

7.2.4 Let $Y = \bar{X} - 6$

$$\mu_X = \frac{a+b}{2} = \frac{(0+1)}{2} = \frac{1}{2}$$

$$\mu_{\bar{X}} = \mu_X$$

$$\sigma_X^2 = \frac{(b-a)^2}{12} = \frac{1}{12}$$

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{\frac{1}{12}}{12} = \frac{1}{144}$$

$$\sigma_{\bar{X}} = \frac{1}{12}$$

$$\mu_Y = \frac{1}{2} - 6 = -5\frac{1}{2}$$

$$\sigma_Y^2 = \frac{1}{144}$$

$Y = \bar{X} - 6 \sim N(-5\frac{1}{2}, \frac{1}{144})$, approximately, using the central limit theorem.

7.2.7

$$\begin{array}{ll} n_1 = 16 & n_2 = 9 \\ \mu_1 = 75 & \mu_2 = 70 \\ \sigma_1 = 8 & \sigma_2 = 12 \end{array} \quad \bar{X}_1 - \bar{X}_2 \sim N(\mu_{\bar{X}_1} - \mu_{\bar{X}_2}, \sigma_{\bar{X}_1}^2 + \sigma_{\bar{X}_2}^2) \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}) \sim N(75 - 70, \frac{8^2}{16} + \frac{12^2}{9}) \sim N(5, 20)$$

a) $P(\bar{X}_1 - \bar{X}_2 > 4)$

$$P(Z > \frac{4-5}{\sqrt{20}}) = P(Z > -0.2236) = 1 - P(Z \leq -0.2236)$$

$$= 1 - 0.4115 = 0.5885$$

b) $P(3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5)$

$$P(\frac{3.5-5}{\sqrt{20}} \leq Z \leq \frac{5.5-5}{\sqrt{20}}) = P(Z \leq 0.1118) - P(Z \leq -0.3354)$$

$$= 0.5445 - 0.3687 = 0.1759$$

7.3.6 a) The average of the 26 observations provided can be used as an estimator of the mean pull force because we know it is unbiased. This value is 75.615 pounds.

b) The median of the sample can be used as an estimate of the point that divides the population into a “weak” and “strong” half. This estimate is 75.2 pounds.

c) Our estimate of the population variance is the sample variance or 2.738 square pounds. Similarly, our estimate of the population standard deviation is the sample standard deviation or 1.655 pounds.

d) The estimated standard error of the mean pull force is $1.655/26^{1/2} = 0.325$. This value is the standard deviation, not of the pull force, but of the mean pull force of the sample.

e) Only one connector in the sample has a pull force measurement under 73 pounds. Our point estimate for the proportion requested is then $1/26 = 0.0385$