./gm.	5. $\frac{\delta W - \delta H}{\delta W} \times 100.$	6. W. cal./gm.	7. W − H. cal./gm.
0·0625	21,300	0·5790	0·5322	8·08	0·5790	0·0468
0·1469	32,600	1·1980	1·0890	8·10	1·7770	0·1558
0·2408	36,400	1·487	1·3480	9·35	3·264	0·295
0·3220	39,550	1·398	1·280	8·44	4·662	0·413
0·4092	41,650	1·580	1·4690	7·03	6·242	0·524
0·506	42,500	1·790	1·700	5·03	8·032	0·614
0·5900	43,400	1·586	1·540	2·90	9·618	0·660

For annealed mild steel, fig. 4, the diagram indicates that at the maximum observed value of ND/l, namely, $0 \cdot 70$, the material is not yet saturated with latent energy, though the proportion of the applied cold work which is absorbed and remains latent is only about half what it was in the initial annealed state. When $ND/l = 0 \cdot 7$, $W - H = 1 \cdot 27$ calories per gram. For copper, fig. 6, shows that the proportion of the applied cold work which remains latent is nearly constant up to $ND/l = 1 \cdot 0$, and that after this stage of the twisting has been reached the absorption of latent energy rapidly decreases till the metal becomes saturated at about $ND/l = 1 \cdot 6$. The maximum measured value of $W - H$ is $1 \cdot 15$ calories per gram at $ND/l = 1 \cdot 45$, and fig. 6 suggests that this may be nearly the maximum possible latent energy which the metal can retain at the temperature (15° C.) at which the measurements were made.

Comparison with Measurements of Farren and Taylor.

It is not possible to compare these results directly with those of Farren and Taylor because the type of distortion was different in the two experiments, but if it be assumed that the condition of the metal depends only on the amount of applied cold work retained latent in it irrespective of the distribution of the

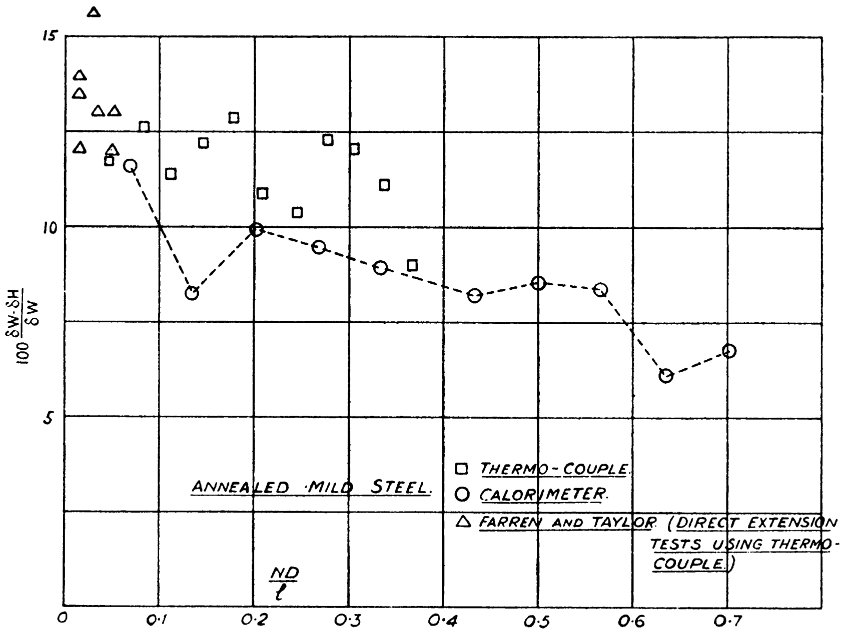


Fig. 4.

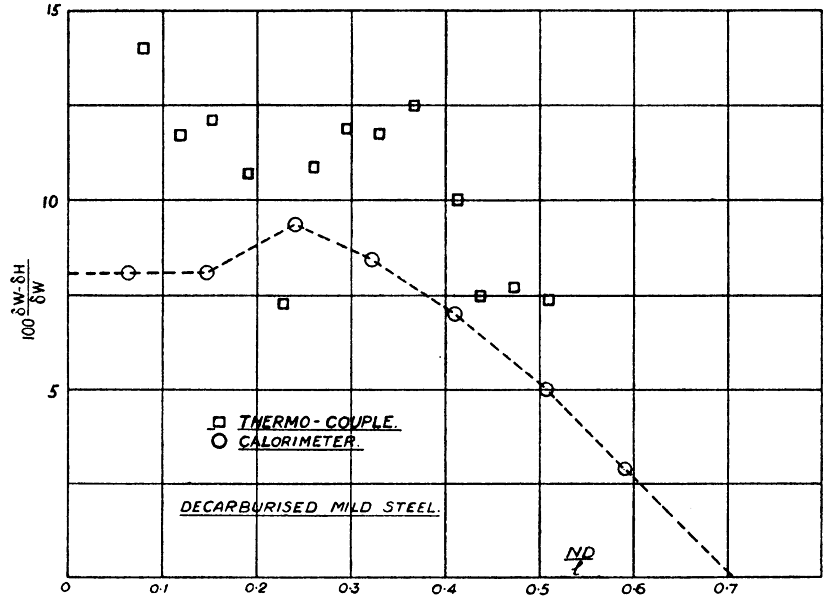


Fig. 5.

Figs. 4, 5.—Percentage of work done which remains latent in specimen subjected to plastic twisting.

applied stresses, then a virtual value of ND/l can be calculated* for which a twisted rod would have received the same amount of cold work as that given by the direct load in Farren and Taylor's experiments. The value of $100(\delta W - \delta H)/\delta W$ given by Farren and Taylor have been plotted in figs. 4 and 6 at the appropriate virtual values of ND/l. It will be seen that the agreement with the present results is good, but that a direct load is capable of giving to these metals only a very small fraction of the latent energy which they can contain.

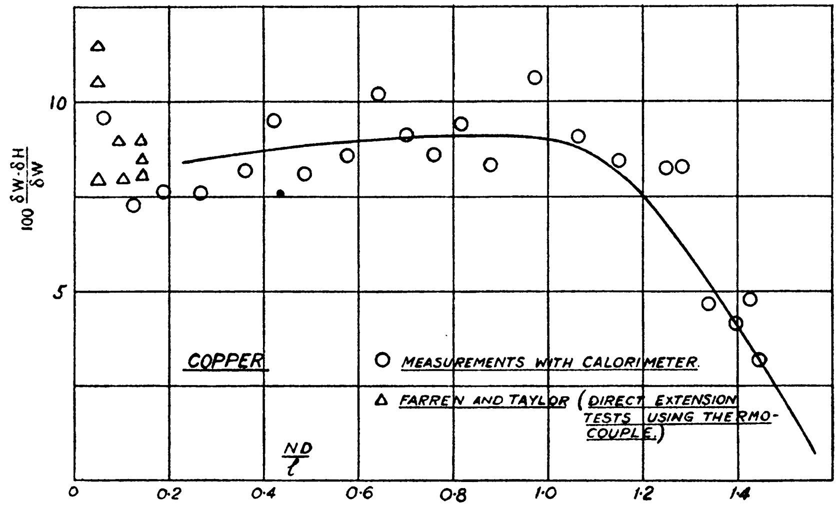


FIG. 6.—Percentage of work done which remains latent in specimen subjected to plastic twisting.

Connection between Strength and Latent Energy.

It appears that both the strength and the latent energy of the material increase with increasing amounts of cold work. For both these, however, there seems to be a limit beyond which there is no further increase with further application of cold work. The question may naturally be asked are these two limits identical? Does the attainment of maximum strength in a metal occur when the absorption of latent energy reaches its greatest possible value?

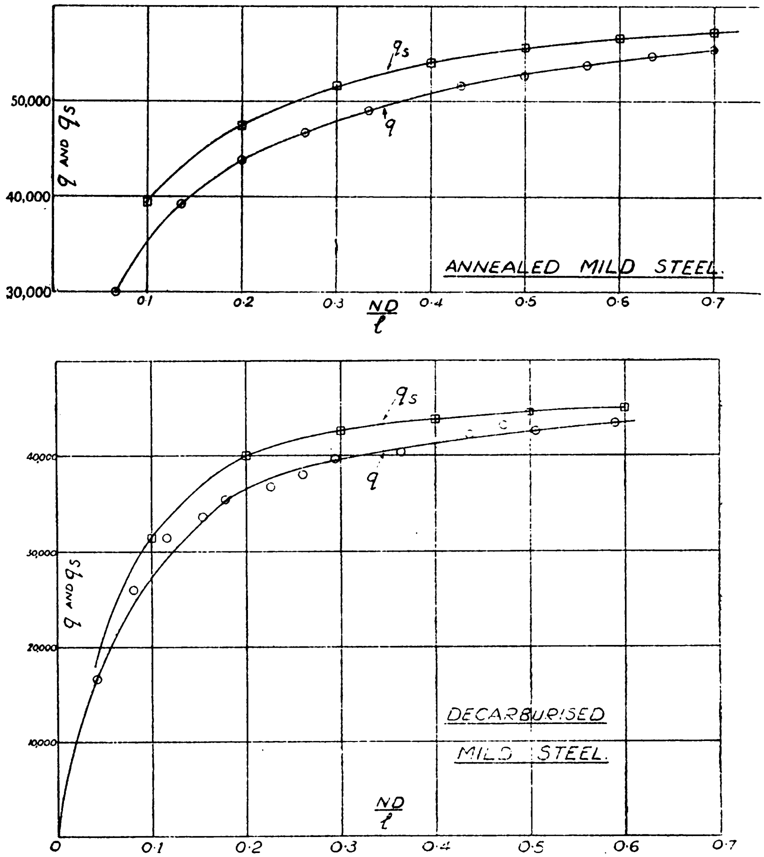
The values of q given in columns 2 of Tables I to V are mean values of the shear stress over the section. The value of the shear stress q_s at the surface of the specimen may be found from the measured values of q by means of the following formula†

$$q_s = q + \tfrac{1}{3}\frac{\mathrm{ND}}{l}\frac{dq}{d(\mathrm{ND}/l)}. \qquad (5)$$

* See p. 315 above.

† This relation is given by Nadai in a different form in his "Plasticity," p. 128.

The relationship between q and ND/l for annealed mild steel and decarburized mild steel is shown in figs. 7 and 8 and the values of q_s found by applying (5) to these curves are also given in the same figure. It will be seen that for decarburized mild steel, fig. 8, q_s increases rapidly up to ND/$l = 0 \cdot 4$, but that at this point the increase practically ceases. Turning to fig. 5 it will be seen that the proportion of applied cold work which becomes latent in the metal



Figs. 7, 8.—Stress-strain curves during plastic twisting.

does not appear to decrease till about ND/$l = 0 \cdot 4$, but that it decreases rapidly when ND/l rises above $0 \cdot 4$. For annealed mild steel, fig. 7, q_s increases up to ND/$l = 0 \cdot 7$ and it will be noticed in fig. 4 that $(\delta W - \delta H)/\delta W$ does not suffer any rapid decrease in this range. There is a gradual decrease, but saturation with latent energy is only reached outside the range of our experiments.

The values of q given in Table III for copper are the figures which must be used in calculating q_s, but they were obtained during the rapid twisting of the specimen. Copper is capable of withstanding a greater stress when the rate of deformation is large than when it is small. This effect seems to be considerably greater in twisted specimens than it is when the distortion is uniform. It is not possible, therefore, to deduce from Table III the point in the test at which the strength of the material ceases to increase with increasing cold work. For this reason independent experiments were made under conditions ensuring uniform distortion to find out how much cold work must be done on copper before it attains its maximum strength.

The Load-extension Curve for Pure Copper.

The two most convenient methods for producing uniform distortion in soft metals are to extend a long bar of uniform section or to compress a short cylinder or disc between parallel plates, the ends being lubricated with grease. The conditions under which the distortion is uniform in the latter method were studied by one of the present writers* in connection with the distortion of single crystals of aluminium. It was then found that if the load was increased only slightly between successive stages of the experiment, uniform distortion was obtained if the specimen was greased before each application of the load. The effect of the friction between the flat faces of the specimen and the parallel steel plates was found to be inappreciable.

It has been pointed out that a uniform bar extended by a direct load necessarily breaks long before the material reaches its maximum strength. A compressed disc, however, can be subjected to far greater amounts of distortion than a bar under direct load or even one subjected to torsion. In carrying out our measurements, therefore, it was necessary first to compare the curve representing T as a function of $\log (l/l_0)$ in an extension experiment with that representing P as a function of $\log (h_0/h)$ in a compression experiment. Here T and P are the stresses (expressed in lbs. per sq. in.), l is the length and l_0 the initial length of the extended bar, h is the thickness and h_0 the initial thickness of the compressed disc. If these curves are identical, it seems that the effect of the friction of the ends of the compressed disc on the steel plates is negligible and the (P, $\log h_0/h$) curve truly represents the relationship between strength and the amount of distortion. The compression experiments can then be continued far beyond the stage at which the material reaches its maximum strength.

The results of such tests are shown in figs. 9 and 10. In the extension experiment the bar was loaded till the extension was 20%. This corresponds

* 'Proc. Roy. Soc.,' A, Vol. 111, p. 531 (1926).

with $\log l/l_0 = 0 \cdot 18$ and is represented by a dotted line in figs. 9 and 10. In fig. 9 the observations in the extension experiments are represented by crosses, while those in the compression experiments are shown as round dots. It will

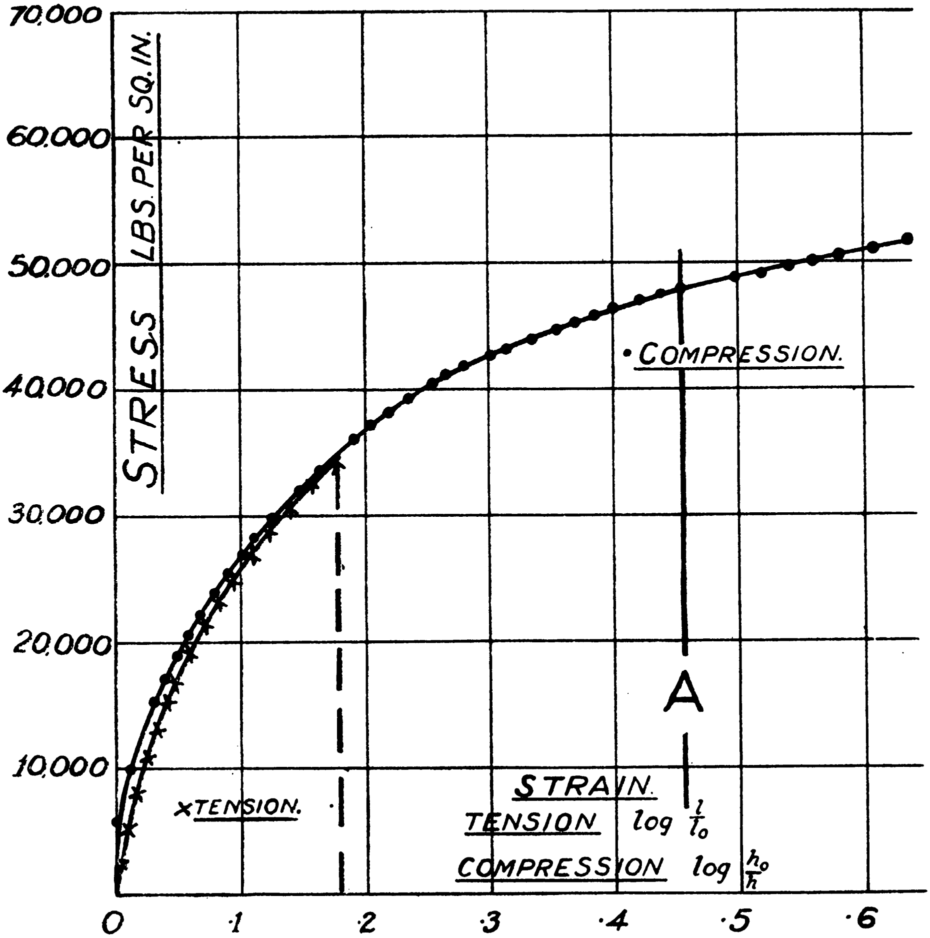


Fig. 9.—Comparison between stress-strain curves for copper in direct extension and direct compression.

be seen that the compression and extension curves nearly coincide so that the compression results may be used with confidence outside the range in which they can be directly compared with those obtained by tensile loading.

The complete stress-strain curve in compression is shown in fig. 10, but for clearness only a few of the points representing the observations are marked.

A short cylinder or disc of annealed copper $0 \cdot 4770$ inches high $\times$ $0 \cdot 4390$ inches diameter was first compressed in 31 stages till its thickness was $0 \cdot 61$ of its original thickness. The results of this test are shown by means of 31 dots in fig. 9 and 11 dots in fig. 10. The line A at $\log h_0/h = 0 \cdot 46$ marks the end of this stage of the experiment.

At this stage the specimen was 0·3007 inches thick $\times$ 0·55 inches diameter and since the effect of the friction on the ends increases as the ratio of the diameter to the thickness increases it was thought better to reduce the radius, accordingly the specimen was cut down to 0·2795 inches, the thickness remaining unaltered.

The reduced specimen was then further compressed till its thickness was 0·1178 inches. The corresponding value of $\log h_0/h$ was then 1·40. The end of this stage of the test is marked in fig. 10 by the line B. The specimen was then cut down to 0·1973 inches diameter and compressed by frequent small increments in load till its thickness was 0·0260. At the end of this third stage $\log h_0/h = 2 \cdot 91$. This is indicated by the line C in fig. 10.

The remainder of the points marked in fig. 10, namely, those corresponding with strains from $\log (h_0/h) = 2 \cdot 91$ to $3 \cdot 98$ were obtained with another specimen cut from the same sample of copper.

Fig. 10 shows that the compressive stress rises with increase in strain till at

$$\log (h_0/h) = 1 \cdot 5, \text{ i.e., } h/h_0 = 0 \cdot 22,$$

the maximum value of 60,000 lbs. per sq. in. is attained. No further increase was observed, although the specimen was compressed till at $\log (h_0/h) = 3 \cdot 98$ its thickness was only 1/53rd of its original thickness. It is worth noticing that 60,000 lbs. per sq. in. is about equal to the tensile strength of hard drawn copper wires.

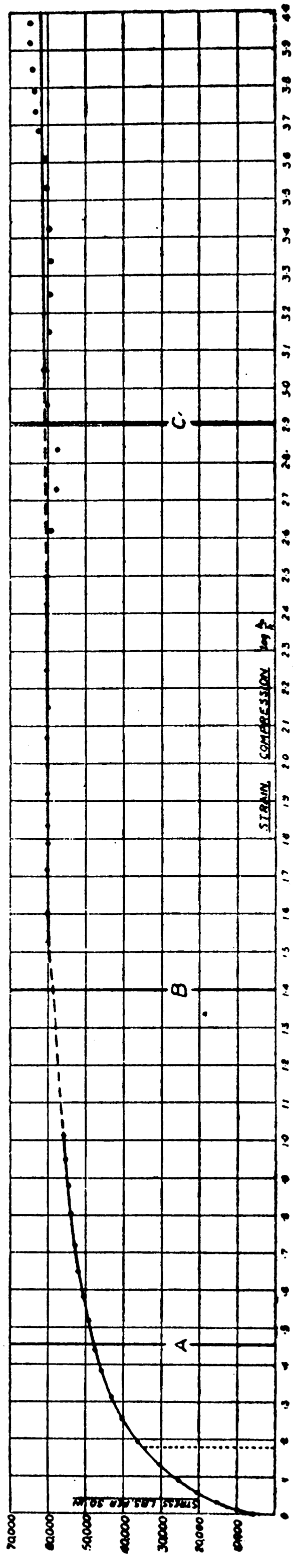


FIG. 10.—Stress-strain curve for copper in compression. At maximum strain ($\log h_0/h = 3 \cdot 98$) height of specimen was 1/53rd of the original height.

Cold Work necessary to raise Strength to Maximum.

The work done on unit volume of material during compression from thickness h_0 to thickness h is

$$\int_h^{h_0} \mathrm{P}\, d\,(\log h_0/h).$$

This can be found by means of a planimeter from the curve of fig. 10. During the course of the compression to $\log h_0/h = 1 \cdot 5$ the work done is found in this way to be $5 \cdot 78 \times 10^9$ ergs. per cubic centimetre. Since the density of copper is $8 \cdot 93$ this is equivalent to $15 \cdot 5$ calories per gram of copper.

Comparison between Cold Work necessary to Saturate Metal with Latent Energy and that necessary to give Maximum Strength.

It has already been pointed out in connection with fig. 6 that the absorption of latent energy has nearly ceased at $\mathrm{N D}/l = 1 \cdot 45$, and from Table III it will be seen that at this stage $\mathrm{W} = 14 \cdot 1$ calories per gram. The cold work necessary to saturate copper with latent energy at room temperature (about 15° C.) is therefore roughly the same as that necessary to raise the metal to its maximum strength, namely, $15 \cdot 5$ calories per gram.

Another way in which this question might be treated is to make use of v. Mises' hypothesis concerning the criterion for plastic deformation and to assume also that the state of the material depends only on the amount of cold work done on it irrespective of whether it has been distorted by pure extension or by pure shear. Using these hypotheses s, the amount of shear equivalent to compression from thickness h_0 to thickness h is $\sqrt{3} \log (h_0/h)$, so that the maximum strength of a twisted tube would be attained when $s = \sqrt{3}(1 \cdot 5) = 2 \cdot 6$. Since $s = \pi \mathrm{N D}/l$ the maximum strength of the copper might be expected to be attained in the outer layers of a twisted copper rod when $\mathrm{N D}/l = 2 \cdot 6/\pi = 0 \cdot 83$. Using Mohr's hypothesis instead of v. Mises', the result would have been $\mathrm{N D}/l = 2\,(1 \cdot 5)/\pi = 0 \cdot 95$. It has been pointed out in discussing fig. 6 that the latent energy absorbed in twisting a copper bar bears an almost constant ratio to the applied cold work up to $\mathrm{N D}/l = 1 \cdot 0$, and that at that point it suddenly begins to decrease. It seems significant that this decrease occurs at a stage of twisting which so nearly coincides with that at which the maximum strength is reached in the outer layers of the specimen.

In conclusion, we wish to express our thanks to Professor Inglis for allowing us to carry out this work in the Engineering Laboratory at Cambridge, and to Mr. Parkes and Mr. Jacobsohn for assistance in carrying out the work.