Nonlinear Finite Element Methods for Elastic-Plastic Problems

ECI212B

Teaching Plan

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Chapter 1

Nonlinear FEM

1.1 Objectives

This course will provide students with state of the art finite element methods, numerical analysis tools and models for solving elastic–plastic problems in mechanics of solids and structures, on sequential and parallel computers. Presented will be computational formulation, numerical techniques and models for static, nonlinear, elastic-plastic finite element methods that are used in professional practice and research.

During this course students will:

- Learn about linear and nonlinear finite element modeling and simulation
- Select and calibrate nonlinear, elastic-plastic models for solids and structures: soil, rock, concrete, steel and interfaces
- Perform linear elastic and nonlinear, elastic-plastic analysis of solids and structures made of soil, rock, concrete, steel and their interfaces on sequential and parallel computers
- Become proficient in performing nonlinear analysis using different levels of sophistication, from simplified models to high fidelity elastic-plastic models.

Who Should Attend?

Students and practicing engineers who want to learn about and expand their knowledge of modeling and simulation for nonlinear/inelastic material behavior for engineering solids and structures.

1.2 Additional Information

Lecture Notes: http://sokocalo.engr.ucdavis.edu/~jeremic/LectureNotes/.
Recorded Lectures: http://sokocalo.engr.ucdavis.edu/~jeremic/Online-Education/.
Computers: Most of the problems in this course will require numerical simulations. Students will use nonlinear FEM programs of their choice. Alternatively, a finite element modeling system called Real-ESSI Simulator is available as well. Please refer to http://real-essi.us to find out how to use Real-ESSI on local computers or on AWS computers.
Problems: Assigned weekly, students are expected to attempt to develop solutions. You are encouraged to discuss the approach to problem solutions with other students in the course as well as with the instructor.
Examples: Model development, finite element models, finite element mesh, boundary conditions, material models, loads, model verification process, linear and nonlinear elastic FEM with solids and structural elements, There are a large number of examples available at the Real-ESSI Simulator web site:
http://real-essi.us/, and in these documents:

- Real-ESSI Simulator Examples Collection
- Real-ESSI Simulator Short Course Examples Collection

Term Project: Term project will involve work related to developing or using numerical models for numerically simulating elastic–plastic problem of your choice, related to your interests. Term projects will be presented at the end of quarter.
Grading: TBD
Examination: TBD
Literature:

- The Finite Element Method, *Olgierd Cecil Zienkiewicz and Robert L. Taylor*, McGraw-Hill Book Company, Volumes 1 and 2, ISBN 0-07-084175-6
- Non - Linear Finite Element Analysis of Solids and Structures Volume 1: Essentials, *Crisfield, M. A.*, John Wiley and Sons, Inc. New York, 1991 , ISBN 0 471 92956 5 v.1
- Finite Element Procedures in Engineering Analysis, *Klaus-Juergen Bathe*, Prentice Hall, ISBN 0-13-301458-4
- Constitutive Laws for Engineering Materials With Emphasis on Geologic Materials *Chandakant S. Desai and Hema J. Siriwardane*, Prentice–Hall, Inc. Englewood Cliffs, NJ 07632, ISBN 0-13-167940-6
- Plasticity for Structural Engineers *W. F. Chen and D. J. Han* , Springer Verlag, 1988 ISBN 0-387-96711-7
- Boris Jeremić, Zhaohui Yang, Zhao Cheng, Guanzhou Jie, Nima Tafazzoli, Matthias Preisig, Panagiota Tasiopoulou, Federico Pisano, Jose Abell, Kohei Watanabe, Yuan Feng, Sumeet Kumar Sinha, Fatemah Behbehani, Han Yang, and Hexiang Wang, Katarzyna Staszewska. Nonlinear Finite Elements: Modeling and Simulation of Earthquakes, Soils, Structures and their Interaction. University of California, Davis, CA, USA, 1989-2024. ISBN: 978-0-692-19875-9

1.3 Teaching Plan, Topics

1.3.1 Section I, Introduction

Introduction: Course objectives, methodology, computer modeling and simulation
Modeling and Simulation System Setup: Introduction to the Real-ESSI Simulator system. Computational Mechanics field of study, kinematics of deformation, strain, stress, linear and nonlinear elasticity, equilibrium relations, finite element method review, nonlinear analysis cycles;
Lectures, recordings and slides:

- Introduction to Modeling and Simulation: PDF slides, MP4 recording
- Introduction to Modeling Simplifications, Epistemic Uncertainty: PDF slides, MP4 recording
- Introduction to Parametric, Aleatory Uncertainty: PDF slides, MP4 recording

Reading: Lecture Notes: 101, 201, 205; Papers/Reports: CM988, CM2714, CM2715, CM3170
Examples: Model development, finite element models, finite element mesh, boundary conditions, material models, loads, model verification process, linear and nonlinear elastic FEM with solids and structural elements. see examples collection at http://real-essi.us/,
Problems:

1. Model a $0.1m \times 0.1m \times 1.0m$ linear elastic cantilever beam, with transversal end forcing, using:
   - Single and ten Bernoulli beam elements
   - Single and ten Timoshenko beam elements
   - 1 $(1 \times 1 \times 1)$; 10 $(1 \times 1 \times 10)$; 24 node solid brick elements
2. Model a $0.1m \times 0.1m \times 1.0m$ simple shear linear elastic test using
   - 1 $(1 \times 1 \times 1)$ and 32 $(4 \times 4 \times 4)$ 8 node solid brick elements
   - 1 $(1 \times 1 \times 1)$ and 32 $(4 \times 4 \times 4)$ 24 node solid brick elements
3. Model a $0.1m \times 0.1m \times 1.0m$ pure shear (no rotations) linear elastic test using
   - 1 $(1 \times 1 \times 1)$ and 32 $(4 \times 4 \times 4)$ 8 node solid brick elements
   - 1 $(1 \times 1 \times 1)$ and 32 $(4 \times 4 \times 4)$ 24 node solid brick elements

1.3.2 Section II, Inelastic Finite Elements

Theory: Expanding the matrix deformation method, linear elastic truss element, Beams (Bernoulli, Timoshenko), solids, plates, walls (plane stress), shells, stiffness, forces, displacements, interpolating functions for displacements. Local and global equilibrium. Internal and external forces,
Lectures, recordings and slides:

- Introduction to the Finite Element Method (FEM): PDF slides, MP4 recording
- Derivation of FEM equations of motions for single phase, dry material: PDF slides, MP4 recording
- Derivation of FEM equations of motions for coupled, two phase, fully and partially saturated material, u-p-U formulation: PDF slides, MP4 recording

Reading: Lecture Notes: 101, 102; Papers/Reports: CM81, CM125, CM1835, CM2714, CM3155, CM3155
Examples: Truss, beam, solid bricks. external forces. internal forces (sectional forces, stresses). Generalized nodal displacements and internal deformation (curvature, axial, shear, volumetric, general strains),
Problems:

1. Develop a set of simple, single element examples using truss, beam and brick finite elements with simple static loads, and extract sectional forces, stress, strain and strain energy from results.

1.3.3 Section III, Micromechanics of Elasto-Plasticity

Theory: Micro-mechanical origins of elasto-plasticity, particles in contact, friction, Hertz, Mindlin-Deresiewicz contact/interface
Lectures, recordings and slides:

- Micromechanical origins of elasto-plasticity:
- Lectures by Prof. Stein Sture (University of Colorado, Boulder) on micromechanical origins of elasto-plasticity are available HERE,

Reading: Lecture Notes: 103; Papers/Reports: CM1000, CM1830, CM1831,
Examples: Particle contact problems.
Problems:

1. Develop a simple, 2D, plane-strain model of two particles in contact and apply normal and then shear forces,
2. For the above model, vary normal and shear forces. Comment on results.

1.3.4 Section IV, Incremental Elastic-Plastic Theory

Theory: Incremental, continuum elasto-plasticity, Material Models, perfectly plastic, hardening and softening. Explicit, forward Euler and Implicit, backward Euler, constitutive integrations,
Lectures, recordings and slides:

- Introduction to the incremental theory of elasto-plasticity: PDF slides, MP4 recording, YouTube video
- Explicit solution to the constitutive elastic-plastic problem: PDF slides, MP4 recording, YouTube video
- Implicit solution to the constitutive elastic-plastic problem: PDF slides, MP4 recording, YouTube video

Reading: Lecture Notes: 104; Papers/Reports: CM3199
Examples: Constitutive integrations, explicit and implicit computations: single element response using select elastic-plastic material models: von Mises, Drucker-Prager, Cam Clay. Perfectly plastic, isotropic hardening, kinematic hardening models and cyclic response. Inelastic, fiber (1D) and 3D structural models for concrete and steel.
Problems:

1. Develop a constitutive only linear elastic example. with monotonic loading and vary elastic modulus and Poisson's ratio. Comment on results

2. Develop a constitutive only elastic-perfectly plastic von-Mises example, for monotonic loading, and vary elastic properties, yield strength. Comment on results.
3. For the above developed example, develop results using explicit and implicit constitutive integrations. Vary step size, integration algorithm, tolerances. Comment on results.
4. For the above developed examples, use two cycles of cyclic loading. Comment on Results.

1.3.5 Section V, Inelastic, Elasto-Plastic Solids Modeling

Theory: Continuation: Incremental elasto-plasticity. Material modeling for practical applications. Advanced topics in constitutive elasto–plasticity, stability and accuracy, errors in constitutive integrations, problematic incremental steps, energy dissipation, sub-incrementation, line search, model calibrations.

Lectures, recordings and slides:

- Choice of elastic-plastic material models for soils and interfaces/contacts/joints: PDF slides, MP4 recording YouTube video
- Calibration of elastic-plastic material models for sand: PDF slides, MP4 recording YouTube video
- Calibration of elastic-plastic material models for clay: PDF slides, MP4 recording YouTube video

Reading: Lecture Notes: 104, 402, 403, 512; Papers/Reports:

Examples: Errors in constitutive modeling, sub-increments. Material model calibration, Constitutive modeling of soil, rock, concrete, steel: von Mises, Drucker-Prager, Cam Clay, SaniSand, rounded Mohr-Coulomb, Pisano. Modeling G/G_{max} and damping response. Nonlinear, elastic-plastic structural models for concrete and steel (1D, 3D).

Problems:

1. Develop a single element, 8 and 24 node brick, linear elastic example. with monotonic loading and vary elastic modulus and Poisson's ratio. Comment on results
2. Develop a single element elastic-perfectly plastic von-Mises example, for monotonic loading, and vary elastic properties, yield strength. Vary number of integration, Gauss points. Comment on results.
3. For the above developed example, develop results using explicit and implicit constitutive integrations. Vary step size, integration algorithm, tolerances. Comment on results.
4. For the above developed examples, use two cycles of cyclic loading. Comment on Results. Use both axial loading and shear loading.

1.3.6 Section VI, Inelastic, Elastic-Plastic Interfaces, Joints, Contacts Modeling

Theory: Interface/Joint/Contact modeling: Hard contact, Soft contact. Axial contact stiffness, shear contact stiffness. Interface gap opening and closing. Saturated contacts, effective stress and buoyant forces on foundations.

Lectures, recordings and slides:

- Choice of elastic-plastic material models for soils and interfaces/contacts/joints: PDF slides, MP4 recording YouTube video

- Calibration of elastic-plastic material models for interfaces/contacts/joints: PDF slides, MP4 recording, YouTube video

Reading: Lecture Notes: 104, 403, 512; Papers/Reports:
Examples: Interface: concrete to soil and rock, steel to soil and rock. Gap opening, closing. Shear interface, slip, no slip.
Problems:

1. Develop a two solid element example that are connected using force based interface elements.
2. Develop a two solid element example that are connected using stress based interface elements.
3. For the above developed example, use normal loading and vary load step size. Comment on results.
4. For the above developed example, use normal and then shear loading and vary load step size. Comment on results.
5. For the above developed example, vary interface properties, use interface/contact/joint properties for soil on concrete, soil on steel, concrete on concrete, &c. Comment on results.

1.3.7 Section VII, Inelastic, Elastic-Plastic Structural Modeling

Theory: Inelastic structural models, beams, plates, walls and shells.
Lectures, recordings and slides:

- Choice of elastic-plastic material models for structural elements, beams and walls/plates/shells: PDF slides, MP4 recording, YouTube video
- Calibration of elastic-plastic material models for concrete, in reinforced beams and walls/plates/shells: PDF slides, MP4 recording YouTube video
- Calibration of elastic-plastic material models for steel, in reinforced beams and walls/beams/shells: PDF slides, MP4 recording YouTube video

Reading: Lecture Notes: 102, 403, 512; Papers/Reports:
Examples: Nonlinear analysis of structures. Steel Frames. Reinforces concrete frames, walls, plates, shells.
Problems:

1. Develop a nonlinear truss model, and load it using monotonic and cyclic loading up to yielding and past yielding. Comment on results.
2. Develop a nonlinear beam model, and load it in bending using monotonic and cyclic loading up to yielding and past yielding. Comment on results.
3. Develop a two solid element example that are connected using stress based interface elements. Comment on results.

1.3.8 Section VIII, Nonlinear Analysis Progress and Parallal Computing

Theory: Analysis Progress. Stages, increments, iterations, elastic–plastic stiffness matrix, pure incremental methods, force residuals, Newton iterative algorithm for finite element level iterations, constraints to the global (force residual) system of equations, equilibrium iterations, convergence, load control, displacement control, arc-length, hyper-spherical constraint, convergence criteria, automatic step size control, line search, stability and accuracy. High Performance Computing (HPC), Parallel analysis.
Lectures, recordings and slides:

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Reading: Lecture Notes: 102, 107, 403; Papers/Reports:
Examples: Nonlinear analysis of structures and solids, elastic plastic solids, structures and contacts. Staged analysis steps, incremental only analysis with no equilibrium enforcement, incremental-iterative analysis, with equilibrium enforcement, convergence criteria (force, displacement), convergence tolerances, step size control. Developed examples will be run on sequential and parallel computers.
Problems:

1. Develop a nonlinear analysis, using all of previous examples, that will feature explicit, no equilibrium check simulation. Comment on results.
2. Develop a nonlinear analysis, using all of previous examples, that will feature implicit, equilibrium check simulation. Comment on results.
3. Run examples on sequential and parallel computers. Comment on observed speed-up.

1.3.9 Section IX, Verification and Validation

Theory: Verification, Validation and Prediction, basic theory, solution verification, manufactured solutions, validation experiments, prediction under uncertainty,
Lectures, recordings and slides:

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Reading: Lecture Notes: 301, 302, 303..., 313, 314...; Papers/Reports:
Examples: Solution verification examples for elements, material models, constitutive integration algorithms, solution advancement algorithms.
Problems:

1. Choose a model of your interest, and develop a list of verification examples for all components of your model.
2. For the above model, develop a list of validation examples.

1.3.10 Section X, Practical Considerations for Nonlinear Analysis

Theory: Elastic–plastic FEM modeling (practical recommendations for development and analysis of nonlinear (elastic-plastic) finite element models, phased development of general FEM (and ESSI in particular) models. Core Functionality for inelastic/nonlinear modeling, Energy dissipation. Sequential and parallel computing.
Lectures, recordings and slides:

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Reading: Lecture Notes: 510, 512; Papers/Reports:
Examples: Illustrations of algorithms and models described above, benefits and detriments of different algorithms and models. of sequential and parallel computing.
Problems:

1. Develop a realistic nonlinear analysis model of your choice, perhaps the one developed in previous section and experiment with all/most above models and algorithms. Use sequential and parallel computing for your analysis. Comment on results.