4.5.4 a) P(X > x) = $P\left(Z > \frac{x-10}{2}\right)$ = 0.5. Therefore, $\frac{x-10}{2}$ = 0 and x = 10.

b) P(X > x) = $P\left(Z > \frac{x-10}{2}\right) = 1 - P\left(Z < \frac{x-10}{2}\right) = 0.95$

Therefore, $P\left(Z < \frac{x-10}{2}\right) = 0.05$ and $\frac{x-10}{2} = -1.64$. Consequently, x = 6.72.

c) P(x < X < 10) = $P\left(\frac{x-10}{2} < Z < 0\right) = P(Z < 0) - P\left(Z < \frac{x-10}{2}\right)$

$= 0.5 - P\left(Z < \frac{x-10}{2}\right) = 0.2.$

Therefore, $P\left(Z < \frac{x-10}{2}\right) = 0.3$ and $\frac{x-10}{2} = -0.52$. Consequently, x = 8.96.

d) P(10 – x < X < 10 + x) = P(–x/2 < Z < x/2)= 0.95. Therefore, x/2 = 1.96 and x = 3.92

e) P(10 – x < X < 10 + x) = P(–x/2 < Z < x/2) = 0.99. Therefore, x/2 = 2.58 and x = 5.16

4.5.6 a) Let X denote the time.
X distributed N(260, 50^2)

$$P(X > 240) = 1 - P(X \le 240) = 1 - \Phi\left(\frac{240-260}{50}\right) = 1 - \Phi(-0.4) = 1 - 0.3446 = 0.6554$$

b) $\Phi^{-1}(0.25) \times 50 + 260 = 226.2755$

$\Phi^{-1}(0.75) \times 50 + 260 = 293.7245$

c) $\Phi^{-1}(0.05) \times 50 + 260 = 177.7550$

4.5.16 a) P(X > 10) = $P\left(Z > \frac{10-4.6}{2.9}\right)$ = P (Z > 1.8621) = 0.0313

b) P(X > x) = 0.25, then $\Phi^{-1}(0.75) \times 2.9 + 4.6 = 0.6745 \times 2.9 + 4.6 = 6.5560$

c) P(X < 0) = $P\left(Z < \frac{0-4.6}{2.9}\right)$ = P (Z < -1.5862) = 0.0563

The normal distribution is defined for all real numbers. In cases where the distribution is truncated (because wait times cannot be negative), the normal distribution may not be a good fit to the data.

4.9.7 a) $P(X > 5000) = 1 - F_X(5000) = e^{-(5000/4000)^2} \approx 0.2096$

b) $P(X > 8000 \mid X > 3000) = \frac{P(X > 8000, X > 3000)}{P(X > 3000)} = \frac{P(X > 8000)}{P(X > 3000)}$

$\frac{1 - F_X(8000)}{1 - F_X(3000)} = \frac{e^{-(8000/4000)^2}}{e^{-(3000/4000)^2}} \approx 0.03214$

c) If it is an exponential distribution, then β = 1 and

$P(X > 8000 \mid X > 3000) = \frac{1 - F_X(8000)}{1 - F_X(3000)} = \frac{e^{-(8000/4000)}}{e^{-(3000/4000)}} = e^{-5000/4000} = P(X > 5000)$

4.10.9 a) $P(X > 5) = 1 - P(W < \ln(5)) = 1 - \Phi\left(\frac{\ln(5)-1}{1}\right) = 0.271$

b) $P(X < 8 \mid X > 5) = \frac{P(5 < X < 8)}{P(X > 5)} = \frac{P(X < 8) - P(X < 5)}{P(X > 5)} =$

$\frac{[\Phi(\ln(8)-1)] - [\Phi(\ln(5)-1)]}{1 - \Phi(\ln(5)-1)} = \frac{0.86 - 0.73}{1 - 0.73} = 0.483$

c) $\mu = \exp(1 + 0.5) = 4.48$

$\sigma^2 = \exp(3)(\exp(1) - 1) = 34.517$