

ECI114: Probabilistic Systems Analysis for Engineers

HW # 3, Due: 05May2023, by midnight on Canvas.

Problem:

1. Assume that X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the value for x that solves each of the following:
 - a $P(X > x) = 0.5$
 - b $P(X > x) = 0.95$
 - c $P(x < X < 10) = 0$
 - d $P(-x < X - 10 < x) = 0.95$
 - e $P(-x < X - 10 < x) = 0.99$
2. The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes.
 - a What is the probability that a battery lasts more than four hours?
 - b What are the quartiles (the 25% and 75% values) of battery life?
 - c What value of life in minutes is exceeded with 95% probability?
3. The length of stay at a specific emergency department in Phoenix, Arizona, in 2009 had a mean of 4.6 hours with a standard deviation of 2.9. Assume that the length of stay is normally distributed.
 - a What is the probability of a length of stay greater than 10 hours?
 - b What length of stay is exceeded by 25% of the visits?
 - c From the normally distributed model, what is the probability of a length of stay less than 0 hours? Comment on the normally distributed assumption in this example.
4. Suppose that the lifetime of a component (in hours) is modeled with a Weibull distribution with $\beta = 2$ and $\delta = 4000$. Determine the following in parts (a) and (b):
 - a $P(X > 5000)$
 - b $P(X > 8000 | X > 3000)$
 - c Comment on the probabilities in the previous parts compared to the results for an exponential distribution.
5. An article in Applied Mathematics and Computation ["Confidence Intervals for Steady State Availability of a System with Exponential Operating Time and Lognormal Repair Time" (2003, Vol. 137(2), pp. 499-509)] considered the long-run availability of a system with an assumed lognormal distribution for repair time. In a given example, repair time follows a lognormal distribution with $\theta = \omega = 1$. Determine the following:
 - a Probability that repair time is greater than five time units
 - b Conditional probability that a repair time is less than eight time units given that it is more than five time units
 - c Mean and variance of repair time