ECI114: Probabilistic Systems Analysis for Engineers

HW # 5, Due: 27May2023, by midnight on Canvas.

Problem:

1. Suppose that the random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1 & ; \quad 0 \leq x \leq 1 \\ 0 & ; \quad \text{otherwise} \end{cases} \tag{0.1}$$

   Suppose that a random sample of $n = 12$ observations is selected from this distribution. What is the approximate probability distribution of $(\bar{X} - 6)$? Find the mean and variance of this quantity.

2. A random sample of size $n_1 = 16$ is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size $n_2 = 9$ is taken from another normal population with mean 70 and standard deviation 12. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means. Find:

   (a) The probability that $\bar{X}_1 - \bar{X}_2$, exceeds 4

   (b) The probability that $3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5$

3. Data on pull-off force (pounds) for connectors used in an automobile engine application are as follows:
   79.3, 75.1, 78.2, 74.1, 73.9, 75.0, 77.6, 77.3, 73.8, 74.6, 75.5, 74.0, 74.7, 75.9, 72.9, 73.8, 74.2, 78.1, 75.4, 76.3, 75.3, 76.2, 74.9, 78.0, 75.1, 76.8.

   (a) Calculate a point estimate of the mean pull-off force of all connectors in the population. State which estimator you used and why.

   (b) Calculate a point estimate of the pull-off force value that separates the weakest 50% of the connectors in the population from the strongest 50%.

   (c) Calculate point estimates of the population variance and the population standard deviation.

   (d) Calculate the standard error of the point estimate found in part (1). Interpret the standard error.

   (e) Calculate a point estimate of the proportion of all connectors in the population whose pull-off force is less than 73 pounds.